Solutions to Chapter 16 Exercises

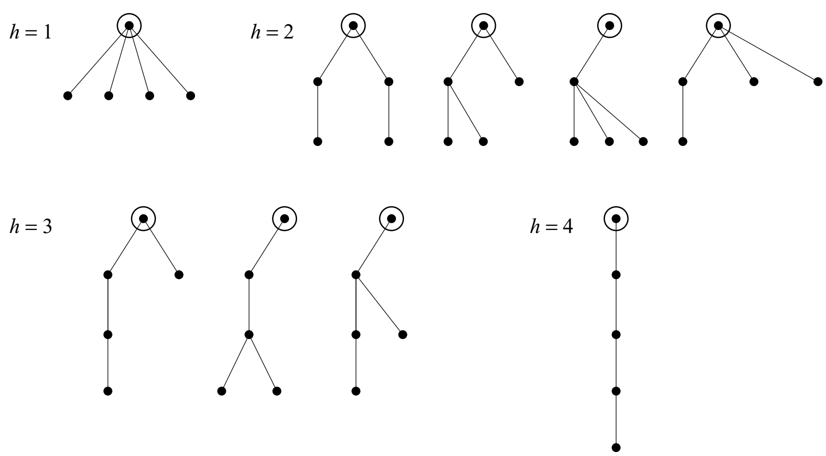
in *Discrete Mathematics* by Norman L. Biggs;
2nd Edition 2002

16.1 Counting the leaves on a rooted tree

16.1.1 In the following table $n_5(h)$ is the number of non-isomorphic *rooted* trees which have five vertices and height h. (Two rooted trees are said to be isomorphic if there is an isomorphism between them (considered as unrooted trees) which also takes the root of the first tree to the root of the second.) Verify the table by sketching the required number of examples in each case.

h :	1	2	3	4
$n_5(h)$:	1	4	3	1

Solution



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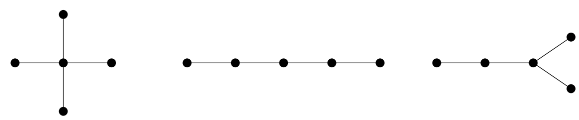
16.1.2 If we consider ordinary (unrooted) trees, what is the number of non-isomorphic types with five vertices? Make a list of these types and hence check that the list obtained in the previous exercise is complete.

Solution



There are three trees with 5 vertices. In order to obtain a rooted tree, the root must be specified. Not all choices give different rooted trees; for example, in the first tree there are only two essentially different choices. In the second tree there are three, and in the third tree there are four, so the total number of rooted trees is $2 + 3 + 4 = 9$, in agreement with Ex. 16.1.1.

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16.2 Trees and sorting algorithms

16.2.1 What is the smallest possible height of the decision tree of an algorithm for sorting four objects using binary comparisons?

Solution

$$\text{Number of outcomes} = 4! = 24.$$

$$\text{Smallest height} = \lceil \log_2 24 \rceil = 5.$$

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16.2.2 Calculate the number of binary comparisons needed (in the worst case) when four objects are sorted

(i) by bubble sort;
(ii) by insertion (sequential method);
(iii) by insertion (bisection method).

Solution

(i) $3 + 2 + 1 = 6$.

(ii) $1 + 2 + 3 = 6$.

(iii) $1 + 2 + 2 = 5$.

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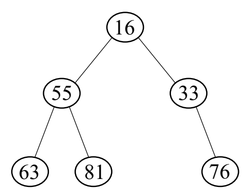
16.2.3 Use the heapsort method to form a heap from the following unsorted lists. In each case illustrate the resulting heap in tree form and write out the corresponding list.

(i) 63, 55, 33, 16, 81, 76.

(ii) 73, 21, 17, 28, 32, 56, 19, 84, 38, 49, 77, 51, 12.

Solution

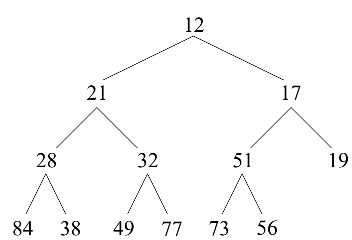
(i)



or, as a list

16, 55, 33, 63, 81, 76.

(ii)



or, as a list

12, 21, 17, 28, 32, 51, 19, 84, 38, 49, 77, 73, 56.

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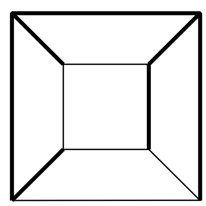
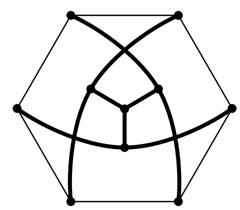
16.3 Spanning trees and the MST problem

16.3.1 Find spanning trees for the cube graph (Fig. 15.12) and Petersen's graph (Fig. 15.14).

Solution





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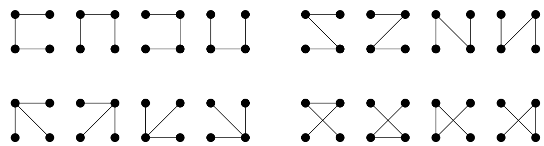
16.3.2 Sketch all the spanning trees for the complete graph K_4 (there are 16 of them).

Solution



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16.4 Depth-first search

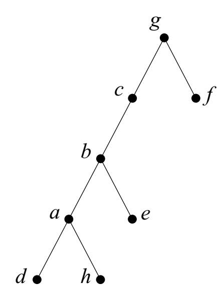
16.4.1 Let G be the graph defined by the adjacency list shown in Table 16.4.2.

Table 16.4.2

a	b	c	d	e	f	g	h
b	a	b	a	b	g	c	a
d	c	d	b			f	g
h	d	g	c			h	
	e						

Sketch the DFS tree in G, starting from g. (Whenever a choice is possible, pick the first vertex in alphabetical order.) Is G connected?

Solution



G **is connected**, since we have reached all vertices.

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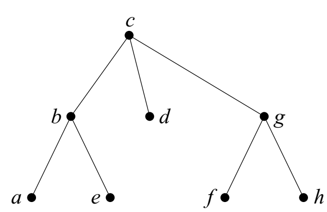
16.5 Breadth-first search

16.5.1 Construct the BFS tree rooted at c for the graph G defined in Ex. 16.4.1.

Solution



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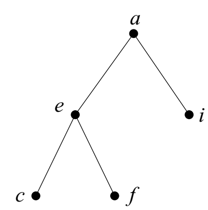
16.5.2 Use BFS to test if the graph defined by Table 16.5.2 is connected.

Table 16.5.2

a	b	c	d	e	f	g	h	i
e	d	e	b	a	c	b	b	a
i	g	f	g	c	e	d	d	c
	h	i	h	f	i			f

Solution



Not all encountered. $\therefore$ **Not connected**.