

Practice exam 3

Paper 1

LEVEL3DIFFICULTY/I/LOVE/WRITING/IB/CODES

MATHEMATICAL STUDIES

STANDARD LEVEL

PAPER 1

(whenever you feel like taking it!)

1 hour 30 minutes

This is NOT an official IB examination. It is written by the author and has not been approved in any way by the IB.

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Paper 1 (continued)

1. The equation $y = x^2 + ax + b$ passes through points A (3, 0) and B (11, 0).

- (a) Find the values of a and b . [3 marks]
 (b) Find the gradient of the function at $x = 1$. [3 marks]

Working:

Answers:

- (a)
- (b)

2. Consider the geometric series $36, x, y, \frac{32}{3}, \frac{64}{9}, \dots$ where the common ratio is r .

- (a) Find the value of r . [2 marks]
 (b) Find the values of x and y . [2 marks]
 (c) Calculate the sum of the first 10 terms of this series. [2 marks]

Working:

Answers:

- (a)
- (b)
- (c)

3. A set of numbers is given: 3, 12, 9, 2, 5, 5, a , b , 8, 8. Given that the mode is 5 and the mean is 6, find

- (a) a and b ; [3 marks]
 (b) the interquartile range of the data. [3 marks]

Working:

Answers:

- (a)
- (b)

Paper 1 (continued)

4. The Hagen–Poiseuille equation can describe the rate of blood flow in vessels. R is the rate of flow in cm sec^{-1} , η is the fluid viscosity, L is the length of the vessel, r is the radius of the tube, and p_1 and p_2 are the two pressures at either end of the vessel.

$$R = \left(\frac{r^4}{\eta}\right)\left(\frac{\pi}{8}\right)\left(\frac{p_1 - p_2}{L}\right)$$

In a particular blood vessel, the viscosity η is 1.03 centipoise, the length is 3 cm, the radius is 0.22 cm, and $p_1 - p_2$ is 1400 Pa.

- (a) Write the value of R as displayed on your GDC. [2 marks]
 (b) Write the value of R in the form $a \times 10^k$, where k is an integer and $1 \leq a < 10$. [1 mark]
 (c) Write the value of R rounded to **4 decimal places**. [1 mark]
 (d) Calculate the percentage error between your answers in parts (a) and (c). [2 marks]

Working:**Answers:**

- (a)
- (b)
- (c)
- (d)

5. A legal definition of battery is as follows:

a crime where there is unconsented physical contact with another person, even when the contact is not violent but merely menacing or offensive

If p = unconsented physical contact was made, q = contact was menacing or offensive and r = the person is guilty of battery, then

- (a) write the following **in symbolic form**:

If unconsented physical contact was made and the contact was menacing or offensive, then the person is guilty of battery. [2 marks]

- (b) write the following **in words**: $(p \vee q) \Leftrightarrow r$; [2 marks]

- (c) complete the truth table for the logical statement in part (b). [2 marks]

p	q	r	$(p \vee q)$	$(p \vee q) \Leftrightarrow r$
T	F	F		
F	F	T		

Working:**Answers:**

- (a)
- (b)

Paper 1 (continued)

6. Two functions $f(x)$ and $g(x)$ are defined as follows:

$$f(x) = \frac{\sqrt{x-4}}{x} \quad g(x) = 0.03x$$

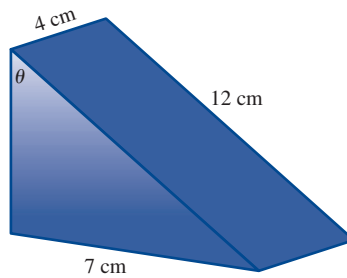
- (a) Write the domain and range of f . [2 marks]
- (b) Use your GDC to find the coordinates of the local maximum of f . [2 marks]
- (c) Find all points where $f(x) = g(x)$. [2 marks]

Working:

Answers:

- (a)
- (b)
- (c)

7. A right-angled prism is shown in the diagram below.



- (a) Find the measure of the angle θ . [2 marks]
- (b) Calculate the surface area of the prism. [2 marks]
- (c) Calculate the volume of the prism. [2 marks]

Working:

Answers:

- (a)
- (b)
- (c)

Paper 1 (continued)

8. Two functions are given: $f(x) = x^2 + x - 2$ and $g(x) = 12x + 1$.

- (a) Factorise $f(x)$. [2 marks]
 (b) Change the function $g(x)$ so that both functions pass through the point $(0, -2)$. [2 marks]
 (c) Write the equation of the axis of symmetry for $f(x)$. [2 marks]

Working:

Answers:

- (a)
 (b)
 (c)

	less than 12 ml of water	between 12 ml and 15 ml of water	between 15 ml and 20 ml of water	more than 20 ml of water
low CO ₂	8	10	18	7
average CO ₂	3	20	16	15
high CO ₂	12	16	6	20

9. A chi-squared independence test is performed on data collected during a series of chemistry experiments. The null hypothesis is that the quantity of the carbon dioxide (CO₂) given off is independent of the volume of water in the beaker. Here are the data:

- (a) Calculate the degrees of freedom of this test. [1 mark]
 (b) Calculate the expected value for low CO₂ and more than 20 ml of water. [2 marks]
 (c) If the p value of this test is 0.0128, determine whether or not to accept the null hypothesis at a 99% level of confidence. Explain your reasoning. [2 marks]
 (d) Give one reason why this test is not valid based on the information given. [1 mark]

Working:

Answers:

- (a)
 (b)
 (c)
 (d)

Paper 1 (continued)

10. At Irene Bank, they are offering a simple interest savings account at 6.2% per annum. Leon decides to open an account at Irene Bank with an initial deposit of €1300.00.
- (a) Write out the balance of Leon's account after 1, 2, 3 and 4 years. [2 marks]
 - (b) Write a formula for B , the balance in Leon's account, after k years. [2 marks]
 - (c) If Leon opened his account in 2009, find the first year in which Leon's account *exceeds* 2000.00. [2 marks]

Working:

Answers:

- (a)
- (b)
- (c)

11. Genzerium, a cool but very hazardous radioactive element, gives off gamma radiation as a function of time. Its half-life, the amount of time it takes for the element to reduce to one-half its mass, is 20 minutes.
- (a) If A represents the amount of genzerium in grams and t represents time in minutes, write a function of A in terms of t . [3 marks]
 - (b) A 400 gram sample of genzerium is obtained via covert means and needs to be delivered to a foreign nation for future study. If the foreign nation needs at least 50 grams, calculate the maximum time that this covert operation can take. [3 marks]

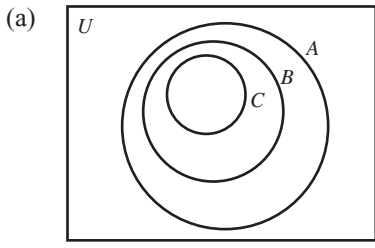
Working:

Answers:

- (a)
- (b)

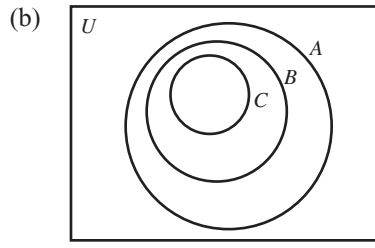
Paper 1 (continued)

12. Shade the following sets on the given Venn diagrams.



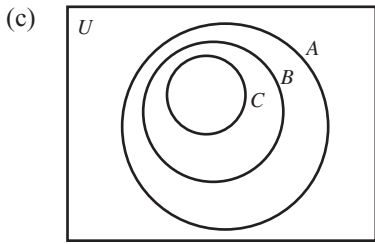
B'

[2 marks]



$(A \cap C) \cup B$

[2 marks]



$(C' \cup A)'$

[2 marks]

Working:

13. An equation of a line l is written as $4x - 7y - 14 = 0$.

- (a) Find the equation of the line m parallel to l that goes through the point $(1, 2)$. Write the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3 marks]
- (b) Find the distance between the x -intercept and the y -intercept of l . [3 marks]

Working:

Answers:

- (a)
- (b)

Paper 1 (continued)

14. The cost C in British Pounds (£) to produce x portable music players can be modelled using the function $C(x) = 3400 + 60x$. The factory decides to sell each music player for £70.
- (a) If P represents the profit of the factory for producing x music players, write an expression for P in terms of x . [3 marks]
- (b) Calculate the number of music players the factory must produce in order to make a profit (P must be greater than zero). [3 marks]

Working:

Answers:

- (a)
- (b)

15. Jochem has fallen in love with Zoë and in order to earn her affection, he delivers roses to her house every day. Zoë dutifully calls Jochem every time to thank him for the beautiful roses and Jochem records the duration of each phone call. He wants to see if the duration of the time Zoë talks to him is dependent on the number of roses delivered.

Number of roses n	8	12	9	6	17	13	4	20	11	10
Duration of call t (in min)	20	32	22	17	40	34	8	43	26	23

- (a) Use your GDC to find
- the mean number of roses and the mean duration of phone calls ;
 - the standard deviation of the number of roses and the standard deviation of the duration of phone calls. [2 marks]
- (b) If the correlation coefficient r is 0.978 18, find the covariance S_{xy} . [2 marks]
- (c) Describe the relationship between the number of roses and the duration of the phone calls. [2 marks]

Working:

Answers:

- (a) (i)
- (ii)
- (b)
- (c)

Paper 2

LEVEL3DIFFICULTY/I/LOVE/WRITING/IB/CODES

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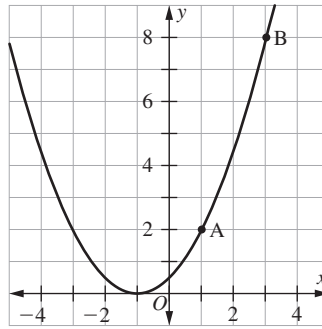
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Paper 2 (continued)

1. [Maximum mark: 20]

It is desired to find the gradient of the tangent line of the function $f(x) = \frac{1}{2}(x + 1)^2$ at the point A with coordinates (1, 2).



(a) If the point B has coordinates (3, 8), find the gradient of the line passing through points A and B. [2 marks]

Point B is now an arbitrary point on f .

(b) If the point B has coordinates $(x, f(x))$, **copy and complete** the table below: [5 marks]

x	2	1.5	1.1	1.01	1.001	1.0001
$f(x)$						
gradient of line \overline{AB}						

(c) From the table, deduce the gradient of the tangent line of the function $f(x) = \frac{1}{2}(x + 1)^2$ at point A. Explain your reasoning. [2 marks]

(d) Rewrite the expression $\frac{1}{2}(x + 1)^2$ in the form $ax^2 + bx + c$ where $a, b, c \in \mathbb{Q}$. [3 marks]

(e) Find $f'(x)$. [3 marks]

(f) **Hence or otherwise** find $f'(1)$. [1 mark]

(g) **Using your result from part (e)**, find the coordinates of the point on f such that the point is a local minimum. [4 marks]

2. [Maximum mark: 17]

The times of the sunrises of Alex's city are modelled by the function

$$h(x) = -2\cos(30x) + 7$$

where the x -axis represents months ($x = 0$ is January 1), and the y -axis represents the time of the sunset ($y = 0$ is 00:00 or midnight).

(a) With help of a GDC, draw a graph of the function $h(x)$ on graph paper, with 1 cm representing 2 months on the horizontal axis, and 2 cm representing 1 hour on the vertical axis. [5 marks]

(b) The function $h(x)$ is a periodic function. Find

(i) the period ;

(ii) the amplitude ;

(iii) $h(8)$;

(iv) one possible value of x when $h = 6$. [6 marks]

(c) Determine, with the use of a GDC, the approximate dates when sunrise is 08:30. [3 marks]

(d) If Alex moves to another city where its sunrises are modelled by the function $g(x) = 3 \sin(30x) + 6$, find the first date of the year when the two cities have the same sunrise time. [3 marks]

3. [Maximum mark: 18]

All answers in this question should be rounded to two decimal places.

Tanikka and Jacobus are headed off to university in two different locations: Tanikka in Antigua and Jacobus to South Africa. The unit of currency in Antigua is the East Caribbean dollar (XCD) and the unit of currency in South Africa is the rand (ZAR). The exchange rates for both currencies against the US Dollar (USD) are shown below:

$$1.0000 \text{ USD} = 2.7000 \text{ XCD} = 7.7397 \text{ ZAR}$$

Paper 2 (continued)

- (a) If both Tanikka and Jacobus go to their respective countries with 700 USD for spending money, calculate the total local currency equivalent for
- Tanikka in Antigua ;
 - Jacobus in South Africa. [2 marks]
- (b) The bank that Tanikka used charged her a flat fee of 20 XCD plus a variable fee of 2% of the value of the transaction. Calculate the total amount of East Caribbean dollars that Tanikka received from her bank. [3 marks]
- (c) The bank that Jacobus used charged him either a flat fee of 30 ZAR or a variable fee of 3% of the value of the transaction, whichever was higher. Calculate the total amount of South African rand that Jacobus received from his bank. [3 marks]
- (d) After converting their money into local currencies, they deposited the money into savings accounts. Tanikka's account paid her 2.5% per annum, compounded monthly. Jacobus's account paid him 2.2% per annum, compounded daily. Calculate the **interest earned in local currency** after six months for
- Tanikka's account ;
 - Jacobus's account. [6 marks]
- (e) After six months, Tanikka and Jacobus decide to meet up in London for a holiday. They both empty their savings accounts after exactly six months and fly to London. The exchange bureau in London Heathrow airport is offering the following rates for British Pounds (GBP):

	BUY	SELL
South African Rand (ZAR)	0.0702	0.0715
East Caribbean Dollars (XCD)	0.1992	0.2104

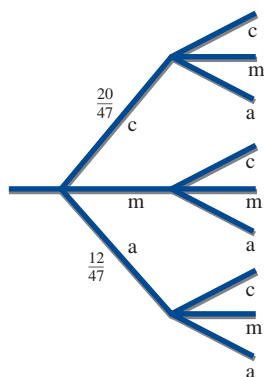
For example, 1 South African Rand (ZAR) will buy 0.0702 British Pounds (GBP).

Calculate the total amount of money that Tanikka and Jacobus have after converting all of their money into GBP. [4 marks]

4. [Maximum mark: 17]

During the holiday of Halloween, children go from house to house and receive candy. At one such house, children close their eyes, reach into a bowl and are allowed to pick two candies. In the bowl there are 20 chocolate candies (c), 15 mint candies (m) and 12 apple candies (a).

- (a) **Copy and complete** the probability tree diagram below.



[5 marks]

- (b) Calculate the probability of drawing
- two chocolate candies ;
 - a chocolate candy and a mint candy ;
 - anything but apple candies ;
 - two chocolate candies, given that neither candy is apple. [10 marks]
- (c) Katie cheats by looking into the bowl while picking the first candy. She takes a chocolate candy and picks the next candy at random. Calculate the probability that she picked two chocolate candies. [2 marks]

Paper 2 (continued)

5. [Maximum mark: 18]

The internationally recognised dimensions of the field for the track and field event called the hammer throw are shown on the right and given below:

- BC = 54.00 metres
- AB = AC = 90.00 metres
- DE = 12.00 metres

It is desired to calculate other measurements of the field to ensure that the field is drawn properly prior to competition.

- (a) Use the cosine rule to calculate the measure of angle BAC. [3 marks]
- (b) Deduce the measures of angles
 - (i) BCA ;
 - (ii) EDA. [3 marks]
- (c) Use the sine rule to calculate the length of AD. [3 marks]
- (d) Calculate the area of triangle ABC. [3 marks]
- (e) Show that the following property is true:

$$\frac{\text{area of triangle ABC}}{\text{area of triangle ADE}} = \left(\frac{\text{length of BC}}{\text{length of DE}} \right)^2 \quad [4 \text{ marks}]$$

- (f) If a hammer is thrown from A directly toward the midpoint of BC at a horizontal speed of 67 km hr⁻¹ and lands 80 metres from A, calculate the number of **seconds** that the hammer was in the air. [5 marks]

