Part 1
Special relativity

The principle of relativity and the speed of light

Imagine you are in a train carriage waiting at a station. Out of the window you see a second train standing alongside yours. The whistle blows, and at last you are on your way. You glide smoothly past the other train. Its last carriage disappears from view, allowing you to see the station also disappearing into the distance as it is left behind. Except that the station is *not* disappearing; it is just sitting there going nowhere – just as you are sitting in the train going nowhere. It dawns on you that you weren't moving at all; it was the *other* train which moved off.

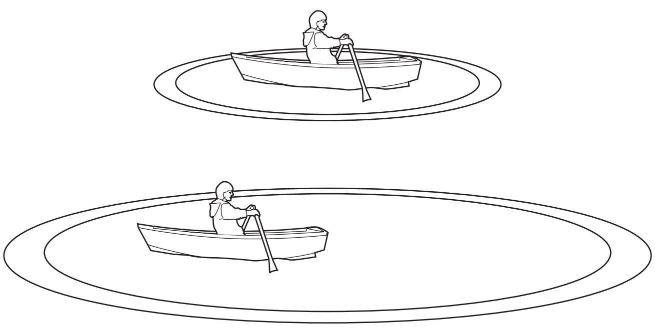
A simple observation. We all get fooled this way at some time or other. The truth is that you cannot tell whether you are really on the move or not – at least, not if we are talking about steady uniform motion in a straight line. Normally, when travelling by car, say, you do know that you are moving. Even if you have your eyes shut, you can feel pushed around as the car goes round corners, goes over bumps, speeds up or slows down suddenly. But in an aircraft cruising steadily, apart from the engine noise and the slight vibrations, you would have no way of telling that you were moving. Life carries on inside the plane exactly as it would if it were stationary on the ground. We say the plane provides an *inertial frame of reference*. By this we mean Newton's law of inertia applies, namely, when viewed from this reference frame, an object will neither change its speed nor direction unless acted upon by an unbalanced force. A glass of water on the tray table in front of you, for example, remains stationary until you move it with your hand.

But what if you look out of the aircraft window and see the earth passing by underneath? Does that not tell you that the plane is moving? Not really. After all, the earth is not stationary; it is moving in orbit about the sun; the sun itself is orbiting the centre of the Milky Way Galaxy; and the Milky Way Galaxy is moving about within a cluster of similar galaxies. All we can say is that these movements are all *relative*. The plane moves relative to the earth; the earth moves relative to the plane. There is no way of deciding who is *really* stationary. Anyone moving uniformly with respect to another at rest is entitled to consider himself to be at rest and the other person moving. This is because the laws of nature – the rules governing all that goes on – are the same for everyone in uniform steady motion, that is to say, everyone in an inertial frame of reference. This is *the principle of relativity.*

And no, it was not Einstein who discovered this principle; it goes back to Galileo. That being so, why has the word 'relativity' become associated with Einstein's name? What Einstein noticed was that amongst the laws of nature were Maxwell's laws of electromagnetism. According to Maxwell, light is a form of electromagnetic radiation. As such, it becomes possible, from a knowledge of the strengths of electric and magnetic forces, to calculate the speed of light, c, in a vacuum. The fact that light has a speed is not immediately obvious. When you go into a darkened room and switch on a lamp, the light appears to be everywhere – ceiling, walls, and floor – instantly. But it is not so. It takes time for the light to travel from the light bulb to its destination. Not much time – it's too fast to see the delay with the naked eye. According to this law of nature, the speed of light in a vacuum, c, works out to be 299,792.458 kilometres per second (or very slightly different in air). And that's what the speed is measured to be.

What if the source of light is moving? One might, for example, expect light to behave like a shell being fired from a passing warship where an observer on the seashore would expect the speed of the ship to be added to the shell's muzzle speed if being fired in the forward direction, and subtracted if being fired to the rear. The behaviour of light in this regard was checked at the CERN laboratory in Geneva in 1964, using tiny subatomic particles called *neutral pions*. The pions, travelling at $0.99975c$, decayed with the emission of two light pulses. Both pulses were found to have the usual speed of light, c, to within the measurement accuracy of 0.1%. So, the speed of light does not depend on the speed of the source.

It also does not depend on whether the observer of the light is considered to be moving or not. Take the case of a moving vessel again. Having already established that light does not behave like a shell being fired from a gun, we might expect it to behave like the ripples on the water. If the observer were now someone aboard a moving boat, the wave front would appear to move ahead of the boat more slowly than the wave front going to the rear – because of the motion of the boat and of himself relative to the water (see Figure 1). If light were a wave moving through a medium



1. Ripples sent out by a boat appear to an observer on the boat to move away more slowly in the forward direction than to the rear

pervading all of space – a medium provisionally called the aether – then, with the earth ploughing its way through the aether, we ought to find the speed of light relative to us observers travelling along with the earth to be different in different directions. But in a famous experiment carried out by Michelson and Morley in 1887, the speed of light was found to be the same in all directions. Thus, the speed of light is independent of whether either the source or the observer is considered to be moving.

So there we have it:

(i) The principle of relativity, which states that the laws of nature are the same for all inertial frames of reference.

(ii) One of those laws allows us to work out the value of the speed of light in a vacuum – a value which is the same in all inertial frames, regardless of the velocity of the source or the observer.

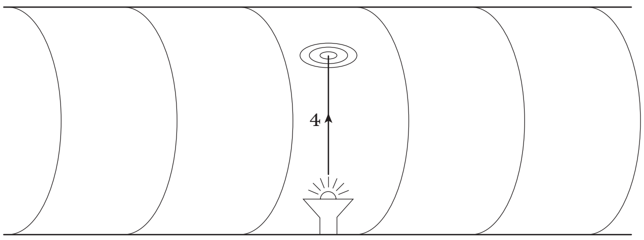
These two statements came to be known as the two *postulates* (or fundamental principles) of special relativity.

These facts had been common knowledge among physicists for a long time. It required the genius of Einstein to spot that although each of the two statements made sense when you thought about them separately, they did not appear to make sense if you put the two ideas together. It seemed as though if the first of them was right, then the second must be wrong, or if the second was right, the first must be wrong. If both were right – which we appear to have established – then something very, very serious must be amiss. The fact that the speed of light is the same for all inertial observers regardless of the motion of the source or observer means that our usual way of adding and subtracting velocities is wrong. And if there is something wrong with our conception of velocity (which is simply distance divided by time), then that in turn implies there must be something wrong with our conception of space, or time, or both. What we are dealing with is not some peculiarity of light or electromagnetic radiation. *Anything* travelling at the same speed as that of light will have the same value for its speed for all inertial observers. What is crucial is the speed (and the implications for the underlying space and time) – not the fact that we happen to be dealing with light.

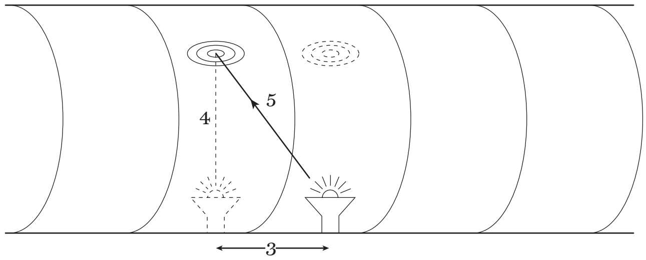
Time dilation

To see what is amiss, imagine an astronaut in a high-speed spacecraft and a mission controller on the ground. They both have identical clocks. The astronaut is to carry out a simple experiment. On the floor of the craft she is to fix a lamp which emits a pulse of light. The pulse travels directly upwards at right angles to the direction of motion of the craft (see Figure 2). There the pulse strikes a bullseye target fixed to the ceiling. Let us say that the height of the craft is 4 metres. With the light travelling at speed, c, she finds that the time taken for this trip, t', as measured on her clock, is given by $t' = 4/c$.

Now let's see what this looks like from the perspective of the mission controller. As the craft passes him overhead, he too observes the trip performed by the light pulse from the source to the target. According to his perspective, during the time taken for the pulse to arrive at the target, the target will have moved forward from where it was when the pulse was emitted. For him,



2. The astronaut arranges for a pulse of light to be directed towards a target such that the light travels at right angles to the direction of motion of the spacecraft



3. According to the mission controller on earth, as the spacecraft passes overhead, the target moves forward in the time it takes for the light pulse to perform its journey. The pulse, therefore, has to traverse a diagonal path

the path is not vertical; it slopes (see Figure 3). The length of this sloping path will clearly be longer than it was from the astronaut's point of view. Let us say that the craft moves forward 3 metres in the time that it takes for the light pulse to travel from the source to the target. Using Pythagoras' theorem, where $3^2 + 4^2 = 5^2$, we see that the distance travelled by the pulse to get to the target is, according to the controller, 5 metres.

So what does he find for the time taken for the pulse to perform the trip? Clearly it is the distance travelled, 5 metres, divided by the speed at which he sees the light travelling. This we have established is c (the same as it was for the astronaut). Thus, for the controller, the time taken, t, registered on his clock, is given by $t = 5/c$.

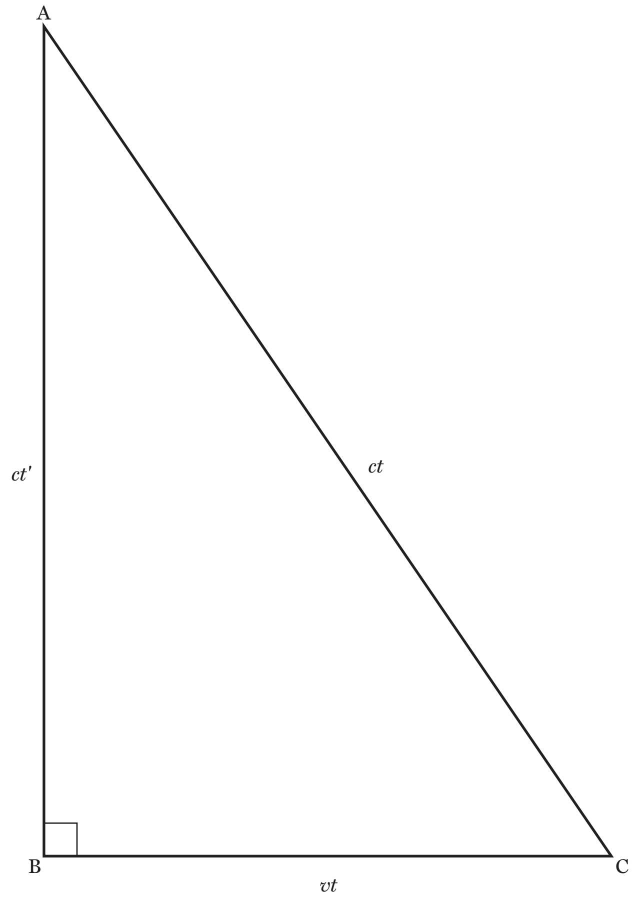
But this is not the time the astronaut found. She measured the time to be $t' = 4/c$. So, they disagree as to how long it took the pulse to perform the trip. According to the controller, the reading on the astronaut's clock is too low; her clock is going slower than his.

And it is not just the clock. Everything going on in the spacecraft is slowed down in the same ratio. If this were not so, the astronaut would be able to note that her clock was going slow (compared, say, to her heart beat rate, or the time taken to boil a kettle, etc.). And that in turn would allow her to deduce that she was moving – her speed somehow affecting the mechanism of the clock. But that is not allowed by the principle of relativity. All uniform motion is relative. Life for the astronaut must proceed in exactly the same way as it does for the mission controller. Thus we conclude that everything happening in the spacecraft – the clock, the workings of the electronics, the astronaut's ageing processes, her thinking processes – all are slowed down in the same ratio. When she observes her slow clock with her slow brain, nothing will seem amiss. Indeed, as far as she is concerned, everything inside the craft keeps in step and appears normal. It is only according to the controller that everything in the craft is slowed down. This is *time dilation*. The astronaut has her time; the controller has his. They are not the same.

In that example we took a specific case, one in which the astronaut and spacecraft travel 3 metres in the time it takes light to travel the 5 metres from the source to the target. In other words, the craft is travelling at a speed of ${}^{3}/{}_{5}c$, i.e. $0.67c$. And for that particular speed we found that the astronaut's time was slowed down by a factor ${}^{4}/{}_{5}$, i.e. 0.8. It is easy enough to obtain a formula for any chosen speed, v. We apply Pythagoras' theorem to triangle ABC. The distances are as shown in Figure 4. Thus:

$$
\begin{aligned}
\mathrm{AC}^2 &= \mathrm{AB}^2 + \mathrm{BC}^2 \\
\mathrm{AB}^2 &= \mathrm{AC}^2 - \mathrm{BC}^2 \\
c^2 t'^2 &= (c^2 - v^2)t^2 \qquad (1) \\
t'^2 &= (1 - v^2/c^2)t^2 \\
t' &= t\sqrt{(1 - v^2/c^2)}
\end{aligned}
$$

From this formula we see that if v is small compared to c, the expression under the square root sign approximates to one, and $t' \approx t$. Yet no matter how small v becomes, the dilation effect is



4. According to the mission controller, BC is the distance travelled by the craft in the time taken for the light pulse to travel to the target, and AC is the distance travelled by the pulse. AB is the distance travelled by the pulse according to the astronaut

still there. This means that, strictly speaking, whenever we undertake a journey – say, a bus trip – on alighting we ought to readjust our watch to get it back into synchronization with all the stationary clocks and watches. The reason we do not is that the effect is so small. For instance, someone opting to drive express trains all their working life will get out of step with those following sedentary jobs by no more than about one-millionth of a second by the time they retire. Hardly worth bothering about.

At the other extreme, we see from the formula that, as v approaches c, the expression under the square root sign approaches zero, and t' tends to zero. In other words, time for the astronaut would effectively come to a standstill. This implies that if astronauts were capable of flying very close to the speed of light, they would hardly age at all and would, in effect, live for ever. The downside, of course, is that their brains would have almost come to a standstill, which in turn means they would be unaware of having discovered the secret of eternal youth.

So much for the theory. But is it true in practice? Emphatically, yes. In 1977, for instance, an experiment was carried out at the CERN laboratory in Geneva on subatomic particles called *muons*. These tiny particles are unstable, and after an average time of 2.2×10^{-6} seconds (i.e. 2.2 millionths of a second) they break up into smaller particles. They were made to travel repeatedly around a circular trajectory of about 14 metres diameter, at a speed of $v = 0.9994c$. The average lifetime of these moving muons was measured to be 29.3 times longer than that of stationary muons – exactly the result expected from the formula we have derived, to an experimental accuracy of 1 part in 2000.

In a separate experiment carried out in 1971, the formula was checked out at aircraft speeds using identical atomic clocks, one carried in an aircraft, and the other on the ground. Again, good agreement with theory was found. These and innumerable other experiments all confirm the correctness of the time dilation formula.

The twin paradox

We have seen how the mission controller sees time passing slowly in the moving spacecraft, while the astronaut regards her time as normal. How does the *astronaut* see the *mission controller's* time?

At first one might think that if her time is going slow, then when she observes what is happening on the ground, she will perceive time down there to be going fast. But wait. That cannot be right. If it were, then we would immediately be able to conclude who was actually moving and who was stationary. We would have established that the astronaut was the moving observer because her time was affected by the motion whereas the controller's wasn't. But that violates the principle of relativity, i.e. that for inertial frames, all motion is relative. Thus, the principle leads us to the, admittedly somewhat uncomfortable, conclusion that if the controller concludes that the astronaut's clock is going slower than his, then she will conclude that his clock is going slower than hers. But how, you might ask, is that possible? How can we have two clocks, both of which are lagging behind the other?!

A preliminary to addressing this problem is that we must first recognize that in the set-up we have described we are not comparing clocks directly side-by-side. Though the astronaut and controller might indeed have synchronized their two clocks as they were momentarily alongside each other at the start of the space trip, they cannot do the same for the subsequent reading; the spacecraft and its clock have flown off into the distance. The controller can only find out how the astronaut's clock is doing by waiting for some kind of signal (perhaps a light signal) to be emitted by her clock and received by himself. He then has to allow for the fact that it has taken time for that signal to travel from the