

Overview

This unit focuses on consolidating and extending students' understanding of fractions, both as part of a whole and an operator. It develops to consider percentages as amounts and as operators and extends to using ideas of ratio and proportion. It also includes using ratio as an operator and how to investigate proportion problems. Spreads N1.1, N1.2, N1.4, N1.5, N1.7 and N1.9 form the focus of the unit.

Framework references

This unit covers objectives from the Framework (Y456) on pages: 23, 25, 27, 33, 66 and 87 and from the Framework (Y789) on pages: 23 and 80.

Introduction

Discuss examples of proportion from everyday life, such as following a recipe, emphasising that you have to keep the proportions the same if you want to increase the overall quantity. Discuss what students know about fractions and percentages, emphasising that percentages are parts of 100. Use the check in activity to show that it is easier to compare and order percentages.

Check in activity

Show students fractions: $\frac{1}{10}$, $\frac{1}{12}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{3}{6}$. As a class, discuss how to order them. You could use a number line (R6), if needed. Discuss the percentage equivalents for each fraction and order the percentages.

Useful resources

Worksheets

- N1.1WS – plotting coordinates
- N1.3WS – fractions
- N1.4WS – fractions of amounts
- N1.5WS – percentages and 10%
- N1.6WS – using a calculator
- N1.8WS – dividing into ratio
- N1.9WS – proportion

OHPs

- N1.2OHP – fraction addition diagrams
- N1.7OHP – ratio examples

General resources

- R4 – place value tables
- R6 – number line
- R8 – coordinate grid
- R21 – multiplication table
- R24 – 10 by 10 grid
- Multilink cubes

Differentiation – spanning the bridge

Spread	Bridge to the Support tier
N1.1	Support students will benefit from the Access discussion on equivalent fractions, but will need to extend to adding fractions with the same and different denominators.
N1.2	Support students need to consider multiplying integers by fractions. Link the word 'of' to multiplying using "3 lots of 4 is 3×4 " to illustrate " $\frac{1}{4}$ of 8 is $\frac{1}{4} \times 8$ ". Progress from unitary fractions to non-unitary fractions.
N1.3	Support students need to extend to using fractions as an operator to do division calculations. Build on use of the word 'of' in N1.2 to develop that 'x' is the inverse of '÷'.
N1.4	Highlight the link between percentages as fractions out of 100 to progress Support students from finding fractions of amounts (Access) to calculating percentages of amounts.
N1.5	Develop the idea that $\frac{1}{10} = 10\%$ to include other fractional equivalences of multiples of 10%. Remind Support students that any percentage can be written as a fraction over 100, and how to simplify a fraction by cancelling common factors.
N1.6	Support students progress from calculating a percentage (Access) to calculating percentage increases and decreases. First calculating the percentage of the amount by converting it to a fraction over 100 (as Access students are doing) before adding or subtracting this from the original amount.
N1.7	Both tiers consider equivalent ratios in this lesson, but Support students will benefit from some discussion about converting quantities to the same unit. It will also be necessary to clearly differentiate between ratio and proportion.
N1.8	Support students focus on equivalent ratios and proportional increase extending to proportional decrease, using the unitary method to calculate more complex direct proportional changes.

N1.1 Equivalent fractions

Access

This spread provides access to the Year 9 objective:
▶ Add and subtract fractions (67).

Lesson Plan

Mental starter

Write some unitary fractions on the board, for example $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. Ask students:

- ▶ Which fraction of a pizza would you rather have?
- ▶ Which fraction would give you the smallest amount?

Highlight that as the denominator increases, each part becomes smaller.

- ▶ Which is bigger: $\frac{1}{2}$ a pizza or 2 slices of a pizza split into quarters?

Repeat with different equivalent fractions.

Introductory activity

Refer to the mental starter and establish that some fractions are **equivalent** and describe **equal-sized parts**.

Ask students to write fractions equivalent to $\frac{1}{2}$ on the board. Highlight the relationship between the denominator and numerator of each.

Discuss how to convert $\frac{1}{2}$ to an equivalent fraction with denominator 6. Emphasise that when the denominator is multiplied by 3, the numerator is also multiplied by 3. Repeat for other fractions equivalent to $\frac{1}{2}$.

Ask students to give equivalent fractions to $\frac{1}{3}$ using this scaling method. Refer to the Students' book diagram that shows $\frac{1}{3} = \frac{2}{6}$.

Discuss how to construct a graph to show the relationship between fractions equivalent to $\frac{1}{3}$.

Show the OHP of **R8** (first quadrant only) and ask students to plot the coordinates (numerator, denominator).

Extend the line and ask students to identify other fractions equivalent to $\frac{1}{3}$ from the line.

Plenary

Ask students to give fractions equivalent to a half, greater than a half and less than a half.

Repeat for thirds.

Homework

- ▶ Ask students to give 5 fractions equivalent to a quarter, 5 less than a quarter and 5 more than a quarter.

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y456) Page 23.

Useful resources

R8 – coordinate grid in four quadrants
N1.1WS – provides further practice of the key ideas.

Differentiation

- ▶ Question 1 focuses on identifying equivalent fractions from diagrams.
- ▶ Questions 2 and 3 focus on the proportional relationship of equivalent fractions.
- ▶ Questions 4–6 focus on calculating equivalent fractions using scale factors.

Support tier: focuses on adding and subtracting fractions.

Misconceptions

Students may confuse the values for denominator and numerator when writing fractions in question 2. Encourage them to check that the equivalent fractions they generate could also be generated by multiplying numerator and denominator by a scale factor. Students may also write the coordinate values in the wrong order in question 3. Remind them that the first value in the pair is the value on the horizontal axis, which is labelled as 'Numerator'.

Links

Proportion: Framework (Y456) Page 27.

Exercise Answers

- 1 a $\frac{3}{6}$ b $\frac{5}{10}$ c $\frac{4}{8}$ d $\frac{6}{12}$
 2 a 6, 3 b $\frac{1}{5}$, $\frac{2}{10}$ c $\frac{1}{10}$ d $\frac{2}{20}$, $\frac{3}{30}$
 3 a (1, 4) (2, 8) (3, 12) b (1, 6) (2, 12) (3, 18)
 c (1, 5) (2, 10) (3, 15) d (2, 5) (4, 10) (6, 15)
 e (3, 5) (6, 10) (9, 15)
 4 a 20 b 10 c $\times 2$, 14 d $\times 5$, 45
 5 Student's work
 6 a $\frac{1}{2}$ b $\frac{2}{3}$ c $\frac{1}{3}$ d $\frac{1}{4}$ e $\frac{1}{4}$ f $\frac{1}{2}$ g $\frac{3}{4}$ h $\frac{1}{3}$ i $\frac{3}{4}$
 j $\frac{1}{3}$ k $\frac{1}{4}$ l $\frac{2}{3}$

Worksheet Answers

- 1 c (5, 10) (6, 12) (7, 14) (8, 16) d $\frac{1}{2}$
 2 a Student's work c A (6, 8) B (4, 12) C (8, 12)

N1.1WS

Name:

- 1 a Plot these coordinates onto this grid.

(0, 0) (1, 2) (2, 4) (3, 6) (4, 8)

- b Join your points with a straight line.

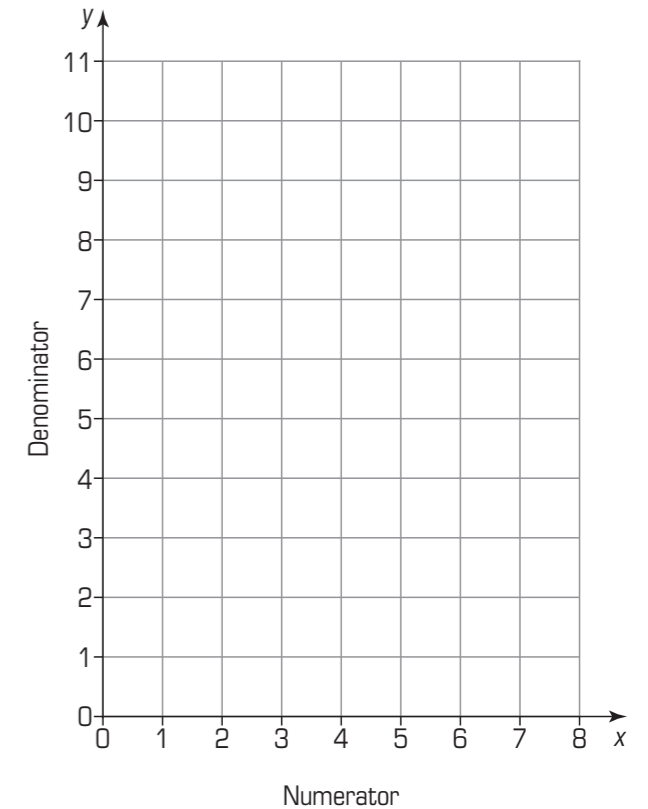
- c Write the next 4 coordinate pairs that are on the line when it is extended.

(5, _____) (_____, _____)

(_____, _____) (_____, _____)

- d What fraction does the line represent?

Fraction: _____.



- 2 a Write 2 coordinates you could plot for each of these fractions.

Line A: $\frac{3}{4}$ (3, 4) (_____, _____)

Line B: $\frac{1}{3}$ (_____, _____) (_____, _____)

Line C: $\frac{2}{3}$ (_____, _____) (_____, _____)

- b Draw each line on the grid.

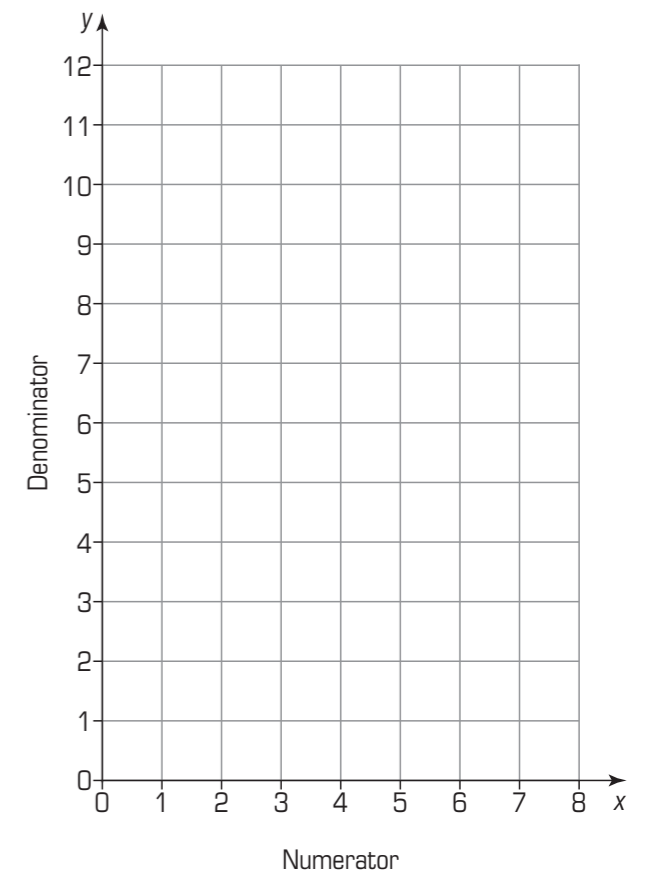
Extend the lines to the edge of the grid.

- c For each line, what is the last whole number coordinate pair on the grid?

Line A: (_____, _____)

Line B: (_____, _____)

Line C: (_____, _____)



N1.2 Adding and subtracting fractions

Access

This spread provides access to the Year 9 objective:

- ▶ Add and subtract fractions (67).

Lesson Plan

Mental starter

Students draw a 3×3 grid and fill it with numbers from 1 to 20. Write on the board and read out string calculations with addition and subtraction, say $3 + 2 + 6 - 4$. Students cross off answers as they appear in their grid. The winner is the first to complete a row.

Introductory activity

Write these calculations on the board:

$$\blacktriangleright \frac{1}{8} + \frac{1}{8} = \frac{2}{8} \qquad \blacktriangleright \frac{1}{8} + \frac{1}{8} = \frac{2}{16}$$

Discuss which calculation is correct. Encourage students to say each fraction out loud (one eighth). Refer to the cake example in the Students' book for a visual representation. Highlight that:

- ▶ The denominator shows the **size** of each part.
- ▶ The numerator indicates the **number** of parts.

Emphasise that when adding fractions, the size of each part remains the same, but the number of parts you have increases; the denominator stays the same – you only add the numerators.

N1.2OHP gives further examples of fraction additions, supported by diagrams. Show the second diagram and discuss what needs to be added to $\frac{3}{8}$ to get 1.

Emphasise that $\frac{8}{8} = 1$ and encourage students to give other equivalent fractions to 1.

Discuss that a whole is equivalent to any fraction where the denominator (total parts) and numerator (number of parts) are equal.

Ask students which of these calculations is correct:

$$\blacktriangleright \frac{2}{8} - \frac{3}{8} = \frac{2}{0} \qquad \blacktriangleright \frac{2}{8} - \frac{3}{8} = \frac{2}{8}$$

Encourage students to say each fraction out loud. Emphasise that only the numerators are subtracted.

Plenary

Discuss $\frac{1}{2} + \frac{1}{4}$. Highlight that the size of the parts is unequal so you cannot just add the number of parts. Encourage students to give equivalent fractions to a half. Emphasise that $\frac{1}{2} + \frac{1}{4}$ is the same as $\frac{2}{4} + \frac{1}{4}$. Repeat for $\frac{1}{2}$ and $\frac{1}{5}$ and link to question 5 which looks at both these fractions as tenths.

Homework

- ▶ Building on question 4, ask students to write 10 fraction calculations where the answer is 1.

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y789) Page 66 and Framework (Y456) Page 23.

Useful resources

N1.2OHP – fraction addition diagrams

Differentiation

- ▶ Questions 1–3 focus on additions and subtractions of fractions from a whole.
- ▶ Question 4 focuses on addition of fractions with the same denominator.
- ▶ Questions 5 and 6 focus on fraction additions and subtractions that equal a whole.

Support tier: focuses on multiplying by fractions.

Misconceptions

Students often add the denominators as well as the numerators. It may help them to say each calculation out loud. Remind them of the roles of the numerator (number of parts) and denominator (size of part) and emphasise that the size of part does not change.

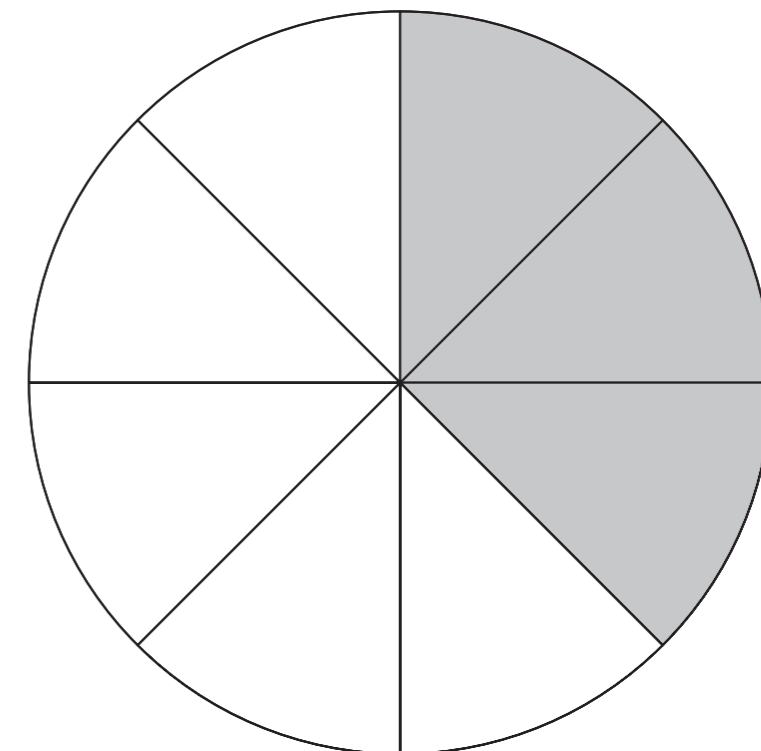
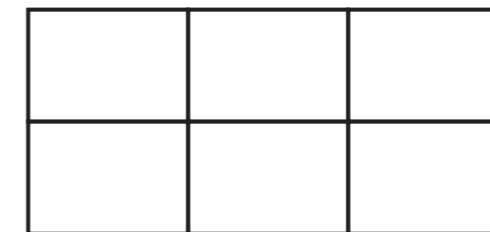
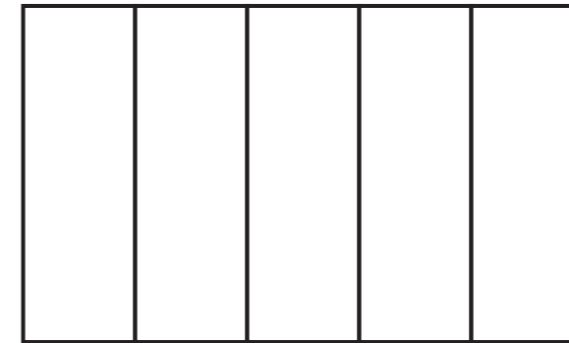
Links

Proportion: Framework (Y456) Page 27.

Exercise Answers

- 1 **b** 6 **c** 20 **d** 3 **e** 5 **f** 14 **g** 5
 2 **a** $\frac{9}{10}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$ **d** $\frac{6}{7}$ **e** $\frac{19}{20}$ **f** $\frac{1}{3}$ **g** $\frac{1}{4}$ **h** $\frac{2}{5}$
i $\frac{4}{9}$ **j** $\frac{1}{10}$
 3 **b** $\frac{2}{3}$ **c** $\frac{7}{8}$ **d** $\frac{3}{5}$ **e** $\frac{7}{9}$ **f** $\frac{17}{20}$ **g** $\frac{6}{7}$ **h** 1
 4 **a** and **c**
 5 **a** $\frac{7}{10}$ **b** Students' work **c** $\frac{3}{10}$
 6 **a** $\frac{2}{5}$ **b** $\frac{1}{6}$

N1.2OHP Adding and subtracting fractions



N1.3 Ordering fractions

Access

This spread provides access to the Year 9 objective:
▶ Order fractions (65).

Lesson Plan

Mental starter

▶ Ask: which is bigger $\frac{5}{10}$ or $\frac{1}{2}$?

Divide the board into 3, with headings: less than $\frac{1}{2}$, more than $\frac{1}{2}$, equal to $\frac{1}{2}$. Ask students to write a fraction in one of the groups, justifying their choices.

Introductory activity

Refer to the mental starter. Discuss how they know that $\frac{7}{10}$ is bigger than $\frac{1}{2}$.

Emphasise conversion to equivalent fractions, recapping the method: multiplying numerator and denominator by the same scale factor.

Discuss how to compare $\frac{1}{2}$ and $\frac{1}{3}$ in the Students' book example. **Emphasise that there is no scale factor to convert halves to thirds, so both fractions need to be converted to equivalent fractions.**

Ask students for fractions equivalent to $\frac{1}{3}$. Compare with the fractions equivalent to $\frac{1}{2}$ from the mental starter.

▶ Which denominator appears in both 'fraction families'?

Emphasise that $\frac{1}{2}$ and $\frac{1}{3}$ can both be converted into sixths, as six is a multiple of 2 and 3.

Highlight that $\frac{3}{6}$ is bigger than $\frac{2}{6}$ and so $\frac{1}{2}$ is bigger than $\frac{1}{3}$. Write this as $\frac{1}{2} > \frac{1}{3}$, reminding students of the notation.

Draw a number line from 0 to 1 or use **R6**. Ask a student to mark $\frac{1}{2}$ on the line. Emphasise that the line has been split into **2 equal parts**.

Repeat for $\frac{1}{3}$, with the line split into **3 equal parts**.

Highlight that $\frac{1}{3}$ is smaller than $\frac{1}{2}$ so $\frac{1}{3} < \frac{1}{2}$.

Repeat to compare $\frac{2}{3}$ and $\frac{3}{4}$.

Discuss using fraction families to change $\frac{5}{2}$ to $2\frac{1}{2}$.

Plenary

Write the fractions $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{3}{4}$ on cards. Shuffle them, face down. Turn up two cards at a time and discuss which fraction is largest. Encourage students to justify their answers.

Homework

▶ Ask students to order the fractions: $\frac{1}{10}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{9}{10}$ by converting them all to equivalent fractions in tenths and then writing them from smallest to largest.

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y456) Page 23.

Useful resources

R6 – number lines

R21 – multiplication table

N1.3WS – provides support for weaker students

Differentiation

▶ Questions 1 and 2 focus on using diagrams to compare fractions with different denominators.

▶ Questions 3 and 4 focus on comparing fractions using a number line.

▶ Questions 5 and 6 focus on mixed numbers and top heavy fractions.

Support tier: focuses on dividing integers by fractions.

Misconceptions

Students may find it difficult to make equivalent fractions due to poor multiplication skills. A multiplication table (**R21**) will give useful support. Students may assume that the largest numerator indicates the largest fraction, or that the largest denominator indicates the largest sized 'part'. Remind them that the denominator represents the number of parts, and the larger the number of parts, the smaller each part will be.

Links

Ratio and proportion: Framework (Y456) Page 27.

Exercise Answers

1 a True b True c True d True

2 a < b > c > d > e > f >

3 a < b > c < d =

4 a < b = c > d < e > f > g > h <

5 a $\frac{3}{2}$ b $\frac{4}{3}$ c $\frac{7}{4}$ d $\frac{11}{16}$ e $\frac{9}{8}$ f $\frac{7}{3}$ g $\frac{5}{2}$ h $\frac{13}{4}$

i $\frac{23}{8}$ j $\frac{23}{9}$ k $\frac{21}{10}$ l $\frac{26}{7}$

6 a $1\frac{1}{5}$ b $1\frac{3}{4}$ c $4\frac{1}{2}$ d $3\frac{2}{3}$ e $3\frac{1}{5}$ f $4\frac{1}{5}$

Worksheet Answers

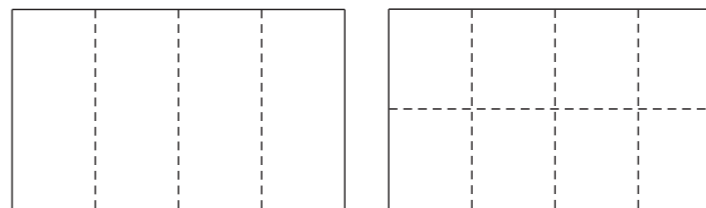
2 b $\frac{3}{15}$ d $\frac{5}{15}$ e i > ii < iii < iv =

3 c i $\frac{4}{20}$, $\frac{2}{10}$, $\frac{8}{40}$ ii $\frac{5}{20}$, $\frac{10}{40}$

N1.3WS

Name:

1 Shade $\frac{1}{4}$ of these rectangles.



2 a Shade $\frac{1}{5}$ of this grid.

b Use your drawing to complete these equivalent fractions.

$$\frac{\quad}{15} = \frac{1}{5}$$

c Shade $\frac{1}{3}$ of this grid.

d Use your drawing to complete these equivalent fractions.

$$\frac{\quad}{15} = \frac{1}{3}$$

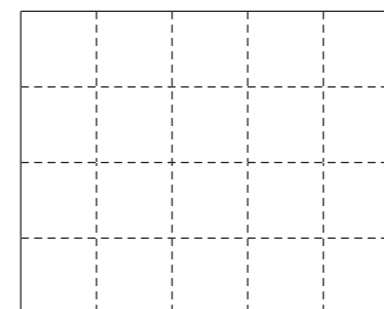
e Use your answers to make these statements correct
Choose a symbol from the box for each space.

i $\frac{5}{15} \square \frac{1}{5}$ ii $\frac{3}{15} \square \frac{1}{3}$ iii $\frac{1}{5} \square \frac{1}{3}$ iv $\frac{3}{15} \square \frac{1}{5}$

> is greater than
< is smaller than
= is equal to

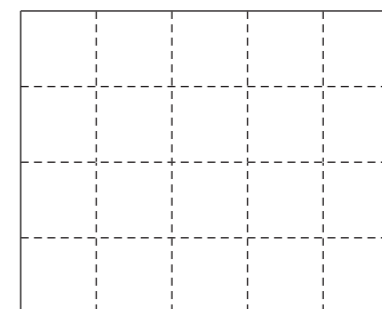
3 a Shade $\frac{1}{5}$ of this grid.

c Complete these equivalent fractions



i $\frac{1}{5} = \frac{\quad}{20} = \frac{\quad}{10} = \frac{\quad}{40}$

b Shade $\frac{1}{4}$ of this grid.



ii $\frac{1}{4} = \frac{\quad}{20} = \frac{\quad}{40}$

N1.4 Fractions of amounts

Access

This spread provides access to the Year 9 objective:
 ▶ Calculate fractions of numbers, quantities or measurements (67).

Lesson Plan

Mental starter

Students draw a 3×3 grid and write in numbers from 1 to 20. Read out fraction questions based on finding halves of amounts: $\frac{1}{2}$ of 30, say, then extend to quarters, such as $\frac{1}{4}$ of 40.

Students cross off answers as they appear in their grid. The winner is the first to cross off a line of three.

Introductory activity

Discuss how to find a half and a quarter of an amount. Emphasise that you divide by 2 or 4 to share the amount into the number of parts given by the denominator. Extend to finding $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{8}$ etc.

Extend to consider non-unitary fractions of amounts, say $\frac{3}{4}$ of 20 oranges.

Encourage students to say the fraction $\frac{3}{4}$ out loud to distinguish the roles of numerator (number of parts) and denominator (size of parts).

Emphasise the strategy of first finding one quarter and then multiplying by three. Use sketches to support the explanation:



Progress to the more complex example of $\frac{4}{5}$ of 4510 in the Students' book. Encourage students to say the calculation out loud to clarify its meaning. Emphasise the strategy of finding $\frac{1}{5}$ and then multiplying by 4. Model the calculation on a calculator, and ask students to check on their own calculators.

Plenary

Write 800 in a circle on the board. Ask students to calculate mentally different fractions of 800 and write them on the board. Encourage them to follow previous calculations, for example if $\frac{1}{10}$ is 80, then $\frac{2}{10}$ must be ...

Homework

▶ Challenge students to find as many different fractions of 300 as possible.

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y456) Page 25.

Useful resources

R21 – multiplication table

N1.4WS – provides support for weaker students.

Differentiation

- ▶ Questions 1–5 focus on finding unitary fractions of amounts.
- ▶ Question 6 focuses on finding non-unitary fractions of amounts.
- ▶ Question 7 extends to using a calculator to find a non-unitary fraction of a large amount.

Support tier: focuses on calculating percentages.

Misconceptions

Students may confuse the numerator and denominator values and multiply and divide randomly (for example, finding $\frac{4}{5}$ they divide by 4 and multiply by 5).

Encourage them to say the fraction out loud to link the denominator with the fraction size (linking $\frac{4}{5}$ to *fifths*). This will also reinforce the unitary method – find one fifth, before progressing to four fifths.

Division and multiplication weaknesses may cause problems and a multiplication table (R21) will provide support.

Links

Ratio and proportion: Framework (Y456) Page 27.

Exercise Answers

- 1 10 2 12 3 a 5 CDs b £4 c 20 cm
 d 5 apples e €9 f 3 mm 4 50
 5 a 4 b 5 c divide, 10 6 a £20 b 12 kg
 c 6 mm d 6 e £36 f €35 g 25 m
 h 6 tonnes 7 a 21 600 b 7200
 c Add to 28 800

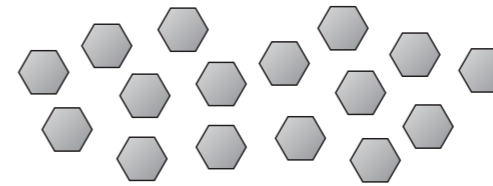
Worksheet Answers

- 1 b 4 2 b 4 c 8 3 20 4 40

N1.4WS

Name:

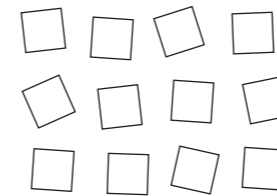
1 a Circle $\frac{1}{4}$ of these tiles.



b Complete this statement:

$$\frac{1}{4} \text{ of } 16 = \underline{\hspace{2cm}}$$

2 a Here are 12 tiles. Shade $\frac{1}{3}$ of them.



b What is $\frac{1}{3}$ of 12? _____

c What is $\frac{2}{3}$ of 12? _____

3 These hexagon tiles are $\frac{2}{5}$ of a larger pattern.

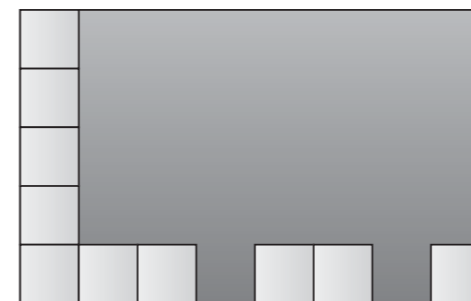
How many tiles are there in the larger pattern?



Answer: _____ tiles

4 One quarter ($\frac{1}{4}$) of the wall has been tiled.

How many tiles will be needed for the whole wall?



Answer: _____ tiles

Remember: to find $\frac{1}{4}$, you divide by 4.

N1.5 Percentages of amounts

Access

This spread provides access to the Year 9 objective:
 ▶ Calculate percentages and use them to solve problems (75).

Lesson Plan

Mental starter

Write the number 350 on the board.
 Discuss how to find $\frac{1}{10}$ of 350.
 Discuss how to use this result to find $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$ and so on, up to $\frac{9}{10}$.

Introductory activity

Show a 10 by 10 grid (R24) and invite students to shade in $\frac{1}{10}$ of the grid.
 Discuss how many parts out of 100 have been shaded.
Emphasise that a percentage is a fraction out of 100, so the shaded amount is $\frac{10}{100}$ or 10% and therefore 10% is the same as $\frac{1}{10}$.

▶ Ask: what percentage is equal to $\frac{2}{10}$, $\frac{3}{10}$, etc.?

Use the 10 by 10 grid as reference.

Refer to the mental starter and recap how to find $\frac{1}{10}$ of an amount, by dividing by 10. Demonstrate on an OHP of a place value table (R4) that each digit moves one place to the right.

Highlight that this is also how you find 10% of an amount and practise with some examples.

Repeat the mental starter with start value £35. Find 10% and then discuss how to use this to find 20%, 30%, etc. Emphasise the strategy of finding 10% and then multiplying by however many 10% you need.

Plenary

Refer to the final activity in the introductory activity.
 Discuss how to find 5% of 35 using the 10% calculation.
 Progress to using 10% and 5% to calculate 15%, 25%, 35%, etc.

Homework

▶ Ask students to start with 250 and calculate 10%, 20%, 30%, etc. Extend confident students to calculating 5%, 15% etc, new facts from their previous calculations.

Worksheet Answers

- 1 b 5 c 10 d 35
 2 49
 3 a 9 b 36

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y456) Page 33.

Useful resources

R4 – place value tables
 R24 – 10 by 10 grid
 N1.5WS – provides support for weaker students.

Differentiation

- ▶ Questions 1–3 focus on calculating 10% of amounts with integer answers.
- ▶ Questions 4 and 5 extend to calculating multiples of 10% with integer answers.
- ▶ Questions 6–8 focus on using place value to calculate 10% of amounts with non-integer answers.

Support tier: focuses on converting between fractions, decimals and percentages.

Misconceptions

Students may think that as they divide by 10 to find 10%, they should divide by 20 to find 20%. Ask them to consider whether 20% is more than 10% and use their intuitive understanding to show them that dividing by 20 does not give a bigger answer. Use the 10 by 10 grid (R24) to link 20% with $\frac{2}{10}$ and emphasise the method of finding $\frac{1}{10}$ and then multiplying by the number of tenths required. Encourage students always to start by finding 10% and to use this to help them derive new facts.

Links

× and ÷ by 10: Framework (Y456) Page 7.

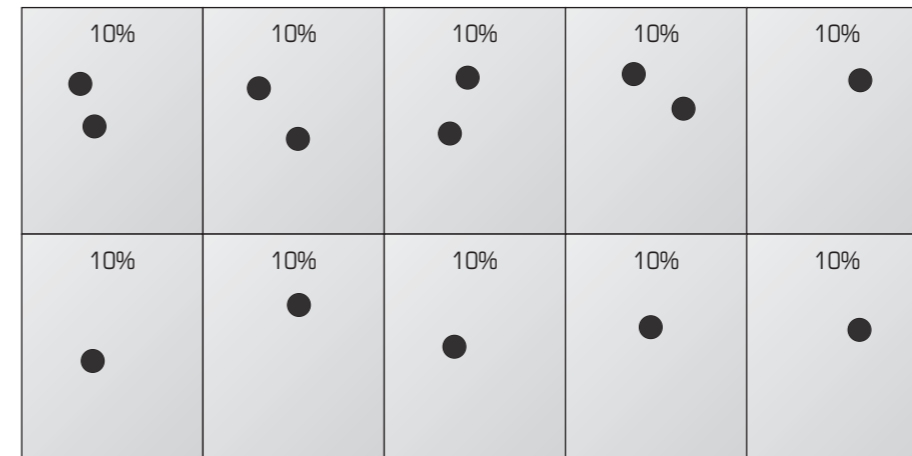
Exercise Answers

- 1 a 60% b 35% c 57%
 2 a $\frac{29}{100}$ b $\frac{39}{100}$ c $\frac{43}{100}$ d $\frac{73}{100}$ e $\frac{99}{100}$
 3 a 5 kg b 6 cm c 9 km d 7 mm e \$10
 f €12 g 15 cm h 17 people i £19 j £20
 4 a 2 cm b 18 cm 5 a £5 b £15 6 £2.50
 7 a £5.50 b £11
 8 a 8 mm b 88 mm c 48 mm

N1.5WS

Name:

1 This box has been divided into 10 equal parts.



Each of the 10 parts is 10%.

- a So far 14 dots have been shared.
 Share out dots equally until you have shared 50 in total.
 The number of dots in each box is 10% of 50.
- b What is 10% of 50? _____
- c What is 20% of 50? _____
- d What is 70% of 50? _____

2 Use this box to work out 70% of 70.



70% of 70 = _____

- 3 a Calculate 10% of 90. 10% of 90 = _____
- b Use your answer to calculate 40% of 90.
 40% of 90 = _____

Hint: First work out 10% of 70. You could share out 70 dots.

Access

This spread provides access to the Year 9 objective:
 ▶ Interpret and use ratio in a range of contexts (81).

Lesson Plan

Mental starter

Students draw a 3×3 grid and write in numbers from 1 to 15. Give students a 'factor' statement:

- ▶ I am a factor of 10.
- ▶ 3 is one of my factors, etc.

Students cross off any number that fits the statement. The first to cross off 3 in a line wins.

Introductory activity

Refer to the mental starter and highlight:

- ▶ A factor of a number divides exactly into the number.
- ▶ A number is divisible by its factors.
- ▶ 1 is a factor of all numbers.
- ▶ All numbers are factors of themselves.

Discuss students' understanding of the keyword: **ratio**. Emphasise that ratio compares parts with each other and can be described with a 'for every' statement. Refer to the 'dogs and owners' example in the Students' book and emphasise that order matters ($3 : 1$ is not the same as $1 : 3$) and the notation. Link to work on fractions and highlight that $12 : 4$ simplifies to $3 : 1$ – they are equivalent ratios.

Link to the mental starter and highlight that 12 and 4 have a common factor, 4, and we can divide each part of the ratio by 4 to simplify. **N1.7OHP** provides more examples to reinforce the key ideas.

Ask students to use multilink cubes to show ratios of red : blue, for example $8 : 6$, $3 : 6$, $6 : 8$

- ▶ What is a factor of both values? (encourage students to give the highest factor).
- ▶ What equivalent ratio do you get when you divide both values by this factor?

Show that dividing by a factor of 1 does not change the ratio.

Plenary

Give students these ratios: $6 : 8$, $8 : 6$, $18 : 24$, $4 : 4$
 Discuss which are equivalent to $3 : 4$, encouraging students to justify their answers.

Homework

- ▶ Give students five ratios, for example $2 : 3$, $4 : 6$, $12 : 18$, $20 : 30$ and $30 : 40$, and ask them to identify the odd one out.

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y456) Page 27 and Framework (Y789) Page 80.

Useful resources

Multilink cubes
R21 – multiplication table
N1.7OHP – ratio examples

Differentiation

- ▶ Questions 1 and 2 focus on writing ratios and simplifying them in a practical context.
- ▶ Questions 3 and 4 focus on simplifying ratios without a practical context.
- ▶ Questions 5 and 6 extend to using ratio to solve 'real life' problems.

Support tier: focuses on sharing a quantity in a given ratio.

Misconceptions

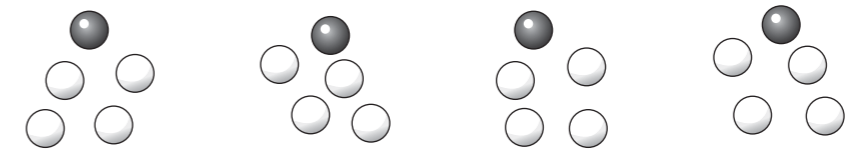
Students may write ratio quantities in the correct order. This is addressed in the introductory activity, but also remind students that red : blue $1 : 2$ is different from $2 : 1$ by asking them to show each ratio using multilink cubes. Some students may find it difficult to identify common factors and may find a multiplication table (**R21**) helpful.

Links

Cancelling fractions: Framework (Y456) Page 23.

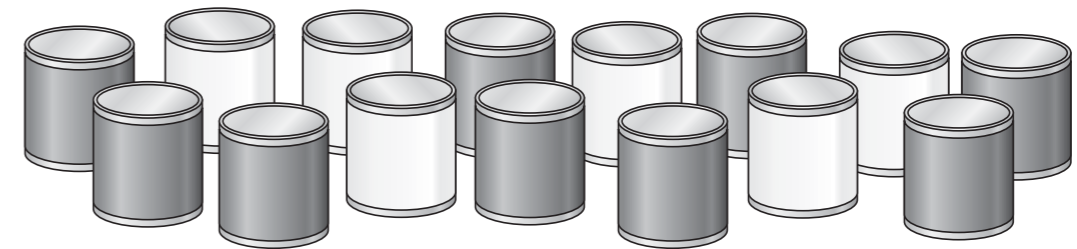
Exercise Answers

- 1 a $2 : 4$ b $2 : 8$ c $3 : 6$ d $3 : 6$ e $9 : 3$
 f $4 : 6$
 2 a $1 : 2$ b $1 : 4$ c $1 : 2$ d $1 : 2$ e $3 : 1$
 f $2 : 3$
 3 a 3 b 2 c 3 d 5 e 1 f 2 g 3 h 5
 4 a $1 : 3$ b $2 : 1$ c $1 : 2$ d $4 : 1$ e $2 : 3$
 f $3 : 2$ g $2 : 3$ h $3 : 4$
 5 a $5 : 7$ b 210
 6 30 gm



black beads	white beads
4	16
1	4

$\div 4$ } } $\div 4$



black	white
9	6
3	2

$\div 3$ } } $\div 3$

N1.8 Dividing using ratio

Access

This spread provides access to the Year 9 objective:
 ▶ Solve problems involving ratio (5, 81).

Lesson Plan

Mental starter

Give students a ratio of pocket money, Said receives £2 for every £1 Kerry receives, say. Ask students to give equivalent ratios. Ask a question based on one of their ratios. For example, if 10 : 5 is suggested, how much each would receive from a total of £15? Encourage students to justify their answers.

Introductory activity

Refer to the mental starter and highlight that in ratio, as well as different sized **parts**, there is a **total** number of parts.

Introduce a problem involving a unitary ratio:

- ▶ £20 is shared between 2 people in the ratio 1 : 3.
How much does each person receive?

Link to the mental starter and highlight that we need an equivalent ratio where the total number of parts is 20.

Discuss strategies, emphasising increasing the ratios systematically and working out the total number of parts each time:

- £1 : £3 £4 in total
- £2 : £6 £8 in total
- £3 : £9 £12 in total
- £4 : £12 £16 in total
- £5 : £15 £20 in total

Repeat for the example of the ratio 2 : 3 in the Students' book. Use diagrams to emphasise the pattern:

- ▶ Blue paint is mixed with yellow in the ratio of 1 : 4 to make green. How many pots of blue and yellow are needed to make 15 pots of green?

Plenary

Work with students to create a spider diagram showing ways of sharing £20 between two people, e.g. £16 and £4. Challenge them to write each as a ratio in its simplest form.

Homework

- ▶ Building on the plenary, challenge students to create a spider diagram showing possible ways of sharing £24 and writing each as a ratio in its simplest form.

Exercise Commentary

Coverage

The questions assess the objectives on Framework (Y456) Page 27.

Useful resources

N1.8WS – provides support for weaker students.

Differentiation

- ▶ Questions 1–3 focus on dividing a total in a given ratio, with diagrams as support.
- ▶ Question 4 focuses on finding an equivalent ratio.
- ▶ Questions 5–7 extend to solving 'real life' problems using ratio.

Support tier: focuses on solving problems involving direct proportion.

Misconceptions

In question 3, students may attempt to divide the number of blue parts by a given ratio (confusing it for the number of total parts). Remind them that the ratio gives the amounts of separate quantities and also the total number of parts. They need to be clear about what information is given and what they need to find out. Encourage students to work systematically and set their working out clearly.

Links

Fractions: Framework (Y456) Page 23; Multiples: Framework (Y456) Page 19.

Exercise Answers

- 1 £8 : £12 2 12 : 6 3 9
 4 No, 25 not divisible by 4 5 12 red 16 black
 6 a 4 : 1 b Sarah 24 Martin 6 7 £9 : £15

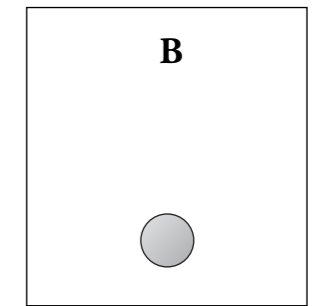
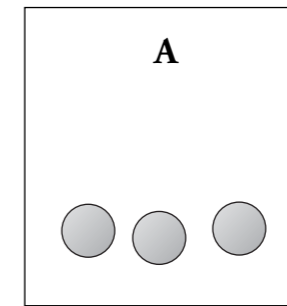
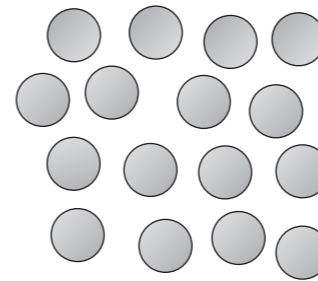
Worksheet Answers

- 1 b 12, 4 2 b 3, 15
 3 £20, 10 : 15 £25, 12 : 18 £30, £12, £18
 4 £20, £4

N1.8WS

Name:

- 1 a Here are 16 coins. Share them into 2 piles in the ratio 3 : 1.



- b Write the number of coins in each pile.

Pile A: ____ coins

Pile B: ____ coins

- 2 a Divide 18 coins into 2 piles in the ratio 1 : 5.
Use this space for your drawing.

- b How many coins are there in each pile? Pile A: ____ coins Pile B: ____ coins

- 3 Divide £30 in the ratio 2 : 3. The answer has been started for you.

2	:	3	→	2 : 3	→	£
			→	2 : 3	→	£5
			→	4 : 6	→	£10
			→	6 : 9	→	£15
			→	8 : 12	→	£ ____
			→	10 : ____	→	£ ____
			→	____ : ____	→	£ ____

£30 shared in the ratio 2 : 3 is £ ____ and £ ____.

- 4 Divide £24 in the ratio 5 : 1.
Use this space for your drawing.

£24 shared in the ratio 5 : 1 is £ ____ : £ ____.

N1.9 Using proportion

Access

This spread provides access to the Year 9 objective:

- Identify when proportional reasoning is required to solve a problem (79).

Lesson Plan

Mental starter

Write '2 packets of crisps cost 72p' and some correct and incorrect equivalences, for example: 1 packet costs 36p; 10 packets costs £3.60; 8 packets cost £2.80, 1.5 packets cost 54p.

Ask students which are correct and encourage them to justify their answers.

Discuss methods for calculating the cost of 1.5 packets – partitioning 1.5, finding a half and adding it to the cost of one.

Introductory activity

Refer to the mental starter. Emphasise that in the correct examples, cost and number of packets were multiplied by the same number.

Write this recipe for 2 sponge cakes:

- 4 eggs; 300g flour; 400g sugar; 100 ml milk.

Discuss how to scale the recipe from 2 to 6 cakes.

To emphasise the link to multiplication:

- Ask how many 'lots of 2 cakes are needed?

Emphasise that the multiplication factor is 3 and work out the new quantities. Highlight that addition would not give the same recipe – an increase of 8 eggs and 8g of flour would not make the same cakes.

Discuss the ingredients needed for one cake and highlight the division factor to reduce quantities proportionally.

Discuss the ingredients needed for 3 cakes.

Emphasise two possible methods:

- Halve the quantities for 6 cakes
- Multiply the quantities for 1 cake by 3.

Plenary

Give students a recipe for omelette for 3 people: 6 eggs, 200 ml milk; 30g butter.

Discuss how to scale the recipe for 4 people.

Emphasise the unitary method:

- Find the quantities for 1 person
- Multiply by 4.

Homework

- Ask students to find a recipe in a book or magazine for 6 people and work out the quantities for 12 people and 3 people. Challenge them to scale the recipe for 9 people and 15 people.

Exercise Commentary

Coverage

The questions assess objectives on Framework (Y456) Page 87.

Useful resources

N1.9WS – provides further practice of the key ideas.

Differentiation

- Question 1 focuses on identifying simple scale factors.
- Questions 2 and 3 focus on increasing two quantities proportionally.
- Question 4 focuses on decreasing two quantities proportionally and using the unitary method.

Support tier: focuses on the order of operations and BIDMAS.

Misconceptions

In question 3, students may have problems multiplying by 1.5. Encourage them to consider 1 part, and half a part and then combine them (using a partition method).

Encourage students to estimate the general size of each part as a checking procedure.

Links

Multiples and factors: Framework (Y456) Page 19.

Exercise Answers

1 a 2 b 3 c 5 d 10

2 8, 800, 120 g

3 a 5 rooms 20 : 30 7 rooms 28 : 42 10 rooms 40 : 60

b 3

4 a 8 b 32 shirts

Worksheet Answers

1 a 200, 50, 250; 4, 1, 100, 25

b 12, 3, 300, 75, 375

2 20, 7; 120, 360, 80, 28

N1.9WS

Name:

- 1 Here is a recipe for bread and butter pudding.

It makes enough for 4 people.

- a Complete this table.

Some parts have been done to help you.

8 slices of bread
2 eggs
200 ml milk
50 g sugar
250 g sultanas

	bread (slices)	eggs	milk (ml)	sugar (g)	sultanas (g)
4 people	8	2			
2 people					125

- b Complete this table for the recipe for 6 people.

	bread (slices)	eggs	milk (ml)	sugar (g)	sultanas (g)
6 people					

- 2 Concrete is made from cement, sand, gravel and water. This mixture makes enough concrete for one quarter ($\frac{1}{4}$) of a shed floor.

Cement 30 kg
Sand 90 kg
Gravel 20 kg
Water 7 litres

Complete the table to work out the mixture to concrete the whole floor.

	cement (kg)	sand (kg)	gravel (kg)	water (litres)
$\frac{1}{4}$ of the floor	30	90		
whole floor				

N1 Proportional reasoning (9 hours)	Teaching objectives	Framework Maths resources	Other resources
Fractions, decimals, percentages, ratio and proportion (22–23)	Identify and understand equivalent fractions.	9A Student Book: N1.1 9A Teacher's Book: N1.1WS, R8	
Fractions, decimals, percentages, ratio and proportion (22–23)	Reduce a fraction to its simplest form by cancelling common factors.	9A Student Book: N1.1 9A Teacher's Book: N1.1WS, R8	
Y7: Fractions, decimals percentages, ratio and proportion (66–67)	Add and subtract fractions with the same denominator.	9A Student Book: N1.2 9A Teacher's Book: N1.2OHP	
Fractions, decimals, percentages, ratio and proportion (22–23)	Order fractions by converting them to fractions with a common denominator and position them on a number line.	9A Student Book: N1.3 9A Teacher's Book: N1.3WS, R6, R21	
Fractions, decimals, percentages, ratio and proportion (24–25)	Use a fraction as an operator to find fractions of number or quantities.	9A Student Book: N1.4 9A Teacher's Book: N1.4WS, R21	
Fractions, decimals, percentages, ratio and proportion (32–33)	Understand percentage as the number of parts in every 100.	9A Student Book: N1.5, N1.6 9A Teacher's Book: N1.5WS, N1.6WS, R4, R24	
Fractions, decimals, percentages, ratio and proportion (32–33)	Find simple percentages of whole-number quantities.	9A Student Book: N1.5 9A Teacher's Book: N1.5WS, R4, R24	
Fractions, decimals, percentages, ratio and proportion (32–33)	Express fractions as percentages.	9A Student Book: N1.6 9A Teacher's Book: N1.6WS, R4	
Y7: Fractions, decimals, percentages, ratio and proportion (74–76)	Calculate simple percentages.	9A Student Book: N1.6 9A Teacher's Book: N1.6WS, R4	
Y7: Fractions, decimals, percentages, ratio and proportion (78–80)	Reduce a ratio to its simplest form by cancelling.	9A Student Book: N1.7 9A Teacher's Book: N1.7OHP, R21	
Fractions, decimals, percentages, ratio and proportion (26–27)	Solve problems involving ratio and proportion.	9A Student Book: N1.7, N1.8, N1.9 9A Teacher's Book: N1.7OHP, N1.8WS, N1.9WS, R21	
Y7: Fractions, decimals, percentages, ratio and proportion (78–80)	Divide a quantity in a given ratio.	9A Student Book: N1.8 9A Teacher's Book: N1.8WS	
Properties of number and number sequences (18–19)	Recognise multiples.	9A Student Book: N1.9 9A Teacher's Book: N1.9WS	
Reasoning and generalising about numbers of shapes (78–79)	Represent problems mathematically	9A Student Book: N1.1–N1.9	