A1 Integers and decimals

Check in

1 −8, −5, −3, −1, 2, 4

2 **a** 7 **b** −10 **c** −12 **d** −3

3 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Ex A1.1

A1.1 Rich task

Correct, incorrect, correct

1 **a** one thousand three hundred and seven

b twenty nine thousand and six

c three hundred thousand

d six hundred and five thousand and thirty

2 **a** 8043 **b** 70 000 000 **c** 200 051 **d** 2010

3 **a** 8000 **b** 40 **c** 70

4 **a** largest – 95, smallest – 59

i largest – smallest = 36 **ii** 9

iii It is always 9

b largest: 821, smallest: 128

i largest – smallest = 693 **ii** Always get 495

iii Students will find the number is always divisible by 99. This can be proven algebraically:

Using a place value table:

100s	10s	1s
x	y	z

$(100x + 10y + z) - (100z + 10y + x) = 99x - 99z$
$= 99(x - z)$

Students may find something similar using numbers rather than algebra.

Ex A1.2

1 **a** 1 **b** −2 **c** −4 **d** 1

e −12 **f** −15 **g** −9 **h** −10

i 7 **j** 15 **k** −19 **l** −12

m −8 **n** −5 **o** 5 **p** 8

2 **a** −12 **b** −35 **c** −32 **d** −27 **e** 20

f 36 **g** 20 **h** 21 **i** −9 **j** −3

k −2 **l** −3 **m** 3 **n** 4 **o** 8

p 9 **q** 77 **r** 12 **s** −5

3 **a** 16, 23, 7; 23, 22, 1 **b** 8°C

4 **a** 36, −24 **b** $4 \times 3 + 6 \times -2 = 0$

c eg. Answer 4, all correct; Answer 9, 6 correct.

Ex A1.3

A1.3 Rich task

Yes, they would both be 180° because 350° F= 176.7°C is 180° to the nearest 10, and 360°F = 182.2°C is also 180° to the nearest 10.

1 **a** 630 m **b** 220 m **c** 200 m **d** 200 m

2 **a** 7700 g **b** 5500 g **c** 3100 g **d** 2600 g

3 **a** 40 730, 40 700 **b** 310, 300

c 590, 600 **d** 150 900, 150 900

4 302 km 254 km

5 **a** 19.37 **b** 19.4 **c** 0.00 752 **d** 0.008

e 153.262 **f** 153.26 **g** 34.16 **h** 4.90

6 **a** 76 000 **b** 76 300 **c** 23 400 **d** 58 000

e 46 000 **f** 23.40 **g** 23.4 **h** 7.184

7 **a** 165 m to 175 m **b** 50, 51, 52, 53, 54 kg

Ex A1.4

1 **a** 61 **b** 23.83 **c** 1.5625 **d** 0.35

e 1.33 **f** 156.25 **g** 500 **h** 7.5

2 **a** 1.736 307 692 1.7 **b** −1.693 877 551 −1.69

c −0.191 345 436 −0.19 **d** 12.68 283 582 12.7

e 3.839 270 764 3.84 **f** 2.370 021 097 2.4

3 **a** 0.25 **b** $0.\dot{1}$ **c** 0.4 **d** $0.2\dot{7}$ **e** 5

f 6.25 **g** 2 **h** 4 **i** 2.5

4 **a** 1296 **b** 78125 **c** 100000

d 262144 **e** 410.0625 **f** 0.004096

5 This is an investigative task and students' answers will vary, it's an opportunity to practice QWC skills so students should be encouraged to write clear instructions both in terms of language used and mathematical notation used.

Exam practice

1 **a** 2.71828 **b** 2.72

2 **a** 1.716961498 **b** 2

3 Missing numbers are bottom row −1 middle row −6, 2

A2 Summary statistics

Check in

1 **a** 44, 47, 48, 55, 56, 59, 61, 65

b 0.5, 0.8, 1.5, 1.7, 2.1, 2.5

c 13.1, 13.2, 21.3, 23.1, 31.2, 32.1

2 **a** 81 **b** 179 **c** 105 **d** 205 **e** 441
f 108 **g** 135 **h** 177 **i** 219 **j** 41

Ex A2.1

1 **a** **i** 1 **ii** 3 **b** **i** 5 **ii** 10
d No. 18 cells, only 16 poems

3 **a** 11 **b** % of Welsh with AB for example

Ex A2.2

1 **a** **i** cont. **ii** disc **b** **i** cont. **ii** disc
c **i** disc **ii** cont **d** **i** disc **ii** cont

2 **a** secondary **b** a survey, primary

Ex A2.3

1 **a** 75 **b** 63 **c** 60.1 **d** 63

2 **a** 71 **b** bimodal **c** 40.4 **d** 37

3 **a** £26 **b** bimodal **c** £84.20 **d** £87.00

4 **a** **i** £72 000 **ii** £27 000 **iii** £46 900 **iv** £34 000
b lowest price

5 **Rich task**

This is an investigative task and students can experiment with different spreads of data and their averages. They should be encouraged to consider whether different averages are representative by choosing data with a large spread, a small spread. It's also an opportunity to practice QWC.

Ex A2.4

1 **a**

4	5 8
5	1 1 2 4 9 9
6	1 2 3 6 9
7	0 1 4 5 7 8
8	1 2 9
9	3

7 | 4 means 74%

b **i** 48% **ii** 66%

2 **a**

2	6 8 9
3	0 3 7 8 9
4	0 1 3 3 4 7 9
5	2 5 8 9
6	0 1 3 6 7 9
7	1 3 5 7
8	0 4

5 | 8 means 58 mins

b **i** 58 mins **ii** 52 mins

3 **a**

0	3 5 6 7 8 9
1	0 1 2 2 5 6 7 8 9
2	0 1 2 4 6 7 7 9 9
3	1 2 3 4 5 7
4	0 1

1 | 7 means 17 mins

b **i** 38 mins **ii** 20.5 mins

4 **a**

3	2 6 7 9
4	2 5 7 8 9
5	1 2 4 6 6 6 7 8 9
6	0 1 3 6
7	0

4 | 5 means 45 kg

b **i** 38 kg **ii** 54 kg

5 **a**

14	6 7 9
15	0 0 2 2 3 4 5 5 7 8 9
16	0 2 3 5 7 8
17	1 2 2

16 | 0 means 160 cm

b **i** 26 cm **ii** 157 cm

6 **a**

8	8 9
9	1 2 4 8
10	1 3 4 5 6 7 8 8 9
11	0 2 4 6 6 7 7 8 9
12	1 5 6 7 9
13	1 3

11 | 2 means IQ score of 112

b **i** 45 **ii** 110

7 **a**

0	3 6 8 9 9
1	0 1 2 5 6 7 8 9
2	1 1 2 3 3 5 7
3	0 2 3

2 | 3 means 23 mins

b **i** 30 mins **ii** 18 mins

A2.5 Rich task

Mike is correct. There is a smaller range in the girls' data. Students may observe that the boys' data is grouped around the higher values on the diagram showing that they tend to spend more time on the internet. This is an opportunity for students to practice QWC in their explanation.

Ex A2.5

1 Boys' range is 4.0 secs, girls' is 4.2 secs so girls' is larger. Median for boys is 5.3 and for girls is 6.3, so on average girls have longer reaction times.

2 **a** Puzzle P Puzzle Z

		9	8	8	6	0	8	9		
9	8	7	7	4	3	2	1	0	1	1 2 4 5 6 7 7 8 9
		9	7	4	2	2	1	3	4	5 8 9
				7	1	3	2	3		

1 | 3 | 2 means puzzle P 31, puzzle Z 32 minute

b **i** P 31, Z 25 **ii** P 2 modes, Z 17

iii P 17.5, Z 19.5 **iv** P 17, Z 18

c not much difference, but range for Z is larger and Z is more difficult because takes more time on average.

3 **a** class X Class Y

	8	7
9 8 6	9	3 7 9
9 8 6 5 5 4 3	10	2 3 4 6 7 7 9
8 8 7 7 6 5 5 2 0 0	11	3 4 4 4 6 8 8 8 9 9
9 6 4 3 3 2 1	12	1 1 4 6 6 8 9
4 1 0	13	0 1 2

3 | 10 | 2 means 103 for X, 102 for Y

b X and Y have same range, 45, median for Y is 116 and for X is 115 so Y have slightly higher IQ.

A2.6 Rich task

If students try calculating summary statistics they may find that the mean and median are very similar. They could use this to support Thao's argument that the recall of both groups is the same.

To argue that 15 yr olds can recall more objects than 11 yr olds, students might argue that the data for 15 yr olds is spread over higher values than the data for 11yr olds. Also the mean is higher and the total number of objects recalled is higher.

Ex A2.6

1 **a** **i** 6 **ii** 6 **iii** 5.8 **iv** 4

b **i** 5 **ii** 5 **iii** 5 **iv** 4

2 **a** and **b** Mean greater for boys (16.2, 16.1) Mode greater for boys (18, 15) Median 16 for both; no evidence girls have better recall.

Ex A2.7

1 **a** $10 < B \leq 20$ **b** £24.20 **c** uses all data

d 11, 45; 12, 46; 13, 47; 14, 48; 15, 49; 16, 50

2 **a** £88.00 **b** $0 < M \leq 40$

c Min value ≤ £40; Max value = £250; Yes, Range 200 – 40 = 160

3 Modal value and median same for both groups; Mean = 11.5 km for teachers, 14.1 km for office workers; conclusion supported.

A2.8 Rich task

This should be a class/student discussion and answers may vary.

Ex A2.8

1 **a** eg. Assumes show is liked

b eg. boxes don't cover all answers, assumes people listen, "per day" not specified.

2 **Rich task**

Students' answers will vary. Attention should be paid to whether they are using open or closed questions, how they are constructing the answer choice boxes, whether their questions are biased or unbiased and so on.

Exam practice

1 **a** Charlie **b** Charlie leaves, Ellen joins

2 **a** 4.13 mins **b** $4 \leq m < 5$

3

	Mean	Median	Range
Girls	64.7	62	46
Boys	64	66	44

Not a great deal of difference – boys did slightly better

A3 Constructions

Check in

1 **a** 45° **b** 135°

2 Check construction

3 **a** 65.5 mm **b** 6.55 cm

Ex A3.1

1 168.2 units

2 **a** 23 cm **b** 38 cm **c** 46 cm

d 162 kg **e** 177 kg **f** 189 kg

g 640 g **h** 780 g **i** 850 g

j 10.4°C **k** 11.9°C **l** 12.6°C

m 3.26 l **n** 3.38 l **o** 3.42 l

p $7\frac{1}{4}$ oz **q** $8\frac{3}{4}$ oz **r** $9\frac{1}{2}$ oz

3 **a** 07 : 10, 19 : 10 **b** 09.35, 21.35

4 17 : 37

5 **a** 1.25 kg **b** 2.65 m **c** 63.5°C

d 36.5 t **e** 39.5 t **f** 135 volts

g 155 volts **h** 0.605 l **i** 0.685 l

A3.2 Discussion

This is a group or class discussion and students answers may vary, encourage them to think about different contexts in which we use measures.

Ex A3.2

1 **a** **i** 6 m **ii** 8 m **iii** 14 m

b **i** 18 feet **ii** 24 feet **iii** 42 feet

2 **a** **i** 8 m **ii** 6 m **iii** 8 m **iv** 20 m

b **i** 20 feet **ii** 15 feet **iii** 20 feet **iv** 50 feet

3 3 km

4 1.189 m by 0.841 m

5 **a** **i** 60 cm **ii** 24 km **iii** 2.5 kg

iv 27 L **v** 2 L

b **i** 3 feet **ii** 25 miles **iii** 18 lbs
iv 36 pints **v** 4 pints

6 196 lbs and 169 lbs, 27 lbs lost

7 **Research**
This is an opportunity to practice QWC and look at different systems of measurement.

Ex A3.3

1 AB = 11 km YZ = 12.1 km

2 $a = 70°$ $x = 5.0$ cm $y = 5.5$ cm
$b = 81°$ $c = 61°$ $z = 4.5$ cm

3 AB = 6.6 km BC = 7.3 km

4 7.3 m, 62°

5 2 km

A3.4 Rich task

The answer is 36 m, students should be encouraged to practice accurate scale drawing but may also spot the shortcut that it's the side length of the equilateral triangle multiplied by three. Both approaches are equally valid but it should be emphasised that this is an opportunity to develop the ability to draw accurate constructions to scales.

Ex A3.4

1 **b** W 41° 56° 83°
X 119° 32° 29°
Y 37° 53° 90°
Z 44° 68° 68°

c Y is right-angled
Z is isosceles

2 39°

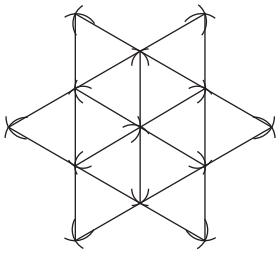
3 1.2 m

4 **d** triangular based pyramid (tetrahedron)

5 **c** parallelogram **d** 29° 29° they are the same

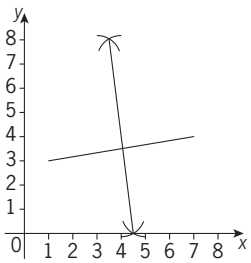
6 Students' own answers, paying attention to QWC.

7 **Challenge**
Students should be encouraged to think about how within a 9 cm equilateral triangle, there are 9 smaller triangles, each of side length 3 cm. They can use this fact to construct a central hexagon of 6 equilateral triangles with a further 6 attached to the outside edges of the hexagon. Once this has been drawn, the construction lines inside the triangle can be erased to show two overlapping equilateral triangles constructing the Star of David.

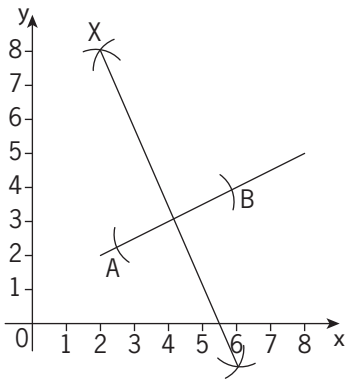


Ex A3.5

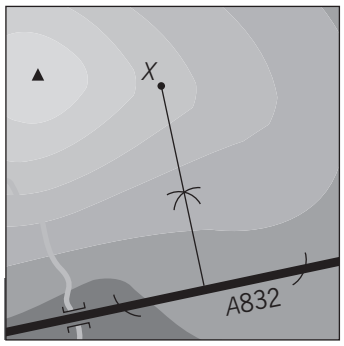
1 **b**



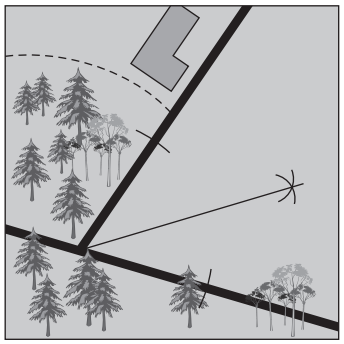
3 **b**



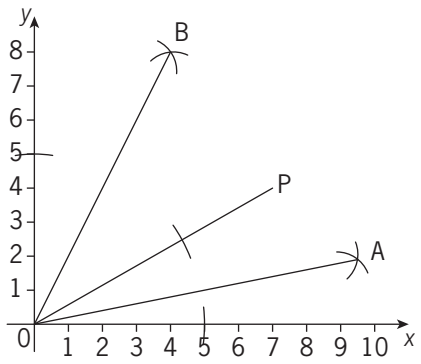
4 **a**



b



5 **a, b**



c 45° This is half of the angle between the x and y axes

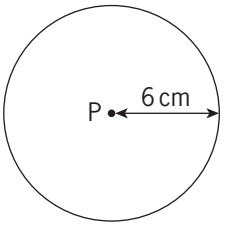
6 5.7 m

A3.6 Rich task

Construct a perpendicular bisector of the line PQ.

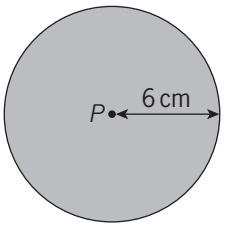
Ex A3.6

1 a



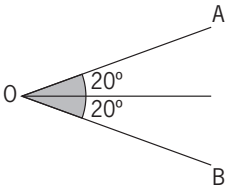
the circle

b



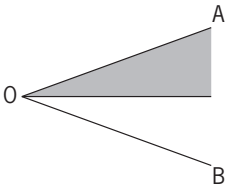
the area inside the circle

c



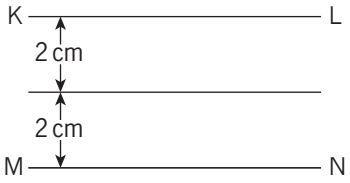
the line bisecting the angle AOB

d



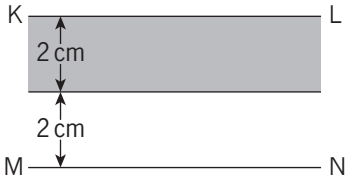
the area between the line OA and the bisector

e



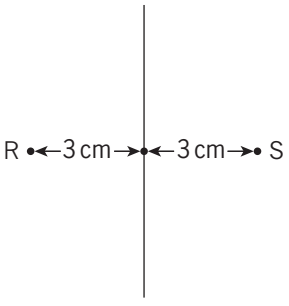
the line parallel to KL and MN and midway between them

f



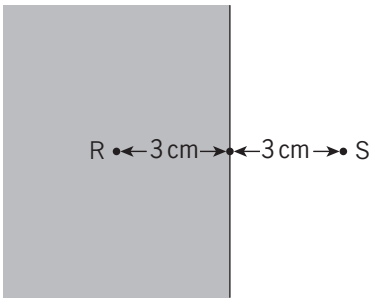
the area between the line KL and the mid-line

g



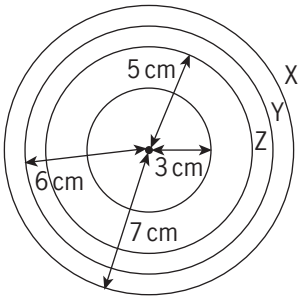
the perpendicular bisector of the line RS

h



the area on the side of the perpendicular bisector that contains R

2



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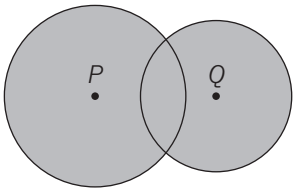

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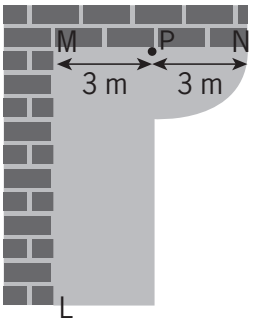

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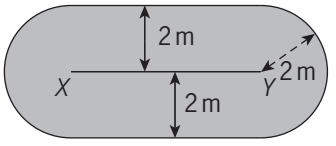
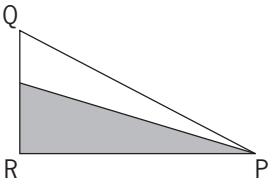
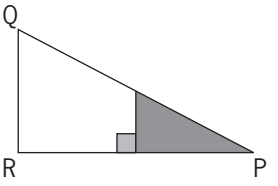


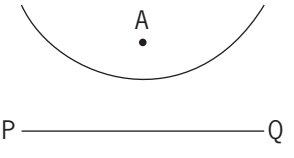
6 a



b



7



8 Challenge

This is an investigative activity, students are encouraged to think about real life examples of loci. They can use the internet to find animations which will help them to visualise loci more easily.
A hypocycloid is a curve generated by the trace of a fixed point on a small circle that moves around a larger circle.

Exam practice

1 3.4 cm, 42°, 88°

2 44 lbs

Case study 1

a Results given to 0.01 s

b Reaction times given to 0.001 s

a Range = 0.76 s

b Median = 9.93 s

c Mean = 9.92 s

a Men: Thompson 0.019 above limit

b Women: Ferguson-McKenzie 0.03 above limit

Range for reaction times:

a Men = 0.046 s

b Women = 0.043 s

Men altered times:

Name	Nationality	Time (s)	Reaction (s)	Time with 0.1 s reaction
Bailey	ANT	9.93	0.129	9.90
Bolt	JAM	9.58	0.146	9.53
Burns	TRI	10.00	0.165	9.94
Chambers	GBR	10.00	0.123	9.98
Gay	USA	9.71	0.144	9.67
Patton	USA	10.34	0.149	10.29
Powell	JAM	9.84	0.134	9.81
Thompson	TRI	9.93	0.119	9.91

Name	Nationality	Time (s)	Reaction (s)	Time with 0.1 s reaction
Fraser	JAM	10.73	0.146	10.68
Stewart	JAM	10.75	0.170	10.68
Jeter	USA	10.90	0.160	10.84
Campbell-Brown	JAM	10.95	0.135	10.92
Williams	USA	11.01	0.158	10.95
Ferguson-McKenzie	BAH	11.05	0.130	11.02
Sturrup	BAH	11.05	0.137	11.01
Bailey	JAM	11.16	0.173	11.09

A4 Factors, multiples and ratio

Check in

1 240 miles **2** 15 miles

A4.1 Activity

Students play the game and try to answers the questions as a result. They may need some specific guidance on some aspects.

Ex A4.1

1 a 3, 4 **b** 5, 25 **c** 3, 21, 24, 30
d 21, 28 **e** 4, 25 **f** 3, 5, 17, 23

2 a 1, 3, 5, 15 **b** 1, 2, 3, 4, 6, 12 **c** 1, 2, 11, 22
d 1, 5, 25 **e** 1, 2, 4, 5, 8, 10, 20, 40

3 55

4 42, 48

5 31, 37

6 a i 1, 2, 11, 22 **ii** 1, 2, 3, 6, 11, 22, 33, 66
b 22 is factor of 66

7 a 1, 3 **b** 1, 2, 3, 6 **c** 1, 7 **d** 1, 2, 3, 6

8 a 4, 8, 12 **b** 2, 4, 6, 8, 10, 12
c 4 is multiple of 2

9 a eg 12, 24 **b** eg 30, 60 **c** eg 21, 42 **d** eg 18, 36

10 a eg 24 **b** eg 1, 4, 16

A4.2 Activity

Students investigate prime factors by playing a game.

Ex A4.2

1 a $2^4 \times 3$ **b** 3^3 **c** 2×3^3 **d** $2^3 \times 3 \times 5$
e $2^3 \times 3^2$ **f** $2 \times 3 \times 7^2$ **g** 7×53

2 a 504, 4 **b** 3960, 3 **c** 3384, 2
d 196, 7 **e** 390, 5

3 a 12 **b** 48

4 The 3 factors are 1, prime number and the number itself

A4.4 Rich task

The ratio tends to 1:1.61803 – the golden ratio.

Ex A4.3

1 a 1 : 3 **b** 2 : 3 **c** 2 : 3 **d** 5 : 8 **e** 3 : 7

2 a 1 : 2.5 **b** 1 : 1.8 **c** 1 : 3.8 **d** 3.3 : 1 **e** 2.5 : 1

3 a £4.95 **b** £24.30 **c** £25.08 **d** £23.45

4 Students play a game to investigate ratios. Their answer will vary. It's an opportunity to practice QWC where they have to comment and explain.

A4.4 Rich task

Fred is 2, Marvin is 6.

Fred will be 8, Marvin will be 12.

Ex A4.4

1 **a** £8 : £40 **b** £56 : £8 **c** £24 : £84
d £56 : £40 **e** £60 : £120 : £240 **f** £75 : £150 : £200

2 £45.30

3 £8.40

4 93.75

5 137.5

6 680 g

7 **a** 6 : 5 **b** 7 : 6 **c** 6 and 3 years old

Exam practice

1 **a** **i** 24 **ii** 25 **iii** 23 **b** $2^2 \times 7$

2 45

3 **a** $\frac{3}{5}$ **b** 5 : 3

A5 Sequences

Check in

1 **a** 3 **b** 6 **c** 5 **d** 6

2 **a** 3 **b** 4 **c** 5 **d** 4

3 **a** 4, 8, 12, 16, 20, 24 **b** 3, 6, 9, 12, 15, 18
c 5, 10, 15, 20, 25, 30 **d** 6, 12, 18, 24, 30, 36

A5.1 Rich task

Students will find answers by investigating using numbers which correspond to 2^n where n is the number of hours that have elapsed.
They can then use the internet to investigate how different types of bacteria grows.

Ex A5.1

1 **a** 80, 75, 70, 65, 60, 55, 50 **b** 16

2 **a** 5, 9, 13, 17 **b** 21, 25, 29 Add 4 each time **c** 81

3 **a**

No. of columns	1	2	3	4	5
No. of tiles used	2	5	8	11	14

b 59. Number of tiles = Treble the number of columns and subtract 1

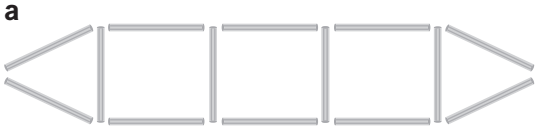
4 **a** 18, 20, 22. 48 **b** 20, 23, 26. 62

A5.2 Rich task

Aya needs 7 rods for the first section, 4 rods for the next section. This is a total of 11 rods. The term-to-term rule is start with 7 and add 4 each time.

Ex A5.2

1 **a**



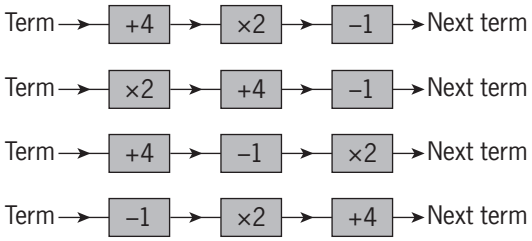
5, 8, 11, 14

b Start with 5 and add 3 each time. 17, 20, 23

2 **a** 3, 8, 18, 38, 78 **b** 1, 6, 21, 66, 201
c $\frac{1}{4}$, $1\frac{1}{2}$, 4, 9, 19 **d** 20, 17, 14, 11, 8
e 600, 280, 120, 40, 0 **f** 4, 4, 4, 4, 4

3 **a** 2, 44, 128, 296 **b** 50, 140, 320, 680
c −30, −20, 0, 40

4



5 **a** Start with 8 and add 3 each time. 20, 23, 26

b Start with 30 and subtract 3 each time. 18, 15, 12

c Start with 1, then double and add 1 each time. 31, 63, 127

d Start with 3, then double and subtract 1 each time. 33, 65, 129

e Start with 1, then treble and add 1 each time. 121, 364, 1093

f Start with 10, then subtract 3 each time. −2, −5, −8

g Start with 200, then halve each time. 12.5, 6.25, 3.125

h Start with $\frac{1}{2}$, then double and add 2 each time. 38, 78, 158

i Start with −7 and add 3 each time. 5, 8, 11

6 5, 11, 23, 47, 95

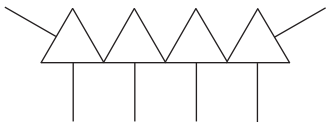
A5.3 Rich task

Students look at Hayley's method for finding a position to term rule. The values in the table for the 4th position are 12 and 11.
Hayley is correct in saying the 100th position will use 299 tiles.

Ex A5.3

1 **a** 4

b



c Start with 6 and add 4 each time

d

Position	1	2	3	4	5
4 × table	4	8	12	16	20
Term	6	10	14	18	22

Multiply position by 4 and add 2

e 202

2 **a** **ii** Multiply the position by 4 and subtract 1.
iii 23, 27, 399

b **ii** Multiply the position by 3 and add 1.
iii 19, 22, 301

c **ii** Multiply the position by 2 and subtract 1.

iii 11, 13, 199

d **ii** Multiply the position by 4 and add 1.

iii 25, 29, 401

e **ii** Multiply the position by 5 and subtract 2.

iii 28, 33, 498

f **ii** Multiply the position by 6 and add 1.

iii 37, 43, 601

g **ii** Multiply the position by $\frac{1}{2}$ and add 3.

iii 6, $6\frac{1}{2}$, 53

h **ii** Multiply the position by −2 and add 8.

iii −4, −6, −192

3 **a** 19, 37, 301 **b** 5, $9\frac{1}{2}$, 77

4 **a** 00139 **b** 00243

5 Multiply the position by 3 and add 12, 102 mm

6 Because it turns 6 revolutions every minute.

Multiply the position by 6 and add 325, 685

A5.4 Rich task

$3n + 1$

Ex A5.4

1 **a** 4

b

Position	1	2	3	4	5	.....	n
4 × table	4	8	12	16	20		$4n$
Term	3	7	11	15	19		$4n-1$

c $4n - 1$ **d** 23, 199

2 **a** $2n + 1$, 201 **b** $3n + 1$, 301

c $4n$, 400 **d** $4n + 1$, 401

3 **a** Multiply the position by 2 and add 5.

$T(n) = 2n + 5$, 17, 85

b Multiply the position by 3 and subtract 1.

$T(n) = 3n - 1$, 17, 119

c Multiply the position by 1 and add 4.

$T(n) = n + 4$, 10, 44

d Multiply the position by 5 and subtract 2.

$T(n) = 5n - 2$, 28, 198

e Multiply the position by 6 and add 1.

$T(n) = 6n + 1$, 37, 241

f Multiply the position by 5 and add 1.

$T(n) = 5n + 1$, 31, 201

g Multiply the position by 4 and subtract 5.

$T(n) = 4n - 5$, 19, 155

h Multiply the position by 7 and add 3.

$T(n) = 7n + 3$, 45, 283

i Multiply the position by 3 and subtract 7.

$T(n) = 3n - 7$, 11, 113

j Multiply the position by −2 and add 10.

$T(n) = -2n + 10$, −2, −70

4 **a** 21 31 501 **b** 32 62 1002

c 15 23 399 **d** 29 45 797

5 4 6 8 10 12. 19

Ex A5.5

1 **a** **i** 10 12 14 16 18

ii 6 4 2 0 −2

iii The first sequence is increasing, the second sequence is decreasing.

b **i** 11 13 15 17 19

ii 7 5 3 1 −1

iii The first sequence is increasing, the second sequence is decreasing.

c $T(n) = 2n + 5$ **d** 0 2 4 6 8

It starts with zero and increases by 2 metres each time.

2 **a** 1 4 9 16 25 36 **b** $T(n) = n^2$

c **i** 81 **ii** 144 **iii** 400 **d** 8

3 **a** 1 3 6 10

b $T(1) = 1$

$T(2) = 1 + 2 = 3$

$T(3) = 1 + 2 + 3 = 6$

$T(4) = 1 + 2 + 3 + 4 = 10$

$T(5) = 1 + 2 + 3 + 4 + 5 = 15$

c $T(1) = \frac{1}{2} \times 1 \times (1 + 1) = 1$

$T(2) = \frac{1}{2} \times 2 \times (2 + 1) = 3$

$T(3) = \frac{1}{2} \times 3 \times (3 + 1) = 6$

$T(4) = \frac{1}{2} \times 4 \times (4 + 1) = 10$

$T(5) = \frac{1}{2} \times 5 \times (5 + 1) = 15$

d $T(6) = \frac{1}{2} \times 6 \times (6 + 1) = 21$

$T(6) = 1 + 2 + 3 + 4 + 5 + 6 = 21$

4 **a** 1 **b** 3 **c** 6

5 people = 10 handshakes, 6 people = 15 handshakes.
It is the same sequence of triangle numbers.
4950 handshakes for 100 people.

5 Using the second method, Sue would get

10p + 20p + 40p + 80p + 160p + 320p + 640p = £12.70 per week.
Sue should opt for this method.

Exam practice

1 **a** 16, 19 **b** double previous term **c** 2, 5, 10

2 **a** **i** 21 **ii** 61 **b** 4n – 3

3 10 more minutes

A6 Representing and interpreting data

Check In

1 **a** 4, 5.5, 6.5, 7, 8 **b** 2.3, 2.4, 3.2, 3.4, 4.2, 4.3

c 7.5, 7.9, 8.3, 8.6, 9.1

2 **a** 6, 6, 7, 8, 8, 9, 9, 9, 9, 9

b 100, 100, 101, 102, 102, 104, 104, 104, 104, 104

A6.1 Activity

China won the most gold medals.
The USA won the most silver and bronze medals.
The USA won the most medals overall.

Rich task

Students might consider using total number of medals won or some kind of points system. Encourage students to discuss answers.

Ex A6.1

1 Bar chart

2 **a** Bar chart

b Higher proportion of girls recalled the Rabbit first recall similar otherwise.

3 eg. most popular; total number sold; preferences by gender

4 **a** **i** car **ii** microwave

Ex A6.2

1 Histogram

2 **a** Histograms **b** $15 \le t < 20$ $20 \le t < 25$

c $15 \le t < 20$ $15 \le t < 20$

d eg. boys quicker – modal class

3 eg range is same, modal class on patio is $12 < m \le 16$, but greenhouse is $4 < m \le 8$ so weights larger on patio.

A6.3 Activity

Elastic band.

Ex A6.3

1 Pie chart

2 **a** pie chart **b** **i** England **ii** 54%

3 **a** 6.5 million tonnes **b** 21.6°

A6.4 Rich task

Students answers may vary but ought to include the fact that the largest segment of the girls' pie chart is a higher earning one.

Ex A6.4

1 **a** baths, showers, hand-washing

b 4100 megalitres

2 **a** leisure, recreation, holidays **b** rail

c no connection between the sets of data.

3 pie charts

Ex A6.5

1 **a** polygon **b** **i** $20 < t \le 30$ $30 < t \le 40$

ii 50 mins, 50 mins

c office workers take longer

2 **a** polygon **b** **i** $40 < m \le 60$, $40 < m \le 60$,

ii 100 miles, 80 miles

c eg smaller range in Jan because no short journeys

3 **a** polygons

b eg modal class D $0 < t \le 5$, P $5 < t \le 10$; range same for both, median in class $10 < t \le 15$ for both; Powerblast better buy because of shape of distribution and modal class.

4 eg 2002–2003 may have range 67, and median in class 50–60, so could be $52\frac{1}{2}$

5 eg higher proportion of 40 – 60 year olds, the highest spending group, on weekdays, so more money spent per person on weekday.

Exam practice

1 **a** **i** 9 **ii** –6 **b** **i** 7 **ii** –3

c **i** –10 **ii** 10

2 **a** 15 **b** £0–£10

c Because we do not know the exact amount spent

3 **a** polygon

b the modal class for both months is 5–10 mins. The range is larger for August than July.

Case study: Sandwich shop

Day	Number of Customers	
	Week 1	Week 2
Monday	50	54
Tuesday	68	60
Wednesday	47	53
Thursday	58	57
Friday	52	56
Saturday	76	70
TOTAL	351	350

a Busiest day: first Saturday

b Range = 76 – 47 = 29

Variety	Mon	Tues	Weds	Thurs	Fri	Sat	Total	Average
Ham	14	16	13	14	17	18	92	15
Cheese	9	11	12	10	8	12	62	10
Hummus	6	5	7	4	6	8	36	6
Tuna	7	6	6	8	6	9	42	7
Chicken	18	22	15	21	19	23	118	20
Total	54	60	53	57	56	70	350	58

Product	Stock (packs)	Portions per pack	Portions left	Stock needed	Amount to order
Bread	6	20	120	18	12
Ham	2.5	10	25	10	8
Cheese	3	10	30	7	4
Hummus	2	14	28	3	1
Tuna	1.5	8	12	6	5
Chicken	1	10	10	12	11

Estimate of stock left on Wednesday morning:

Bread 4
Ham –5 (will have none left)
Cheese 10
Hummus 16
Tuna –2 (will have none left)
Chicken –30 (will have none left)

A7 Formulae and equations

Check in

1 16 cm

2 **a** x **b** m **c** $3n$ **d** $2p$

A7.1 Activity

You subtract 7 from both sides.

The solution is $x = 2$

Activity 2

You substitute 2 for A and 1 for C.

P = ((21 × 2) + (8 × 1))/2 = 50/2 = 25

Ex A7.1

1 **a** formula **b** equation **c** expression

2 **a** 14 **b** 2 **c** 14

3 **a** formula **b** equation **c** expression

4 **a** formula **b** expression **c** equation

5 **a** **i** 16 **ii** 5

b **i** 2 **ii** 5

c **i** $5x + 3$ **ii** $5x - 1$

Ex A7.2

1 $4x + 5y$

2 **a** $6x + 8y$ **b** $8x + 8x^2$ **c** $4x + 3y$

d $x^2 + 8$ **e** $8m + 2n$ **f** $6p + 2q + 7$

g $6a - 2b + 9$ **h** $9x^2 + 8x + 7$

3 **a** $6x + 10$ **b** $10x + 20$ **c** $18y - 12$

d $8z - 12$ **e** $15 + 20a$ **f** $10 - 6c$

g $2a + 10b$ **h** $8x - 12y$ **i** $18a - 6b + 3c$

j $12 + 6x^2$ **k** $15y^2 + 10y$ **l** $4 + 8x + 12x^2$

4 **a** $23x + 12$ **b** $18x + 10$ **c** $9p + 4$

d $18q + 8$ **e** $13m + 3$ **f** $19n$

5 **a** $2(3x + 4)$ **b** $3(3x + 2)$ **c** $4(2x + 3)$

d $5(x + 2)$ **e** $3(x + 3)$ **f** $6(x - 3)$

g $4(3y + 2)$ **h** $5(3a - 2)$ **i** $2(2z - 1)$

j $2(4c + 3d)$ **k** $5(2x + 3y)$ **l** $3(3s - 2t)$

6 **a** $5(2x + 3)$ **b** $4(2y + 3)$ **c** $16(z + 1)$

d $4(2p + q)$ **e** $3(3a + 2b)$ **f** $2(4s + t)$

g $8(1 + m)$ **h** $6(3c - 1)$ **i** $7(2 + z)$

7 3 ways using whole numbers. If fractions are used there are an infinite number of ways.

8 Length could be $(12x + 24)$, $(6x + 12)$, $(4x + 8)$, $(3x + 6)$, $(2x + 4)$, or $(x + 2)$ with corresponding widths 1, 2, 3, 4, 6, and 12

9 $4x + 3y$.

The different lengths are:

$3x + 3y$	$2x + 3y$	$x + 3y$	$3y$	
$4x + 2y$	$3x + 2y$	$2x + 2y$	$x + 2y$	$2y$
$4x + y$	$3x + y$	$2x + y$	$x + y$	y
$4x$	$3x$	$2x$	x	

A7.3 Activity

$I^2 = 25$
$10 \times 25 = 250$
E = 250

Ex A7.3

1 **a** £22 **b** £31

2 **a** £636 **b** £1237.75

3 **a** X 41.5 mpg
Y 38.8 mpg
Z 39.9 mpg

b Z likely to be more accurate as it uses a longer distance and therefore is likely to be based on more varied driving conditions. Yes, the journeys are consecutive so you could use the total mileage and the total fuel used to get a more accurate result.

4 **a** 42 **b** 30 **c** 19.6 **d** 9.4

5 **a** 18 **b** 11 **c** 8.4 **d** –2

6 **a** 48 **b** 10 **c** 37.5

7 25.5

A7.4 Rich task

P = 12 + 16 + 10 + 2x + 4
This simplifies to P = 2x + 42
If P = 50, $x = 4$

Ex A7.4

1 **a** 2 **b** 1.5 **c** 4
d 5.5 **e** 4 **f** 10

2 **a** 2 **b** 6 **c** 5 **d** 4
e 10 **f** 7 **g** 4 **h** 4

3 **a** 5 **b** 4 **c** 32 **d** 4.56

4 $x = 20$, $q = 56$, $a = 5\frac{2}{3}$

A7.5 Rich task

Students explore the area of the shape. They will find that if $x = 5$ then the four squares will meet in the middle.

Ex A7.5

1 **a** $n = x + 1$

b 5 Yes, if you like strong tea

2 **a** Area = $20 + 2x$ cm^2 **b** Area = $8 + 6x$ cm^2
c Area = $10 + 7x$ cm^2 **d** Area = $24 + 5x$ cm^2

3 **a** 45910 **b** $45670 + 50x$

4 **a** 8 hours **b** $5 + x$ hours

5 100 × integer + my age

A7.6 Rich task

$3x + 8 = 50$

$x = 14$, $14 < 15$, therefore the Smith's luggage is allowed on the plane.

Ex A7.6

1 **a** 5 **b** 13 **c** 5 **d** 20
e 1 **f** 5 **g** 6 **h** 8
i −3 **j** 15 **k** 3 **l** 4

2 **a** 4 **b** $\frac{1}{2}$

3 **a** 2 **b** 5 **c** 1 **d** 3
e 9 **f** $1\frac{1}{2}$ **g** $1\frac{1}{4}$ **h** 0
i 4 **j** $\frac{1}{2}$ **k** $\frac{1}{2}$ **l** $16\frac{1}{2}$

4 7

5 **a** 9 **b** 7

6 12 years old

7 $x = 13$
$y = 12$

A7.7 Rich task

A brick weighs a kilo.

Ex A7.7

1 **a** 2 **b** 1 **c** 4 **d** 3 **e** 1
f 3 **g** 2 **h** 4 **i** 4 **j** 6
k $\frac{1}{2}$ **l** $1\frac{1}{4}$

2 **a** 2 **b** 1 **c** 0 **d** 3 **e** 2
f 3 **g** 1 **h** 0 **i** 10 **j** 4

3 **a** 5 **b** $3\frac{1}{2}$ **c** $1\frac{1}{2}$ **d** $2\frac{1}{3}$ **e** 3
f 6 **g** 9 **h** 3 **i** 2 **j** $1\frac{1}{4}$
k $3\frac{1}{2}$ **l** $1\frac{1}{4}$ **m** $2\frac{1}{2}$ **n** $2\frac{2}{3}$ **o** $6\frac{1}{2}$
p 2 **q** 1 **r** 5

4 $3(x - 4) = 24$
$x = 12$

5 $30x + 40 = 220$
£6 an hour

6 $2(x - 12) - 10 = 16$
$x = 25$
£13

7 B $2x + 6$
C $x + 8$
$2x + 6 + x + 8 = x + 100$
$x = 43$

A7.8 Activity 1

$11 - 4x = 2$

$9 = 4x$

$\frac{9}{4} = x$

$= 2\frac{1}{4}$

Activity 2

$2(4x + 3) - 3(x - 5) = 36$

$8x + 6 - 3x + 15 = 36$

$5x + 21 = 36$

$5x = 15$

$x = 3$

Activity 3

$8 = \frac{x}{3}$
$x = 24$

Activity 4

$3 = \frac{12}{x}$
$3x = 12$

$x = 4$

Ex A7.8

1 **a** 20 **b** 15 **c** 16 **d** 30
e 7 **f** 3 **g** 5 **h** 5
i 4 **j** 6 **k** 4 **l** 2

2 **a** 3 **b** 4 **c** 6 **d** $1\frac{1}{2}$
e $\frac{1}{2}$ **f** $1\frac{1}{4}$ **g** 3 **h** $3\frac{1}{3}$
i 3 **j** 4 **k** $3\frac{1}{2}$ **l** $2\frac{1}{4}$
m −2 **n** −2 **o** −1 **p** −3
q $-1\frac{1}{2}$ **r** −7 **s** $-1\frac{1}{4}$ **t** $-1\frac{1}{3}$

3 **a** 2 **b** 1 **c** 3 **d** 5 **e** 1
f 2 **g** 4 **h** 2 **i** −2 **j** −3

4 **a** 3 **b** −1 **c** 5 **d** −2
e $1\frac{1}{2}$ **f** −3 **g** $\frac{1}{2}$ **h** $1\frac{1}{4}$
i $\frac{1}{4}$ **j** 0 **k** $1\frac{1}{2}$ **l** $-1\frac{1}{4}$

5 £40

6 16 years old

Ex A7.9

1 **a** (2, 3) **b** (3, −2) **c** (−3, 2)
d (−4, −3) **e** (−3, −5) **f** Post Office
g Rail bridge over road **h** Crossroads **i** School

2 **a** Yes **b** Yes
c **i** (2, 4) **ii** (5, 7) **d** S(3, 2.5) T(6.5, 3)

3 **a** (10, 10) **b** (4, 9)

4 (30, 25) (30, 25) It is a parallelogram

5 **a** (14, 14) **b** (16, 9) A quarter of the total length

Exam practice

1 **a** $3a + 4b$ **b** **i** $5(x - 3)$ **ii** $x(x + 4)$ **c** $4\frac{3}{5}$

2 $b = \frac{4}{5}(e - 32)$

3 **a** No
since $3 + 2 \neq 3$
b $2n - 3$

A8 Bearings and Pythagoras

Check in

1 **a** 180° **b** 180° **c** 360°

2 **a** 50° **b** 70°

Ex A8.1

1 **a** **i** 035° **ii** 130° **iii** 207°
iv 225° **v** 035° **vi** 0°
b **i** 067° **ii** 247° **c** 15 km
d **i** 57 km **ii** 69 km **iii** 33 km **iv** 73 km

2 **a** 180° **b** 045° **c** 225°

3 **a** 33 m **b** 14 m

4 **a** 24 km **b** 51 km

5 **a** 49 km **b** 70 km

6 **a** Drogheda to Douglas: bearing 066°, distance 140 km
Douglas to Heysham: bearing 097°, distance 110 km
b 260°

7 95 m

8 150°, 240° 330°, just over 1 hour

9 **a** 500 km **b** 2500 km **c** 3200 km

A8.2 Rich task

Number of tiles inside

Pattern	A small square	The other small square	The large square
A	2	2	4
B	8	8	16
C	4	4	8
D	16	16	32

Students should notice that the total number of tiles in the small squares is the same as the number of tiles in the large square

Activity

Students should find that X and Y should fit together inside square Z.

They should conclude from this that the area of X plus the area of Y equals the area of Z. This ought to lead them into thinking about Pythagoras' theorem

Ex A8.2

1 **a** $p = 5$ cm **b** $q = 13$ cm **c** $c = 10$ cm
d $r = 6.4$ cm **e** $s = 7.8$ cm **f** $t = 7.3$ cm

2 6.8 m

3 5.57 m

4 6.5 miles

5 2.28 km

6 Jim's error is in his first line as $x^2 = 3^2 + 5^2$
Jan's error is in her last line as $x = \sqrt{34} = 5.83$ cm

Ex A8.3

1 **a** $w = 9.2$ cm **b** $x = 8.7$ cm
c $y = 9.4$ cm **d** $z = 9$ cm

2 **a** $p = 12$ cm **b** $q = 6$ cm
c $r = 4.9$ cm **d** $s = 7$ cm

3 3.04 m

4 1.4 km

5 21 m

Exam practice

1 9.49 cm

2 **a** 110 km, 100° **b** 170 km, 71° **c** 95 km, 246°

3 31

B9 Fractions, decimals and percentages

Check in

1 0.1, $\frac{1}{4}$, $\frac{1}{2} = 0.5$, $23\% = \frac{23}{100}$

2 **a** 0.375 **b** $0.8\dot{3}$

Ex B9.1

1 **a** $\frac{2}{3}$ **b** $\frac{1}{3}$ **c** $\frac{3}{8}$ **d** $\frac{3}{5}$ **e** $\frac{2}{3}$ **f** $\frac{5}{7}$

2 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{3}{5}$

3 **a** $\frac{3}{8}$ **b** $\frac{2}{3}$ **c** $\frac{2}{5}$

4 $\frac{9}{30} = \frac{3}{10}$

5 $\frac{30}{40}, \frac{6}{8}, \frac{9}{12}$

6 **a** $\frac{1}{9}$ $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ **b** $\frac{2}{7}$ $\frac{4}{7}$ $\frac{5}{7}$

c $\frac{3}{8}$ $\frac{1}{2}$ $\frac{5}{6}$ **d** $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$

7 $\frac{21}{40}$

Ex B9.2

1 **a** 25% **b** 60%

2 0.5 50%, $\frac{1}{4}$ 25%, $\frac{3}{4}$ 0.75, $\frac{1}{10}$ 0.1, 0.2 20%, $\frac{7}{10}$ 70%, $\frac{23}{100}$ 0.23, 0.6 60%

3 **a** $\frac{1}{4}$ 40% 0.7 **b** 40% 0.45 $\frac{4}{5}$

c 0.09 15% $\frac{2}{5}$ **d** 7% $\frac{11}{20}$ 0.6

4 0.625 62.5%

5 $\frac{2}{5}$

6 15%

7 **a** 45% **b** 80%

8 32%

9 60%

Ex B9.3

1 **a** 0.205 0.3 0.43 0.61 **b** 0.205 0.701 0.84 0.9

c 0.405 0.41 0.415 0.45 **d** 0.809 0.89 0.908 0.98

e 0.2 0.205 0.25 0.5 0.52

2 **a** 9.3 **b** 10.44 **c** 9.49 **d** 9.25

e 4.2 **f** 22.89 **g** 141.65 **h** 15.14

3 **a** 4.3 **b** 11.2 **c** 27.7 **d** 3.2

e 1.5 **f** 3.4 **g** 2.45 **h** 4.74

4 **a** 0.06 **b** 0.48 **c** 1.15 **d** 0.48

e 0.6 **f** 0.72 **g** 0.126 **h** 0.345

5 **a** 3 **b** 52 **c** 200 **d** 72.8

e 32 **f** 272.5 **g** 100 **h** 49

6 **a** 0.2, 0.25 **b** $2 \div 0.2 = 10$

7 **c** 19.38

B9.4 Activity

Students play game, outcomes will vary.

Ex B9.4

1 **a** 53 **b** 21 **c** 8 **d** 22.5

e 4.2 **f** 50 **g** 5 **h** 3.2

2 **a** 18 **b** 800 **c** 10.8 **d** 25.5

e 8.4 **f** 50 **g** 20 **h** 3

3 **a** 67.2 **b** 45 **c** 66 **d** 10.4

e 1.26 **f** 98 **g** 60 **h** 28.8

4 25% of 60

5 50% of 30

6 25

7 **a** 400 **b** 350

B9.5 Activity

a six squares **b** 4 squares **c** 2 squares

Ex B9.5

1 **a** £22.50 **b** £21 **c** £10 **d** £16

2 **a** $\frac{4}{5}$ **b** $\frac{1}{2}$

3 **a** $\frac{1}{2}$ **b** $\frac{11}{16}$ **c** $\frac{27}{70}$ **d** $\frac{59}{60}$

4 **a** $\frac{1}{4}$ **b** $\frac{16}{27}$ **c** $\frac{1}{10}$ **d** $\frac{5}{12}$

Ex B9.6

1 **a** $\frac{1}{6}$ **b** $\frac{3}{35}$ **c** $\frac{24}{35}$ **d** $\frac{1}{6}$

2 **a** $\frac{1}{10}$ **b** $\frac{1}{4}$ **c** $\frac{3}{20}$ **d** $\frac{3}{20}$

e $\frac{2}{5}$ **f** $\frac{8}{9}$ **g** $\frac{15}{16}$ **h** $\frac{3}{4}$

3 **a** **i** $\frac{1}{10}, \frac{3}{8}$ **ii** $\frac{1}{2}, \frac{2}{5}, \frac{1}{10}$

b **i** $\frac{1}{5}, \frac{2}{7}$ **ii** $\frac{1}{2}, \frac{1}{5}, \frac{3}{10}$

Ex B9.7

1 **a** $1\frac{3}{5}$ **b** $7\frac{1}{2}$ **c** $7\frac{1}{2}$ **d** $\frac{3}{4}$

e $9\frac{1}{6}$ **f** $2\frac{2}{3}$ **g** $12\frac{1}{2}$ **h** $\frac{6}{7}$

2 6 cans

3 **a** 5 **b** 6 **c** 13

4 **a** $3\frac{31}{40}$ **b** $11\frac{3}{4}$ **c** $8\frac{1}{12}$ **d** $5\frac{2}{5}$

e $2\frac{5}{26}$ **f** $1\frac{13}{20}$ **g** $-\frac{5}{12}$ **h** $-\frac{11}{12}$

B9.8 Rich task

Students may notice that all of these fractions convert to recurring decimals.

Ex B9.8

1 **a** $0.\dot{1}$ $0.\dot{2}$ $0.\dot{3}$ $0.\dot{4}$ $0.\dot{5}$ $0.\dot{6}$ $0.\dot{7}$ $0.\dot{8}$

b All digits the same as numerator

2 3.14, π

3 a i $0.1\dot{6}$ **ii** $0.8\dot{3}$

b i 0.167 **ii** 0.833

4 a i 4 **ii** 3.996 **iii** 3.6

b i 0.333 **ii** 0.3

Exam practice

1 a 13 **b** £80

2 $\frac{7}{30}$

3 a i $\frac{1}{8}$ **ii** $\frac{5}{8}$

b i 0.125 **ii** 0.625

Business

Original table:

	January (£)	February (£)	March (£)
Quantity of standard	22	28	25
TOTAL INCOME	56.10	71.40	63.75
Materials used	6.60	8.40	7.50
Wages	22.00	28.00	25.00
Craft fair fees	10.00	10.00	10.00
Advertising	5.00	5.00	5.00
TOTAL EXPENDITURE	43.60	51.40	47.50
NET CASH SURPLUS/ DEFICIT	12.50	20.00	16.25
CASH BALANCE BROUGHT FORWARD	–	12.50	32.50
CASH BALANCE TO CARRY FORWARD	12.50	32.50	48.75

Fee fairs £15:

	January (£)	February (£)	March (£)
Quantity of standard	22	28	25
TOTAL INCOME	56.10	71.40	63.75
Materials used	6.60	8.40	7.50
Wages	22.00	28.00	25.00
Craft fair fees	15.00	15.00	15.00
Advertising	5.00	5.00	5.00
TOTAL EXPENDITURE	48.60	56.40	52.50
NET CASH SURPLUS/ DEFICIT	7.50	15.00	11.25
CASH BALANCE BROUGHT FORWARD	–	7.50	22.50
CASH BALANCE TO CARRY FORWARD	7.50	22.50	33.75

Materials £0.40:

	January (£)	February (£)	March (£)
Quantity of standard	22	28	25
TOTAL INCOME	56.10	71.40	63.75
Materials used	8.80	11.20	10.00
Wages	22.00	28.00	25.00
Craft fair fees	10.00	10.00	10.00
Advertising	5.00	5.00	5.00
TOTAL EXPENDITURE	45.80	54.20	50.00
NET CASH SURPLUS/ DEFICIT	10.30	17.20	13.75
CASH BALANCE BROUGHT FORWARD	–	10.30	27.50
CASH BALANCE TO CARRY FORWARD	10.30	27.50	41.25

Selling price £2.75:

	January (£)	February (£)	March (£)
Quantity of standard	22	28	25
TOTAL INCOME	60.50	77.00	68.75
Materials used	6.60	8.40	7.50
Wages	22.00	28.00	25.00
Craft fair fees	10.00	10.00	10.00
Advertising	5.00	5.00	5.00
TOTAL EXPENDITURE	43.60	51.40	47.50
NET CASH SURPLUS/ DEFICIT	16.90	25.60	21.25
CASH BALANCE BROUGHT FORWARD	–	16.90	42.50
CASH BALANCE TO CARRY FORWARD	16.90	42.50	63.75

Data for breakeven charts

Original:

Number of cards	Fixed cost	Total cost	Revenue
0	15	15.00	0.00
5	15	21.50	12.75
10	15	28.00	25.50
15	15	34.50	38.25
20	15	41.00	51.00
25	15	47.50	63.75
30	15	54.00	76.50

Fair fees £15:

Number of cards	Fixed cost	Total cost	Revenue
0	20	20.00	0.00
5	20	26.50	12.75
10	20	33.00	25.50
15	20	39.50	38.25
20	20	46.00	51.00
25	20	52.50	63.75
30	20	59.00	76.50

Materials £0.40:

Number of cards	Fixed cost	Total cost	Revenue
0	15	15.00	0.00
5	15	22.00	12.75
10	15	29.00	25.50
15	15	36.00	38.25
20	15	43.00	51.00
25	15	50.00	63.75
30	15	57.00	76.50

Selling price £2.75:

Number of cards	Fixed cost	Total cost	Revenue
0	15	15.00	0.00
5	15	21.50	13.75
10	15	28.00	27.50
15	15	34.50	41.25
20	15	41.00	55.00
25	15	47.50	68.75
30	15	54.00	82.50

B10 Circles, angles and lines

Check in

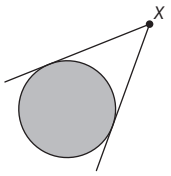
1 **a** Diameter **b** Circumference **c** Radius

2 360°

Ex B10.1

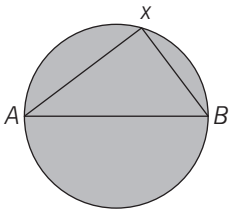
1 **a** semicircle **b** quadrant

2



AX = BX. Symmetry about the line joining X to the centre of the circle.

3



90°, yes

4 Cone, yes

5 **a** **i** 2 **ii** 4 **iii** 7 **iv** 11

6 They are all prehistoric constructions based on circles and connected with the movement of the sun. Stonehenge is another famous example.

Ex B10.2

1 **a** acute 20° **b** right-angle 90°
c acute 60° **d** obtuse 140°
e Straight line 180° **f** reflex 220°

2 **p** right-angle 90° **q** acute 70°
r right-angle 90° **s** obtuse 110°
t acute 20° **u** reflex 210°
v acute 20° **w** obtuse 110°

3 **a** obtuse 180° **b** right-angle 90°
c reflex 270° **d** obtuse 135°

4 **a** $x = 70°$ **b** $y = 160°$ **c** $z = 130°$
d $u = 180°$ **e** $v = 80°$ **f** $w = 100°$

5 120°

6 4 angles 72° 144° 216° 288°

Ex B10.3

1 **a** 50° **b** 100° **c** 55°
d 110° **e** 70° **f** 70°
g 30° **h** 150° **i** 150°

2 **p** 45° **q** 25° **r** 125° **s** 55°
t 60° **u** 40° **v** 70° **w** 60°
x 120° **y** 80°

3 **p** 50° **q** 130° **r** 90°
s 130° **t** 200° **u** 220°

Ex B10.4

2 **a** $a = 50°$ $b = c = 130°$ **b** $d = 120°$ $e = f = 60°$
c $h = 70°$ $g = i = 110°$ **d** $l = 65°$ $j = k = 115°$

3 $a = 120°$ $b = c = 60°$ $d = e = f = 110°$
$g = h = 70°$ $i = 120°$ $j = 50°$ $k = 130°$

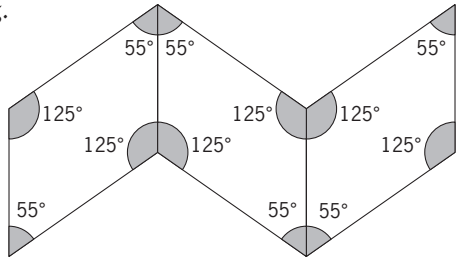
4 $a = 120°$ $b = 140°$ $c = 60°$ $d = 130°$
$e = 50°$ $f = 125°$ $g = 55°$
$h = j = 120°$ $i = 60°$ $k = l = 70°$
$m = n = 50°$ $p = q = 65°$ $r = s = 115°$

5 $p = 135°$
$q = 45°$

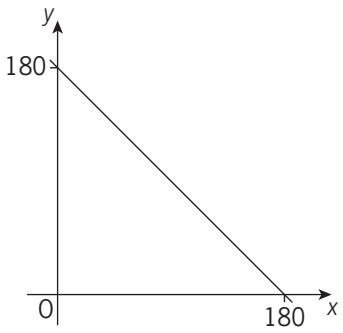
6 $x = 145°$
$y = z = 35°$

7 $p = q = t = 125°$
$r = s = u = 55°$

8 e.g.



9 $y = 180° - x$



Closed when
$x = 0$ or 180
$y = 180$ or 0

Ex B10.5

1 $a = 35°$

2 $m = 20°$ $n = 20°$ $p = 70°$ $q = 70°$

3 $e = 20°$ $f = 80°$ $g = 20°$

4 $x = 50°$

5 $v = 60°$

6 $x = y = 38°$

7 $u = 30°$ $v = 60°$ $w = 60°$

8 $a = 60°$ $b = 30°$

9 $c = 90°$

10 $43\frac{1}{3}°$

11 24° 48° 96° 192°

Ex B10.6

1

Regular polygon	Number of sides	Each interior angle	Each exterior angle	Sum of all interior angles
Triangle	3	60°	120°	180°
Quadrilateral	4	90°	90°	360°
Pentagon	5	108°	72°	540°
Hexagon	6	120°	60°	720°
Octagon	8	135°	45°	1080°
Decagon	10	144°	36°	1440°

2 **a** $a = 120°$ $b = 60°$ $c = 60°$

b $x = 140°$ $y = 110°$ $z = 40°$

c $s = 30°$ $t = 120°$ $u = 150°$

3 **a** 103°, 103°, 103°, 132°, 93°, 186°

b 102°, 102°, 112°, 112°, 112°

4 40°, 40°, 40°, 30°, 30°

Exam practice

1 No, because $66° + 71° + 53° \neq 180°$

2 $x = 24°$
$y = 48°$

3 102°

B11 Straight lines and inequalities

Check in

1 **a** 6 **b** 11 **c** 9 **d** 6

2 **a** $3x + 3$ **b** $2x - 2$ **c** $8x + 12$ **d** $12x - 6$

3 **a** 4 **b** 6

4 **a** $4(x + 2)$ **b** $3(y + 6)$ **c** $3(y - 3)$

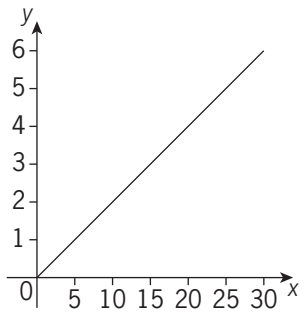
B11.1 Activity

Students should see the equivalence between the formula and flow diagram.
By substituting zero into the formula, they find the temperature at which water freezes in Fahrenheit. By substituting 100°C into the formula, they find that water boils at 212°F.

Ex B11.1

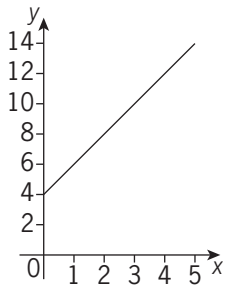
1

No. of seconds, x	5	10	15	20	25	30
No. of miles away, y	1	2	3	4	5	6



2

No.of hours, x	1	2	3	4	5
Cost, £C	6	8	10	12	14

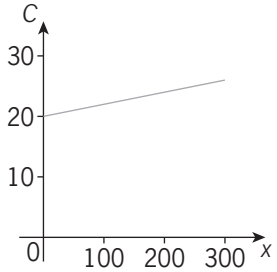


a 3.5 hrs

b 8.5 hrs

3

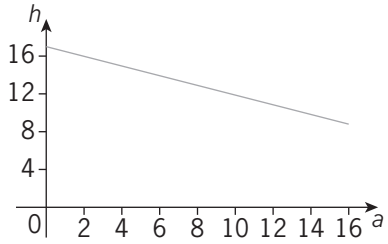
x, miles	50	100	200	300
C, £	21	22	24	26



£23

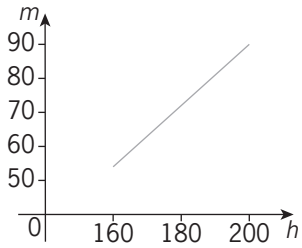
4 For example:

***a*, age in years**	0	4	8	12	16
***h*, no. of hours sleep required**	17	15	13	11	9



5 For example:

***h*, height in cms**	160	170	180	190	200
***m*, mass in kg**	54	63	72	81	90



B11.2 Activity

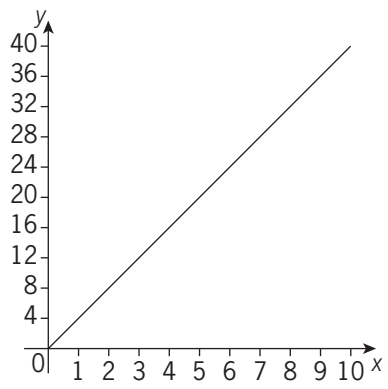
If $x = 60$, $y = 120$
The values are correct because they all add up to 180°
x cannot have values greater than 180, because $x + y = 180$.
This would be the equivalent of the windscreen wiper going off the windscreen!

Ex B11.2

1 a

Length of sides, *x*	5	6	7	8	9	10
Length of edging, *y*	20	24	28	32	36	40

b



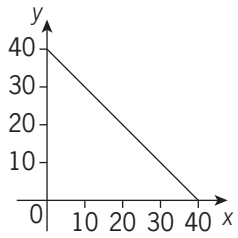
$y = 4x$

c 100 m

2 a

Amount used, *x* litres	0	10	20	30	40
Amount left in tank, *y* litres	40	30	20	10	0

b



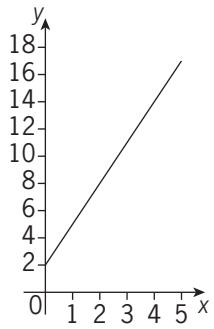
$y = 40 - x$

c 5, 420 miles

3 a

Length of journey, *x* km	1	2	3	4	5
Cost, £*y*	5	8	11	14	17

b

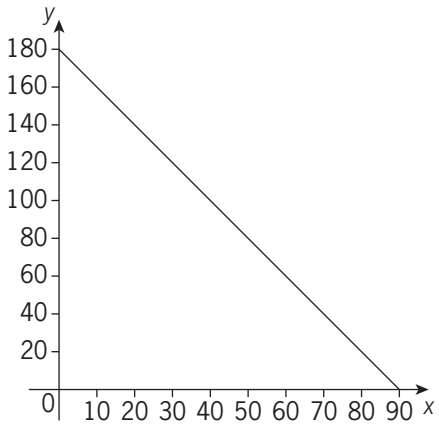


$y = 3x + 2$

c 23, No

4 For example:

***x*, in degrees**	10	20	30	40	50	60	70	80
***y*, in degrees**	160	140	120	100	80	60	40	20



$y = 180 - 2x$

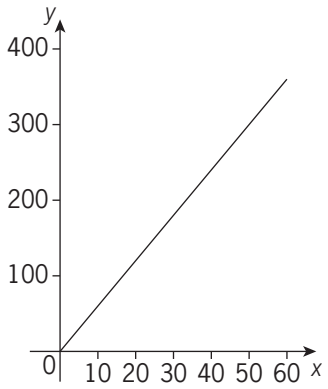
5 a 360°

b 180°

c 6°

For example:

***x*, minutes**	10	20	30	40	50	60
y, degrees	60	120	180	240	300	360



$y = 6x$

B11.3 Rich task

A leaves at 2pm, B leaves at 1pm. A takes 2 hours, B takes 4 hours. A sets off last and gets there first. A travels faster. They cross at 3pm, 50 miles away from HQ.

Ex B11.3

1 **a** 20° C **b** 50° C **c** 40° C

d 8 mins **e** 5° C

2 **a** 750 L **b** 250 L **c** 8 weeks

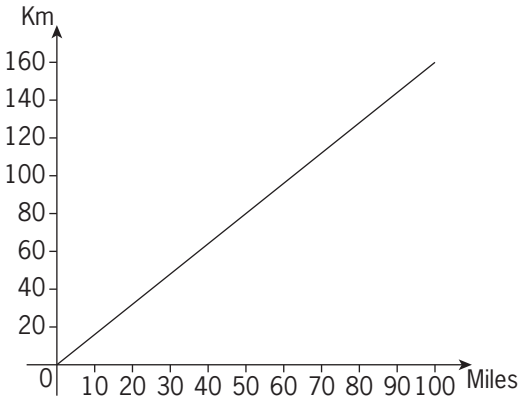
d 16 weeks **e** After 12 weeks

3 **a** **i** \$70 **ii** £25

b No, because the exchange rates are different. She will get less than £40 back.

4 50 miles

a



b 48 km, 128 km

B11.4 Rich task

Constructed as an inequality; $4x + 3 > 2x + 11$ so $2x > 8$, so $x > 4$

Ex B11.4

1 **a** equation **b** identity

c inequality **d** inequality

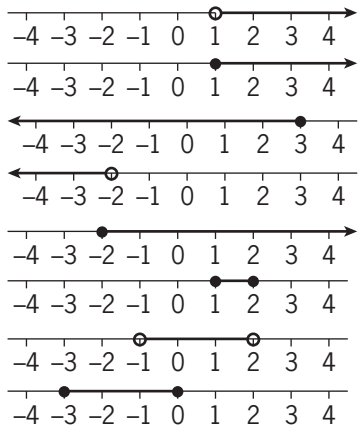
2 **a** $x < 7$ **b** $x > 7$ **c** $x \geq 7$ **d** $x \leq 7$

3 **a** $x > 2$ **b** $x \geq 0$ **c** $x \geq 1$

d $x < 1$ **e** $x < -2$ **f** $x \geq -1$

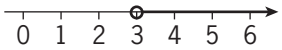
g $x < 0$ **h** $1 \leq x \leq 2$ **i** $-1 \leq x \leq 2$

4 **a**

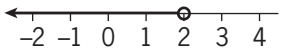


b **c** **d** **e** **f** **g** **h**

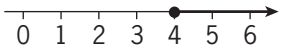
5 **a** $x > 3$



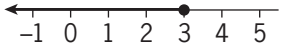
b $x < 2$



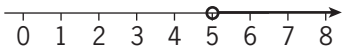
c $x \geq 4$



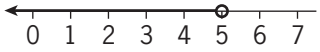
d $x \leq 3$



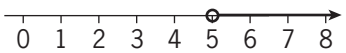
e $x > 5$



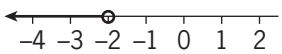
f $x < 5$



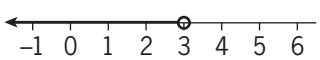
g $x > 5$



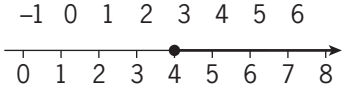
h $x < -2$



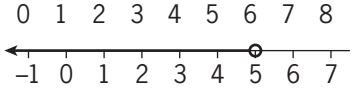
i $x < 3$



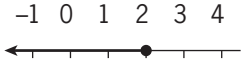
j $x \geq 4$



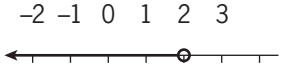
k $x < 5$



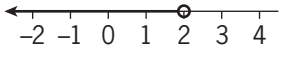
l $x \leq 1$



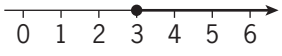
m $x < 2$



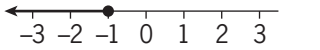
n $x \geq 3$



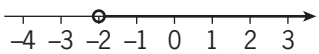
o $x \leq -1$



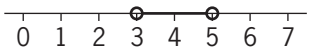
p $x > -2$



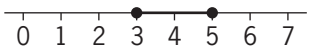
6 **a** $3 < x < 5$



b $3 \leq x \leq 5$



c $5 < x < 7$



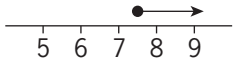
7 **a** 6, 7, 8 **b** 5, 6, 7, 8, 9 **c** –1, 0, 1

8 **a** 7, 9 **b** 1, 3, 5 **c** –3, –1, 1

Exam practice

1 **a** $x \geq 7.5$

b **i**



ii 8, 9

2 **a** graph horizontal for call times between 0 and 1 minute

b $C = 10 + 30\,m$ (only valid when $m \geq 1$)

3 **a** at least 16 times

B12 Transformations

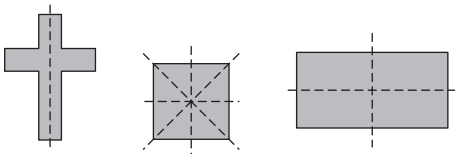
Check in

1 **a** 180° **b** 270° **c** 90° **d** 180°

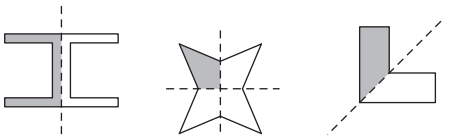
2 **a** Clockwise **b** Anti-clockwise

Ex B12.1

1 **a**



b



2 **a** 4 **b** 2 **c** 1 **d** 3

3 1 line: A C D E M T U V W Y K B

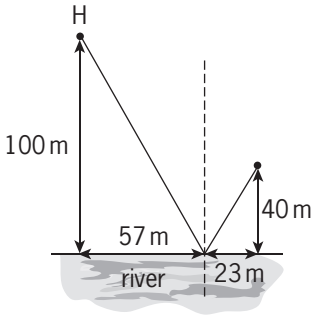
0 line: L Q F G J N P R S Z

2 lines: H I X

2 or infinite lines: O

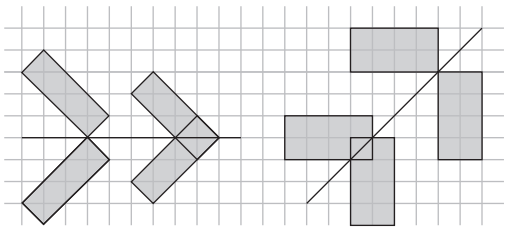
dependent on the shape of font

4

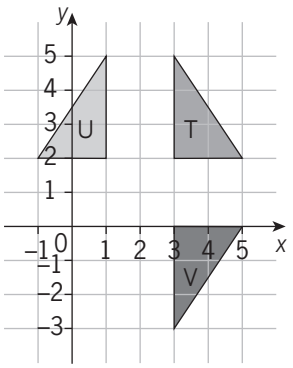


Ex B12.2

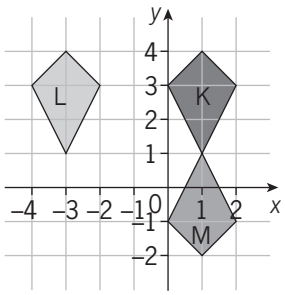
1



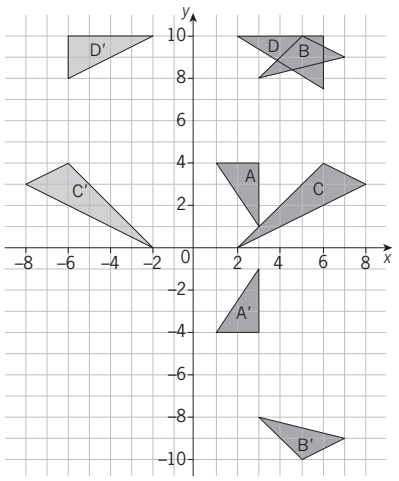
2 **a**



3



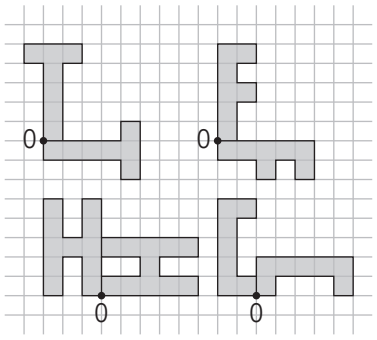
4



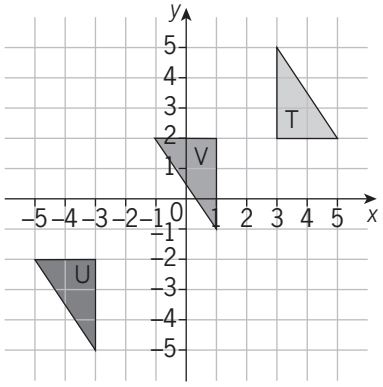
Ex B12.3

1 **a** 180° **b** 90° anticlockwise

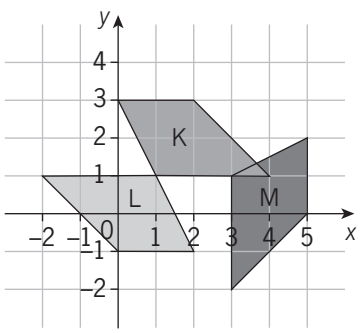
2



3



4



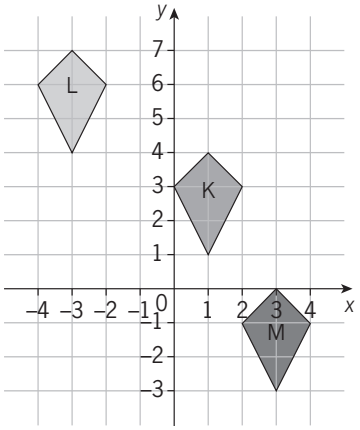
Ex B12.4

1 **a** (4, 5), (7, 3), (8, 2), (5, 9), (6, −2), (6, −1)

b $\begin{pmatrix}4\\2\end{pmatrix}, \begin{pmatrix}2\\-2\end{pmatrix}, \begin{pmatrix}4\\0\end{pmatrix}, \begin{pmatrix}-7\\-4\end{pmatrix}, \begin{pmatrix}-2\\2\end{pmatrix}, \begin{pmatrix}-4\\0\end{pmatrix}$

2 **a** $\begin{pmatrix}3\\2\end{pmatrix}$ **b** $\begin{pmatrix}5\\1\end{pmatrix}$ **c** $\begin{pmatrix}2\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-3\\-2\end{pmatrix}$

3 **a**



4 $\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}2\\-1\end{pmatrix}, \begin{pmatrix}-2\\1\end{pmatrix}, \begin{pmatrix}-2\\-1\end{pmatrix}, \begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}1\\-2\end{pmatrix}, \begin{pmatrix}-1\\2\end{pmatrix}, \begin{pmatrix}-1\\-2\end{pmatrix}$

Exam practice

1 **a** **i** no line symmetry

ii rotational symmetry of order 1

b **i** no line symmetry

ii rotational symmetry of order 5

2 **a** (0, 1) (3, 2) (2, 3) R **b** (1, 0) (2, 3) (3, 2) S

3 **a** S is (1, −2) (1, 0) (2, 1) (4, 1)

b T is (0, 0) (0, 2) (1, 3) (3, 3)

Case study: Art

1 **a** 30 **b** 4.44 m

c length = 40 hands; height = 29.6 hands

2 **a** **i** 2 trapeziums, 1 parallelogram, 2 rectangles

ii 3 trapeziums, 1 parallelogram, 1 rectangle

b **i** (−2, 0), (0, 5), (1, 5), (1, 0), (0.4, 0), (0.4, 2), (−0.6, 2), (−0.6, 0)

ii (2, −1), (1.6, 1), (1.6, 6), (3, 6), (3.6, 5), (2.4, 5), (2.4, 1), (3.6, 1), (3.8, 2), (3.4, 2), (3, 3), (4.4, 3), (3.6, −1)

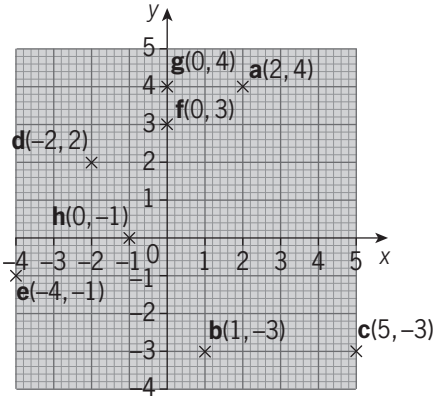
3 **a** 2 **b** scale factor 4

c **i** 120 cm^2 **ii** 480 cm^2

B13 Bivariate data and time series

Check in

1



2 **a** 10 **b** 25 **c** 20 **d** 12.5 **e** 27.5

B13.1 Rich task

Yes, where you live does affect your lung capacity. From the graph you can see that those living at a higher altitude tend to have better lung capacity.
Students answers will vary regarding other possible data to collect.

Ex B13.1

1 graph

2 graph

3 more alcohol associated with longer reaction times
More time on games associated with shorter reaction times. Longer reaction times at 3.0 am for all subjects

Ex B13.2

1 Positive; people from higher altitudes have larger lung capacity.

2 Positive; good at both subjects
Positive; tall people have long arms

3 **b** positive

c all except one subject take longer with left hand than right.

4 **b** no correlation. Number of deaths not related to magnitude. Density of population and type of buildings very important.

Ex B13.3

1 **a** £108 **b** £97

2 **a** 4.9 litres

b **i** beyond data range

ii relationship only established for men

3 **a** graph

b If results for black bear and horse are ignored, correlation very weak so estimates are inaccurate

Ex B13.4

1 Highest in June and December, which are holiday months.

2 2005 higher than 2004, but same monthly pattern. Cost per unit may have increased. Highest in Jan-March then Oct-Dec, the coldest months.

3 Highest in warmer months of August, July and June. Also peak in December, a holiday period.

4 Earns most in May-August, maybe school holidays. Earnings increased each year, maybe better rates as older.

5 Highest in April-June, Lowest in Oct-Dec. 2005 less than 2004, maybe economising.

Exam practice

1 No consistent pattern. Average temperature ranged from 13.6°C to 16.7°C

2 More marriages in summer months.

3 Question not valid as data is artificially paired

B14 More straight-line graphs

Check in

1 1, 3, 5, 7, 9

B14.1 Activity

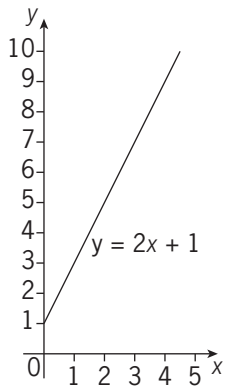
When $x = 2$, $y = 8$. When $x = 3$, $y = 10$.

The coordinates are (0, 4), (1, 6), (2, 8), (3, 10)

Ex B14.1

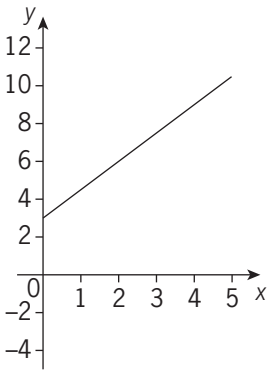
1

x	0	1	2	3
y	1	3	5	7



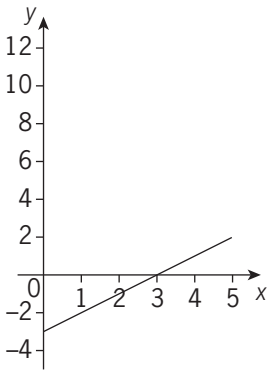
2 a

x	0	1	2	3	4
y	3	4	5	6	7



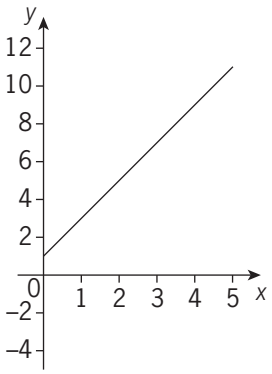
b

x	0	1	2	3	4
y	−3	−2	−1	0	1



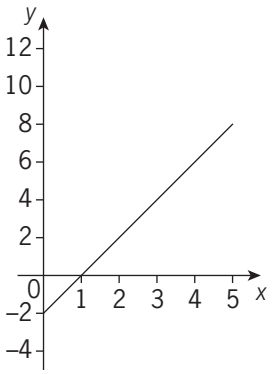
c

x	0	1	2	3	4
y	1	3	5	7	9



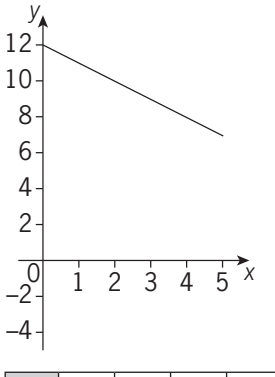
d

x	0	1	2	3	4
y	−2	0	2	4	6



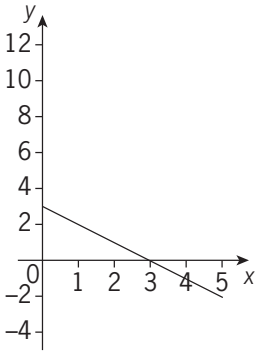
e

x	0	1	2	3	4
y	12	11	10	9	8



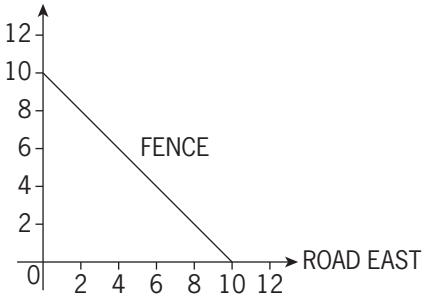
f

x	0	1	2	3	4
y	3	2	1	0	−1



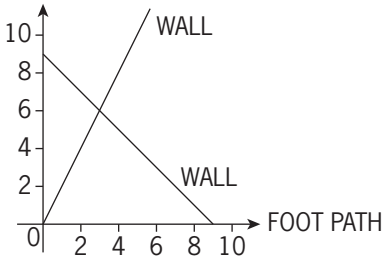
3 **a** Yes **b** Yes **c** No
d Yes **e** Yes **f** No

4 **a**



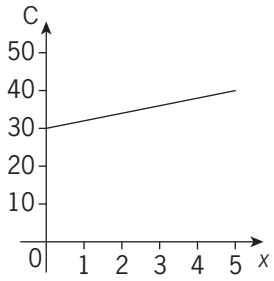
b 50

5 **a**



b 27

6



3.5 hours

7 (20, 52)

B14.2 Rich task

The ratio is often said to be "rise over run". Run is the *horizontal distance* and rise is the *vertical distance* travelled. Cyclists and cars can typically cope with a gradient of no more than 1:10.
The steepest roads are in Rosedale Abbey, N Yorks which is 1:3 and Ffordd Penllech, Harlech, Wales (officially described as not suitable for motor vehicles) which is 1:2.91 at its steepest point.

Ex B14.2

1 P 3 Q 2 R 1 S –2
T 0.5 U –1 V 0

2 **a** 2 **b** 3 **c** 0.5
d 0 **e** –1 **f** –2

3 **a** 5 **b** 4 **c** 1 **d** $\frac{1}{2}$
e –6 **f** –3 **g** –8 **h** –8
i –4 **j** –1 **k** $-\frac{1}{2}$ **l** 3

4 $y = 2x + 1$ and $y = 2x - 1$
$y = 4 - x$ and $y = -x + 5$
$y = \frac{1}{2}x + 3$ and $y = 1 + \frac{1}{2}x$
$y = -2x + 7$ and $y = 6 - 2x$

Rich Task

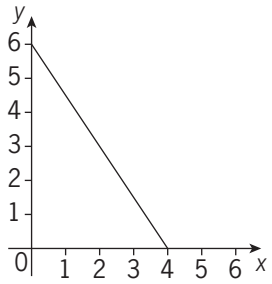
The graphs of $y = mx + 3$ all pass through (0, 3)
The graphs of $y = 2x + c$ are all parallel lines

B14.3 Rich task

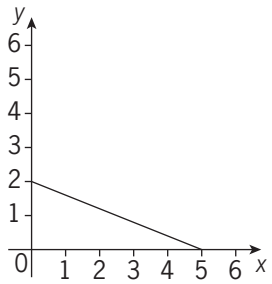
There are 19 combinations of seedlings (where he buys both apple and pair seedlings at the same time) he can buy ranging from $x = 1$, $y = 38$ to $x = 19$, $y = 2$
The graph of $10x + 5y = 200$ will intersect the y axis at (0, 40) and the x axis at (20, 0).

Ex B14.3

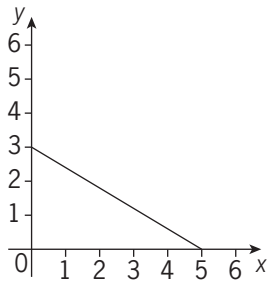
1 **a** (4, 0) (0, 6)



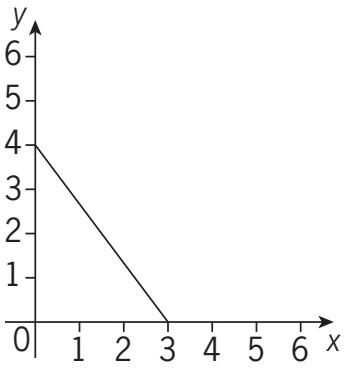
b (5, 0) (0, 2)



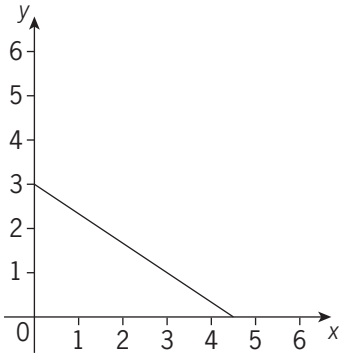
c (5, 0) (0, 3)



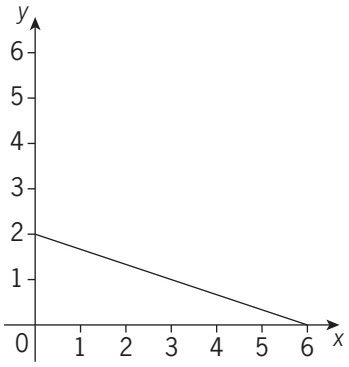
d (3, 0) (0, 4)



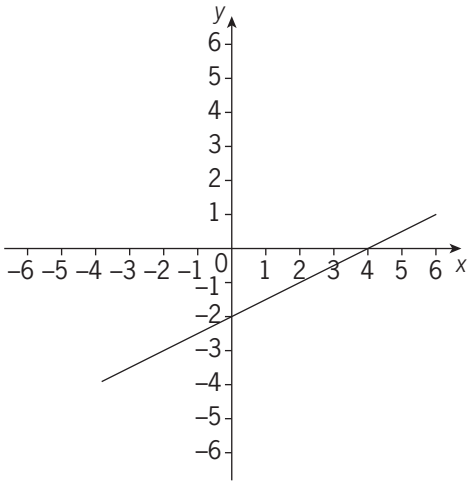
e (4.5, 0) (0, 3)



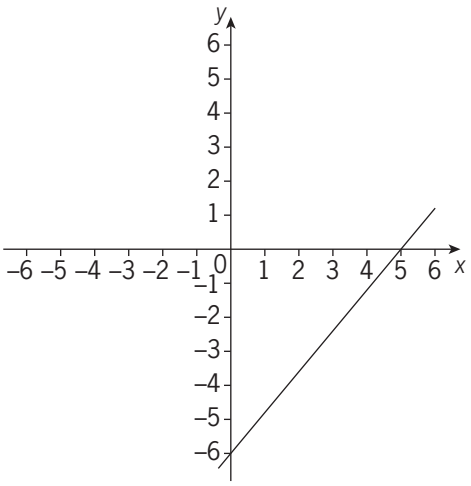
f (6, 0) (0, 2)



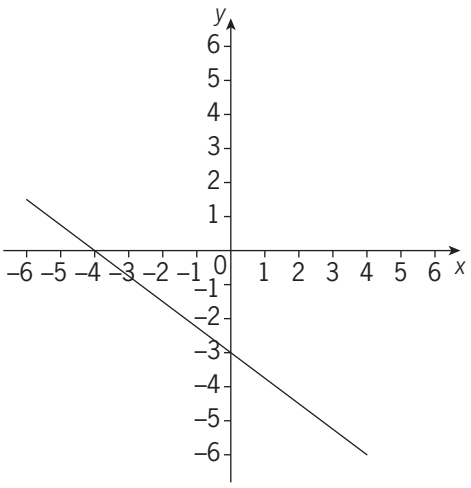
2 a (4, 0) (0, −2)



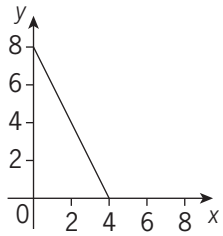
b (5, 0) (0, −6)



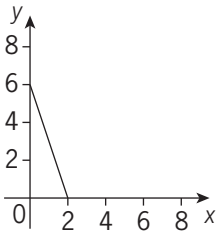
c (−4, 0) (0, −3)



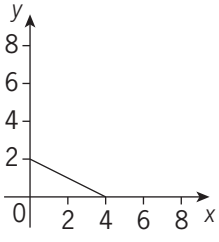
3 a gradient = −2



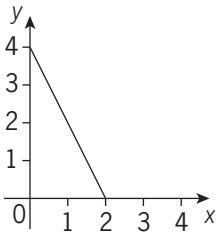
b gradient = −3



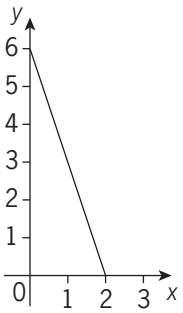
c gradient = $-\frac{1}{2}$



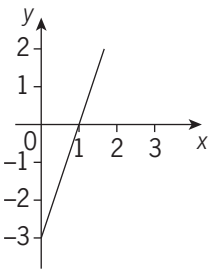
4 a gradient = −2



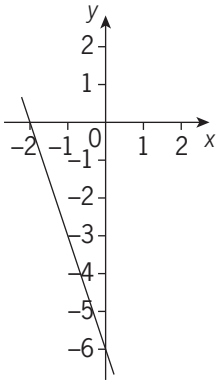
b gradient = −3



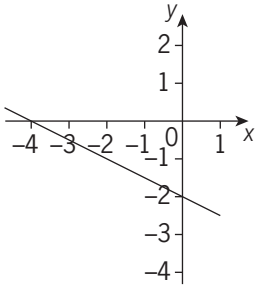
c gradient = 3



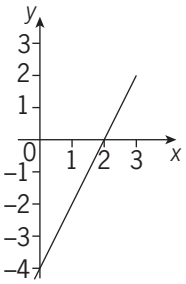
d gradient = −3



e gradient = $-\frac{1}{2}$



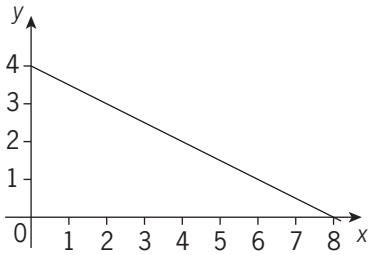
f gradient = 2



5 Graph will be a straight line through (3, 0) and (0, 6)

Possible pairs are: (1, 4) and (2, 2)

6



Possible combinations are (0, 4), (2, 3), (4, 2), (6, 1) and (8, 0)

4 Christmas cards and 2 birthday cards

B14.4 Rich task

Cab A Cost = $3x$

Cab B Cost = $x + 6$

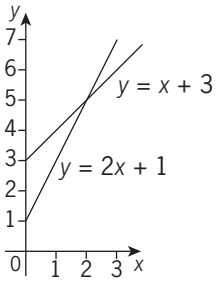
For less than 3 mile journeys, Cab A would be cheaper and for journeys of more than 3 miles, Cab B would be cheaper.

Activity

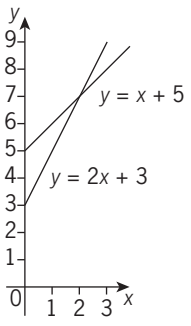
The point is (4, 2).

Ex B14.4

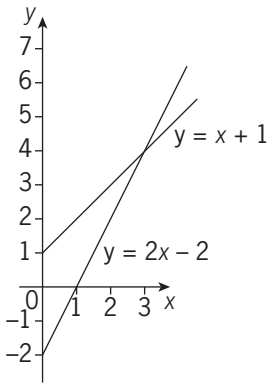
1 a $x = 2$ $y = 5$



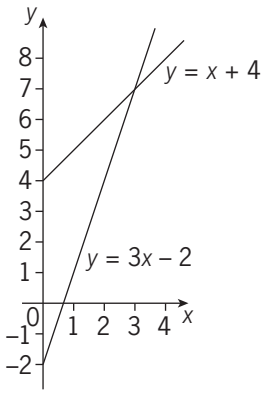
b $x = 2$ $y = 7$



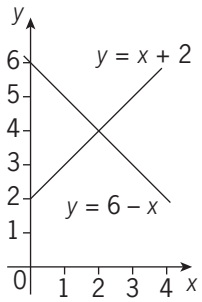
c $x = 3$ $y = 4$



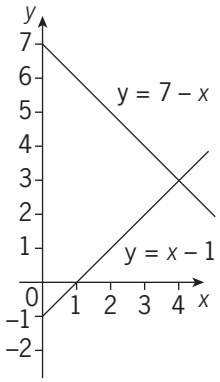
d $x = 3$ $y = 7$



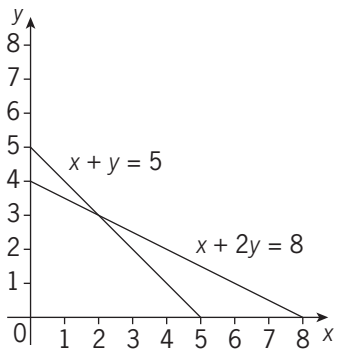
e $x = 2$ $y = 4$



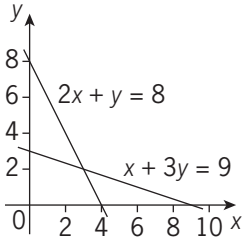
f $x = 4$ $y = 3$



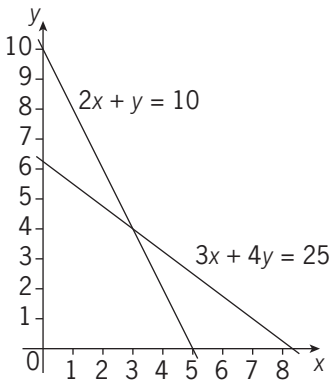
2 a $x = 2$ $y = 3$



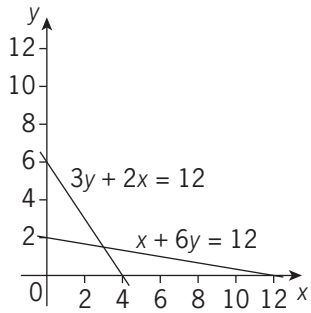
b $x = 3$ $y = 2$



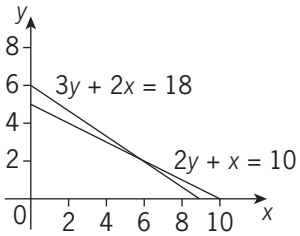
c $x = 3$ $y = 4$



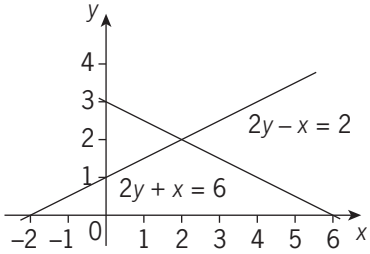
d $x = 3$ $y = 1.5$



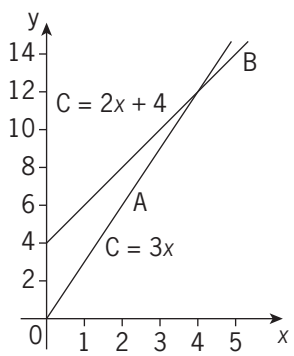
e $x = 6$ $y = 2$



f $x = 2$ $y = 2$



3

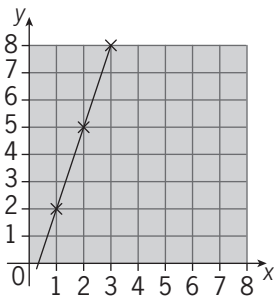


Firm A is cheaper for up to 4 spanners. For more than 4 spanners firm B is cheaper

Exam practice

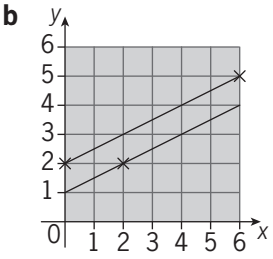
1 a (2, 5)

b



2 a $\frac{1}{2}$

b



3 a $x = 1\frac{1}{3}$ $y = 3\frac{2}{3}$

B15 Estimation and indices

Check in

1 a 49 **b** 8 **c** 1000 **d** 0.32

2 5

3 yes

Ex B15.1

1 a 292 **b** 601 **c** 1739 **d** 273

e 158 **f** 235 **g** 20250 **h** 96

i 2496 **j** 3.3 **k** 86.17 **l** 36.25

2 a i 13.2 **ii** 0.87 **b i** 21.4 **ii** 19.902

c i 7600 **ii** 0.045 **d i** 22.5 **ii** 7.2

3 a £2.76 **b** £7.77

4 a 33 mins **b** 35 mins

5 a 49, 51 **b** 77, 27 or 84, 34

c $98 \times 84 = 8232$

B15.2 Activity 1

Students will find square numbers

Activity 2

Students will find ±6

Ex B15.2

1 a 343 **b** 400 **c** 225 **d** 216 **e** 125

2 a 7 **b** 100 **c** 5 **d** 4 **e** 9

3 a 10, –10 **b** 11, –11 **c** 3, –3 **d** 14, –14

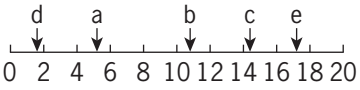
4 a i 13 **ii** 10 **b** 1.3

5 a i 12 **ii** 1.2 **b i** 15 **ii** 1.5

6 a 10^3 **b** 10^6 **c** 10^6

7 a 27 **b** –198 **c** 15 **d** 56 **e** 7

8



9 From 1 + 8 2 + 7 3 + 6 4 + 5 1 + 15 2 + 14 3 +13 4 + 12 5 + 11 6 + 10 7 + 9 7 + 18 8 + 17 9 + 16 10 + 15 11 + 14 12 + 13 1 + 3

10 Rich task

The next number will be 25, these are square numbers.

B15.3 Activity

Students answers will vary, encourage class discussion.

This is also a potential opportunity for QWC practice.

Ex B15.3

1 a 600 **b** 8000 **c** 16 000 **d** 100

e 100 **f** 13 **g** 8 **h** 50

2 a 5 **b** 11

3 a 9 **b** 10 **c** 25

d 64 **e** 7

4 a 4 **b** 100 **c** 2 **d** 10

e 7 **f** 0.5 **g** 12 **h** $\frac{1}{14}$

i 2 **j** 15 000 **k** 1.3 **l** 100

m 0 **n** 440

5 7

6 12

7 40

B15.4 Rich task

Dexter should accept the second option.

Rich task 2

a i 32 **ii** 2^5

b i 81 **ii** 3^4

c Students should experiment with this and their answers will vary.

Ex B15.4

1 a 64 **b** 81 **c** 243 **d** 625

e 72 **f** 256 **g** 16

2 i 8 **ii** 5

3 **a** **i** 5^3 **ii** 5^3 **iii** $2^6, 4^3$

b **i** $4^6 = 16^3$ **ii** all squares **iii** $4^6, 16^3$

4 **a** $2^4 \times 3$ **b** $2 \times 3 \times 7$ **c** $2^3 \times 3 \times 5^2$

d $2^8 \times 5$ **e** $3 \times 5 \times 7^2$

5 **a** 7^9 **b** 3^8 **c** 8^9 **d** 15^8

e 4^{12} **f** 3^6 **g** 8 **h** 9^5 **i** 6^4

6 2, 4

7 **Problem**

$x = 2, y = 4$

Exam practice

1 9 **2** odd (odd × odd = odd) **3** She has enough

Case study 5: Recycling

	Waste not recycled/ composted	Waste recycled/ composted	Total waste	Percentage recycled
1983/84	394	3	397	0.76
1984/85				
1985/86				
1986/87				
1987/88				
1988/89				
1989/90				
1990/91				
1991/92	417	11	427	2.57
1992/93				
1993/94	430	15	445	3.37
1994/95				
1995/96	423	27	450	6.00
1996/97	438	32	469	6.82
1997/98	441	36	477	7.55
1998/99	443	40	483	8.28
1999/00	457	48	505	9.50
2000/01	455	52	507	10.26
2001/02	456	60	516	11.63
2002/03	449	71	521	13.63
2003/04	425	87	512	16.99
2004/05	404	113	517	21.86
2005/06	376	135	511	26.42
2006/07	359	157	516	30.43
2007/08	334	173	507	34.12

a 27.7%

b 5 667%

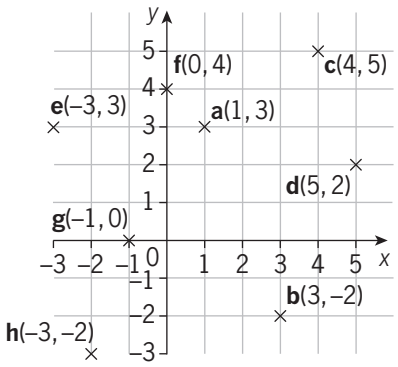
New bottle 26 g; 7.7% reduction

Tray = 576 cm², lid = 375 cm², total = 951 cm²
39.4% lighter

B16 Similarity and enlargement

Check in

1



2 **a** 1 : 3 **b** 1 : 6 **c** 2 : 3 **d** 5 : 2

B16.1 Rich task

a and **j**, **c** and **I**, **f** and **g** are all congruent shapes.

b, **c** and **i**, **f**, **g**, and **h**, and **a**, **d**, **e**, **j** are all either congruent or similar.

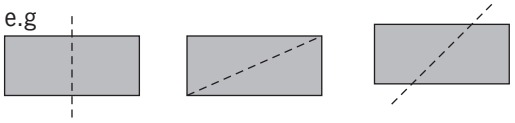
Ex B16.1

1 **a** congruent

b not congruent

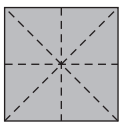
c congruent

2 **a** e.g

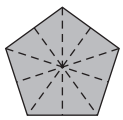


b The vertical line is; the other lines are not

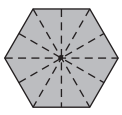
3 **a**



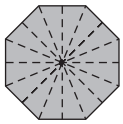
b



c



d



4 **a** similar

b not similar

c similar

5 **a** congruent **b** congruent **c** similar

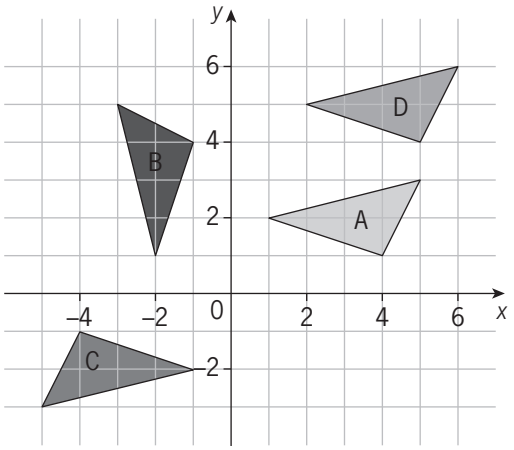
d neither **e** similar **f** congruent

g neither

6 4.40 m long, 1.80 m high

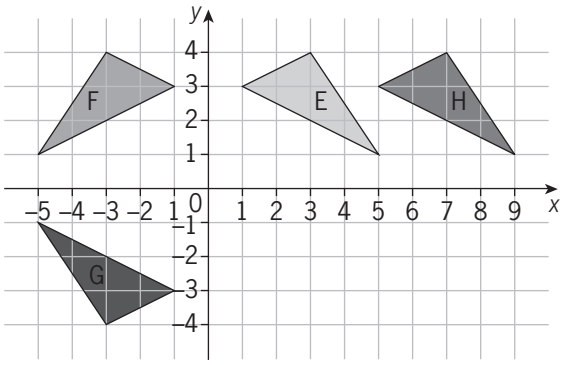
Ex B16.2

1 a, b, d



c Rotation of 180° about (0, 0) **e** Translation $\begin{pmatrix}1\\3\end{pmatrix}$

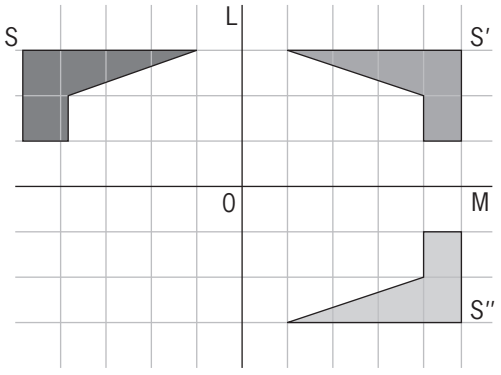
2 a, b, d



c Rotation of 180° about (0, 0) **e** Translation $\begin{pmatrix}4\\0\end{pmatrix}$

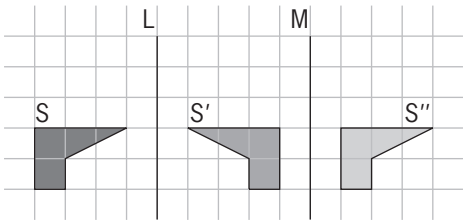
3 Translation by the vector $\begin{pmatrix}4\\-2\end{pmatrix}$

4 a



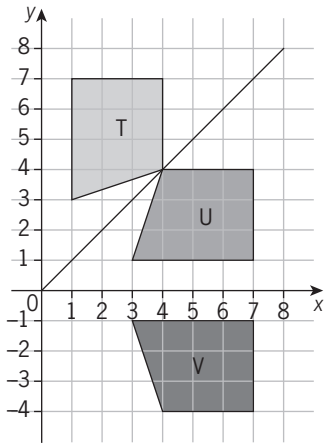
Rotation of 180° about point O

b



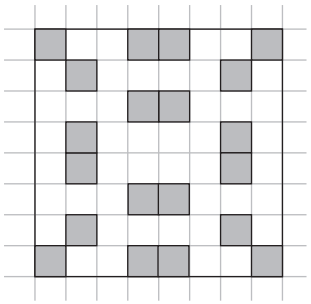
Translation of $\begin{pmatrix}10\\0\end{pmatrix}$

5



Rotation of 90° clockwise about the origin

6

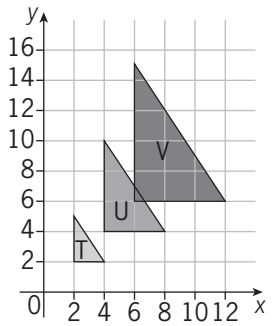


B16.3 Discussion

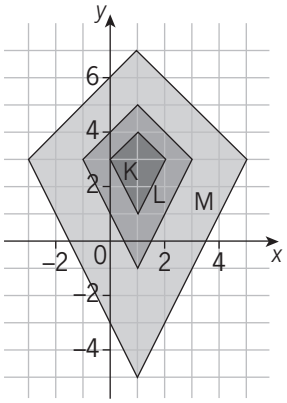
Class or group discussion indicating that the picture will get smaller the closer the projector is to the screen and larger the further away it is.

Ex B16.3

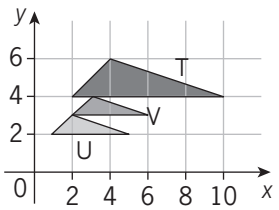
1



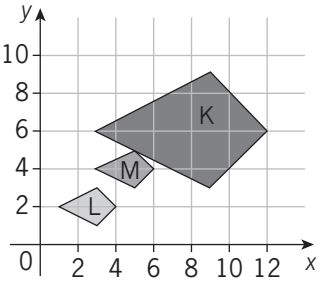
2



3



4

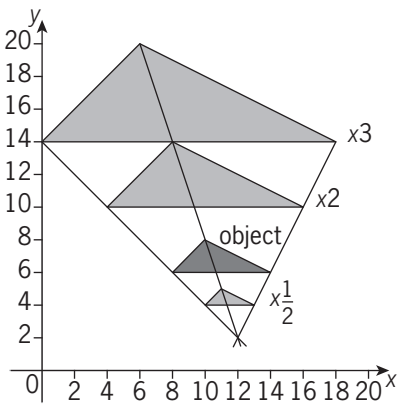


5 Vertices of images are

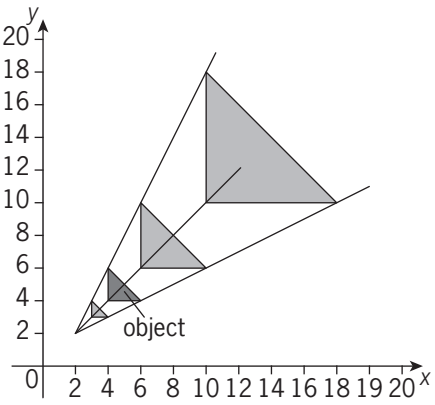
a (4, 6) (8, 6) (4, 12) **b** (18, 0) (24, 6) (15, 9)

c (3, 16) (11, 16) (11, 20) (3, 22) **d** (18, 17) (22, 17) (20, 15)

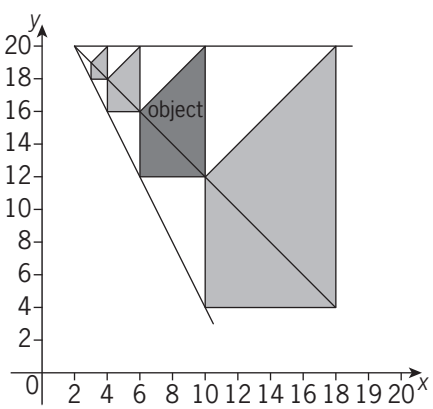
6 a



b



c



Ex B16.4

1 a (10, 1)

b R scale factor 2; S scale factor 3; Q scale factor $\frac{1}{2}$

2 3

3 a i (0, 3) **ii** 2

b i (12, 1) **ii** 3

c i (5, 7) **ii** 4

d i (1, 24) **ii** 5

4 a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{4}$

5 a (1, 7), $\frac{1}{2}$ **b** (4, 8), $\frac{1}{4}$

6 200

7 A4 to A3 has a scale factor 2

An enlargement (scale factor 2) is used for each successive paper size

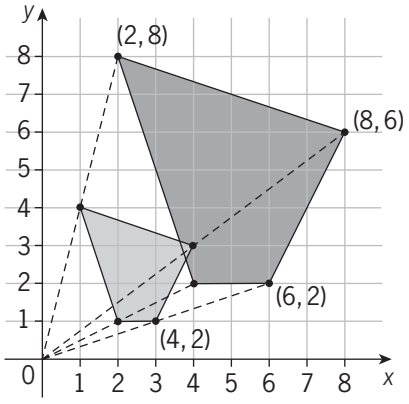
8 180 × 130 mm

9 12 inches × 8 inches and 18 inches × 12 inches are enlargements.

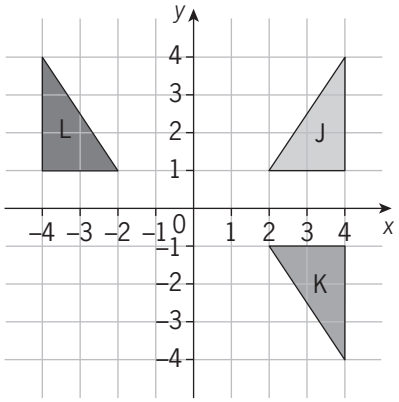
10 inches × 8 inches and 7 inches × 5 inches will need to be cropped.

Exam practice

1



2 a, b



c reflection in y-axis

3 $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$

C17 Percentages and proportional change

Check in

1 **a** 10 **b** 4

2 3

3 0.7 0.75 0.8 0.875

Ex C17.1

1 **a** £7 **b** £18 **c** £112

2 £58.24

3 **a** £1905 **b** £6705 **c** £502 **d** £29 130

4 **a** £74.25 **b** £6000 **c** £11 200

d £10 500 **e** £120 **f** £3.30

C17.2 Problem

The trader made a £12.50 profit.

Rich task

0.65

Ex C17.2

1 **a** £374.40 **b** £48.95

2 **a** 35p **b** £7.80

3 £82.68

4 £36 016.80

5 £124.67

6 **Problem**
They should reduce the price of the bar.

7 **Rich task**
Students should experiment with different numbers and methods to get a feel for how percentage increases and decreases work.

C17.3 Problem

Stan got the better deal.

Ex C17.3

1 **a** 13.7% **b** 14.3%

2 73.3%

3 60%

4 5% profit

Ex C17.4

1 **a** £4800 **b** £19 800

2 **a** £36 **b** £136

3 **a** £1008 **b** 1008

4 £412.50

5 **a** £200 **b** £192 option **c** by £8

C17.5 Rich task

Students should practice on paper and realise that the thickness doubles at every fold which quickly makes it impractical to keep folding in practice. They should think about what happens if you repeatedly double a number.

Ex C17.5

1 **a** £40 **b** £43.20 **c** £583.20

2 **a** **i** £4000 **ii** £36 000

b **i** £7600 **ii** £32 400 **c** £29 160

3 **a** 5.4 m **b** 4.86 m

4 **a** 1.68 million

b 2.36 million (3 sig. figs.)

5 **a** 25 760 **b** 21 803

6 **a** £20 320.50 **b** £21 732.77 **c** £26 586.28

7 **a** £44 184 **b** £48 898.61 **c** £56 930.33

C17.6 Problem

If you work out the area of the photos, then the 30% bigger size is actually 39% bigger. The 100% bigger size is actually 99% bigger.

Problem 2

15 years.

12000 slaves.

Ex C17.6

1 **a** 76p **b** 88p

2 **a** **i** 3 hours **ii** 5 hours

iii 7 hours 30 minutes

b **i** 10.5 days **ii** 16.8 days

3 **a** 5 litres for £1.29

b 200 ml for £2.89

4 26p per biscuit

5 14

Exam practice

1 **a** 160ml

b 17.5g

2 1.375%

3 6 eggs

C18 Area and perimeter

Check-in

1 **a** 600 **b** 71 000 **c** 48
d 2630 **e** 4500 **f** 6
g 75 **h** 6.5 **i** 3.2

2 **a** 9.6 **b** 22.5 **c** 10.5

3 **a** 7.8 **b** 15.5 **c** 9.0

Ex C18.1

1 **a** 9.6 cm **b** 9.3 cm

2 **a** 15.3 cm **b** 17.5 cm

3 **a** 19.2 metres **b** £13.00

4 **a** 5 cm^2 **b** 7 cm^2

5 4.5 cm^2 360 km^2

6 **a** 20 cm^2 **b** 6 cm^2 **c** 27 cm^2
d 14 cm^2 **e** 14 cm^2 **f** 8.55 cm^2

7 **a** 4560 cm^2 **b** 2280 cm^2

8 Colorado, Wyoming
Multiplying length by width which is area is not equal to the actual area, because the shapes of the states are not true rectangles. The shapes are on the earth's sphere and are defined by the earth's meridian and so the shapes taper slightly towards their northern sides.

C18.2 Activity

You need pi squares to cover the circle.

Ex C18.2

1 **a** 16 cm **b** 25 cm **c** 19 cm **d** 63 cm

2 **a** 89.2 mm **b** 14.1 metres
c 75.4 cm **d** 21.4 cm

3 **a** 125.6 cm **b** 125.6 metres

4 **a** 40 000 km (4 sig. figs.)
b **i** 584 million miles **ii** 1.6 million miles
iii 67 000 miles

5 **a** 113 m^2 **b** 78.5 cm^2
c 707 cm^2 **d** 5.31 cm^2
e 1260 cm^2 **f** 7850 cm^2
g 1810 cm^2 **h** 24.6 cm^2

6 452 m^2

7 16.6 m^2

8 **a** 615 m^2 **b** £52300

9 **a** 1.3 cm, 0.8 cm **b** 2.01 cm^2 **c** 3.30 cm^2

10 141 cm^2

C18.3 Rich task

a 51 cm^2 **b** 17 cm^2 **c** 34 cm^2

Ex C18.3

1 **a** 56 cm^2 **b** 55 cm^2 **c** 48 cm^2 **d** 38 cm^2
e 80 cm^2 **f** 36.5 cm^2 **g** 49 cm^2 **h** 36 cm^2

2 **a** 66 m^2 **b** £534.60

3 36 m^2

Ex C18.4

1 **a** 20 cm^2 **b** 30 cm^2 **c** 21 cm^2 **d** 15 cm^2

2 **a** 20 cm^2 **b** 15.6 cm^2

3 2400 cm^2

4 A square of side $2a$

Trapeziums with parallel sides

i a and $7a$, **ii** $2a$ and $6a$.

Ex C18.5

1 **a** w = 5 cm **b** h = 5 cm **c** h = 7 cm
d h = 6 cm **e** r = 3.6 cm **f** r = 3.0 cm

2 10 metres

3 2.5 metres

4 1.4 cm

5 **a** **i** 15.9 cm **ii** 31.8 cm **iii** 207 mm
b **i** 2.5 cm **ii** 5.2 m **iii** 5.6 mm

6 333 cm

7 1 × 36, 2 × 18, 3 × 12, 4 × 9, and if you include a square, 6 × 6

The shape with the smallest perimeter would be a square of 6 × 6, with a perimeter of 24cm.

Ex C18.6

1 **a** (2, 0) 2 4 **b** (1, 1) 3 9

2

	a	b	c	d
Length SF	2	3	4	2
Area SF	4	9	16	4

3 135 cm^2

4 **a** 9 **b** 3 **c** 21 cm

5 Yes, 2.5, 6.25

Exam practice

1 **a** 44 cm^2 36 cm
b 72 cm^2 40 cm

2 **a** **i** C **ii** none

3 65.94 cm^2

Case study: Holiday

1 £32.63 (assuming 13 weeks)

2 £4 per hour
£52 (assuming 13 weeks)

3 CDs £2.50 each; DVDs £3.50 each

4 **a** 219 euros

b £82.19 (2dp)
exchange rate £1 : 1.31 euros (2dp); 90.78 euros

Max temp in Rome = 37.2°C
Min temp in Rome = 17.2°C
Max temp in London = 77°F
Min temp in London = 60.8°F

C19 Algebraic manipulation

Check-in

1 **a** 4 **b** 5.061 **c** 26.368

2 **a** $x = 7$ **b** $x = 10$

Ex C19.1

1 **a** $2(x + y)$ **b** $2x + 3y$

2 **a** $5x + y$ **b** $7p + 10q + 2$ **c** $m - 1$

d $3(x - y + z)$ **e** $5t$ **f** $4 + 3p - 3q$

g 0 **h** $5t$

3 **a** $a = 10\ b = 10$ **b** $a = 5\ b = 3$ **c** $a = 7\ b = -5$

d $a = 2\ b = 6$ **e** $a = 3$ b $= -1$ **f** $a = 3\ b = -2$

g $a = -5$ b $= 8$ **h** $a = 0\ b = 2$

4 **a** $24x + 10$ **b** $14x + 9$ **c** $8p + 6$

d $11q + 17$ **e** $11r$ **f** $24n$

g $2x^2 + 6x$ **h** $2z^2 + 4z$

i $x^2 + 8x + 12$ **j** $y^2 - y + 6$

5 **a** $2(2x + 3)$ **b** $3(2y + 3)$ **c** $4(3z + 2)$

d $3(x + 5)$ **e** $5(x + 4)$ **f** $3(6z - 5)$

g $4(2 + 3y)$ **h** $5(3 - 2y)$ **i** $2(2 - t)$

j x(x + 3) **k** $y(y - 5)$ **l** $z(4 - z)$

m $p(8 - p)$ **n** 3b(b + 2) **o** $4c(2c - 1)$

6 $4(6x + 3y)$

Any three from $24x + 12y$, $2(12x + 6y)$, $3(8x + 4y)$, $6(4x + 2y)$, $12(2x + y)$

C19.2 Rich task

Students should try trial and improvement methods.

Ex C19.2

1 **a** 8 **b** 9 **c** 7 **d** 8

e 6 **f** 8 **g** 7 **h** 9

2

x	x^3	Too high or two low?
2	8	too low
3	27	far too high
2.1	9.26	too low
2.2	10.64	too high
2.15	9.93	too low

Therefore the solution lies between 2.15 and 2.2.

The solution is 2.2 to 1 decimal place

3 8.3

4 **a** 4.3 **b** 6.3 **c** 5.4

d 20.5 **e** 26.4 **f** 0.6

Ex C19.3

1 **a** 8 **b** 8.1

2 $x(x + 4) = 70$

6.6

3 $30x - x^2 = 90$

3.4

4 $n^3 - n^2 = n$

1.6 (note that $n = 0$ and $n = -0.6$ are also solutions)

5 **a** 10 **b** 8.3

6 **a** 5 **b** 2.9 **c** 4.8

d 5.9 **e** 4.5 **f** −9.7

7 $n + 4$

35, 37, 39

Exam practice

1 if question is $x^3 - 3 = 17$, answer is 2.7

if question is $x^3 - 3x = 17$, answer is 3.0

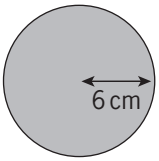
if question is $x^3 - 3x^2 = 17$, answer is 4.0

2 7.7

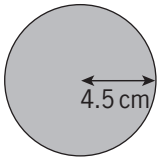
C20 Measures and accuracy

Check-in

1 **a** a circle radius 6 cm



b a circle radius 4.5 cm



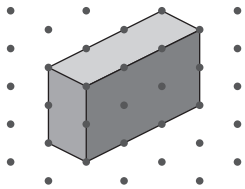
2 a 28 cm^2 **b** 10 mm^2

C20.1 Rich task

Cross section is E is a rectangle.

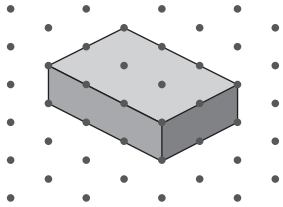
Ex C20.1

1 a i



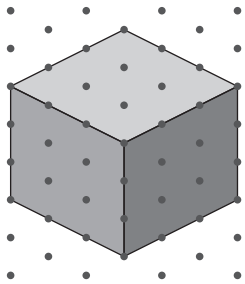
Six faces, twelve edges

ii



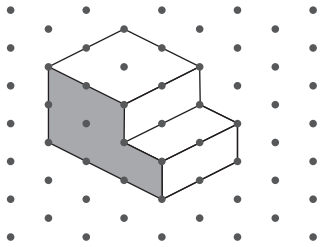
Six faces, twelve edges

iii



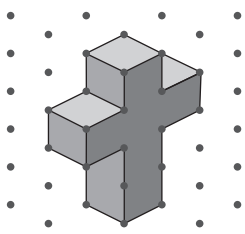
Six faces, 12 edges

iv



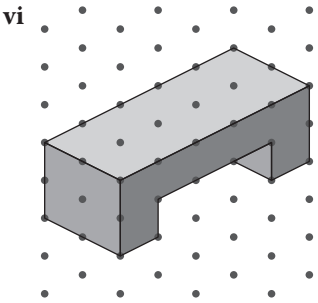
Eight faces, 18 edges

v



14 faces, 36 edges

vi



10 faces, 24 edges

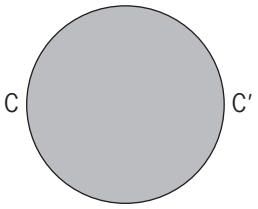
2 a AA′

rectangle

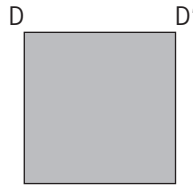
BB′

rectangle

CC′ circle



DD′

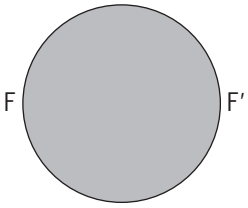


square

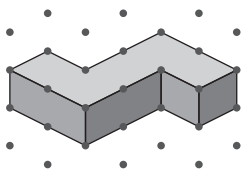
EE′

rectangle

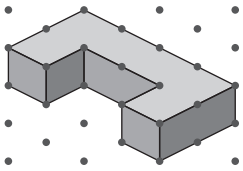
FF′ circle



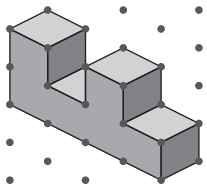
3 a



b



c

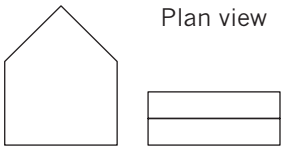


4 3 × 3 cube 26 small cubes have painted faces
4 × 4 cube 56 -do-
5 × 5 cube 98 -do-

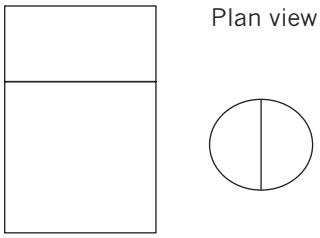
Ex C20.2

1 A view 2; B view 1; C view 3 for both buildings

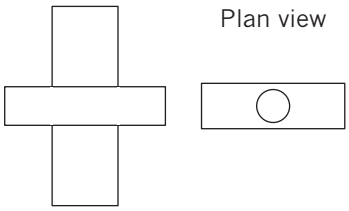
2 a



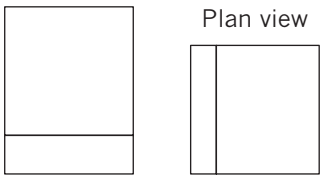
b



c

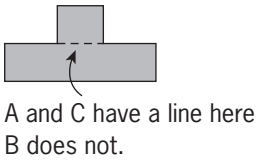


d

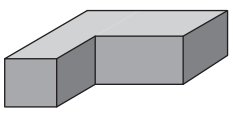


3 a pyramid **b** cone **c** cylinder **d** sphere
e a half cylinder cut along its axis
f a triangular prism

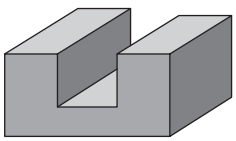
4 B;



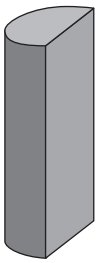
5 a



b



c



7 cube; sphere

C20.3 Rich task

Students should sketch out different possibilities of nets thinking about how they will fit together.

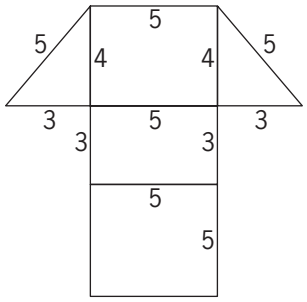
Ex C20.3

1 a 1st and 3rd **b** pyramid

c prism with a triangular cross-section

2 b The faces of a cuboid must be in pairs

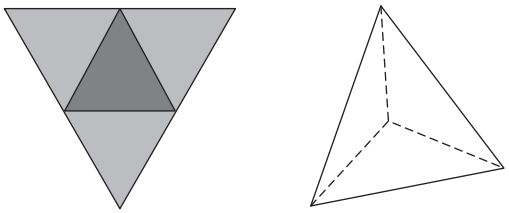
3 For example,



prism with a triangular cross-section

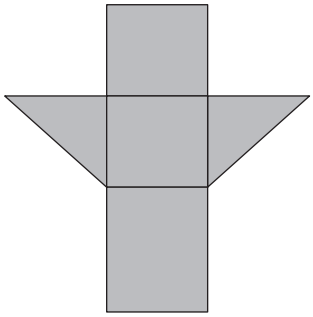
4 Cube A: **a** t **b** s **c** u
Cube B: **a** u **b** s **c** t

5 For example,



Tetrahedron (pyramid with a triangular base)

6 a For example,



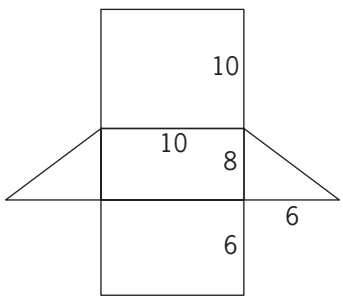
Ex C20.4

1 a 700 cm³ **b** 504 cm³ **c** 538 cm³

2 a 168 cm² **b** 64 cm³

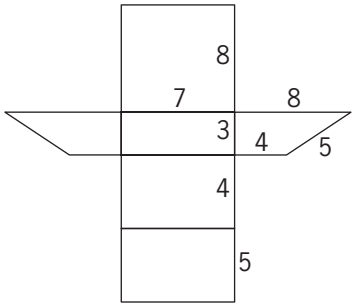
3 a 2.96 m² **b** £36.85 **c** £6.50

4 For example,



surface area 288 cm²

For example,



surface area 176 cm²

5 **a** 400 cm²; 25 cm × 20 cm **b** 20%

Ex C20.5

1 **a** 188 cm² **b** 151 cm² **c** 126 cm² **d** 126 cm²

2 3140 cm² or 0.314 m²

3 **a** 251 cm² **b** 151 cm² **c** 151 cm² **d** 220 cm²

4 **a** 314 cm² **b** 603 cm²

5 848 cm²

6 50%

7 **a** has the smaller surface area of 603 cm².

b has surface area of 730 cm²

8 A sheet 52 cm × 15 cm (area 780 cm²)

Exam practice

1 for example

2 84 cm²

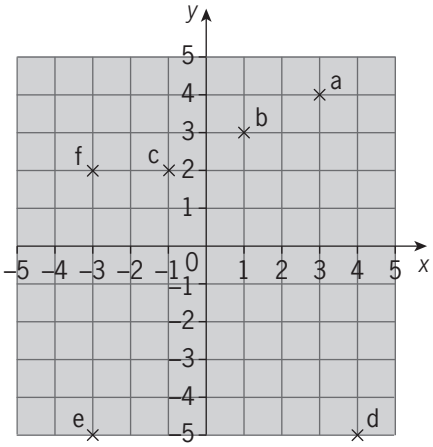
3 Join faces which are 5 cm × 4 cm 56 cm²

C21 Graphs

Check-in

1 **a** 9 **b** –2 **c** 12 **d** 2

2



C21.1 Activity

a It's a straight line. **b** Answer given.

c Because the line is flat at the top – their distance from London is unchanged for 3 hours.

d The line between b and c

e Their speed on the return journey was 50 mph.

Ex C21.1

1 **a** 200 m **b** 4 mins **c** 4 mins

d 100 m **e** 12 mins

2 **a** Twice **b** 300 m **c** 2 mins **d** 8 mins

3 **a** Y **b** Z **c** X

4 **a** A Aircraft B Train C Lorry
D Bicycle E Pedestrian

b A 50 m/s B 25 m/s C 10 m/s
D 6 m/s E 2 m/s

5 **a** 1 km **b** A to B **c** 4 km, 10 mins

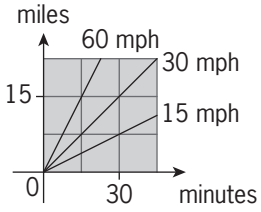
d 24 km/h **e** 6 km, 30 mins **f** 12 km/h

6 **a** 150 m/min or 2.5 m/s **b** 2 mins

c 500 m **d** 5 min **e** 100 m/min or 1.67 m/s

f 83 m/min or 1.4 m/s

7



8 **a** Stephen, 20 mins **b** 50 mins

c 30 km/h **d** 30 km/h

9 **a** X Table tennis ball
Y Cricket ball

b The table tennis ball bounces 5 times, bouncing lower each time. The cricket ball bounces twice only.

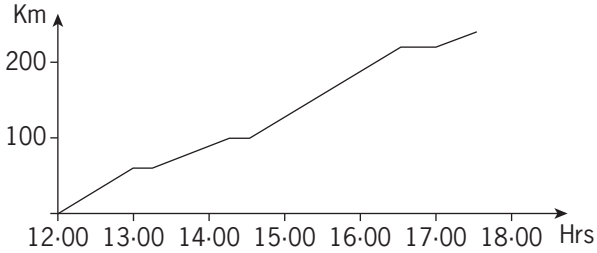
10 The oil level decreases steadily for 3 weeks, then rapidly for 1 week. Then no oil is used for 2 weeks. Then, after 6 weeks, the oil is refilled to the original level. Then the level reduces steadily again.
The weather was probably a little cold until week 3, then very cold for week 3 to 4, warm for weeks 4 to 6 and then a little cold for weeks 6 to 8.

11 The jogger starts running at about 12 km/h. After 10 mins he turns and runs back at about the same speed. He slows and turns again after 20 mins. He then runs forward more slowly, stopping and turning after 30 mins. He then runs back to the starting point, arriving after 40 mins.

12 The machine moves quickly away for 2 secs, then turns and moves quickly back for 3 secs, then turns and moves away again more slowly. After 10 secs it has stopped moving.

13 **a** The coach leaves London, calling at Scalsby, Garthorpe, Skeltoft and Lincoln, It stops for 15 mins at Scalsby and Garthorpe, and it stops for 30 mins at Skeltoft. The whole journey takes $5\frac{1}{2}$ hours.

b



c 60 km/h, 40 km/h, 60 km/h, 50 km/h

d 44.5 km/h

C21.2 Rich task

Students should notice that the green line implies that the person's average heart rate is lower both before and after exercise perhaps implying that the person is fitter.

Ex C21.2

1 **a** **i** £15 **ii** £15 **iii** £65 **b** 23 kg

2 **a** L Firm B **b** Firm A charges £10
M Firm A Firm B charges £15

c Firm B is cheaper for longer journeys, 20 miles

3 **a** P Elex **b** Elex charges £60
Q Trizity Trizity charges £65

c 400 units **d** Trizity

4 **a** **i** The most economical speed is about 55 mph for petrol engines and 60mph for diesel engines.
ii The miles travelled per gallon decreases.
iii Diesel engines do more miles to the gallon than petrol engines at all speeds.

b

removing a bike rack	Increases fuel efficiency
accelerating fast	Decreases fuel efficiency
eating while you drive	No effect on fuel efficiency
tyres inflated at correct pressure	Increases fuel efficiency
having a clean air filter	Increases fuel efficiency
breaking hard and often	Decreases fuel efficiency

5 **a** 500 mins **b** £20 **c** 550 mins

d For less than 100 minutes per month.

6 **a** £80

b Over 6 m and less than or equal to 7 m **c** £150

7

a the distance travelled by a car driven at constant speed	Graph R
b the temperature of a bowl of soup left uneaten	Graph P
c the amount of money left in a purse on a shopping trip.	Graph S
d the temperature outdoors on a cloudy day	Graph Q

8 eg. on day 3 sheena uses half the oil for frying. When the remaining oil is cold she puts it back in the bottle. She uses more oil on day 6 and almost empties the bottle.

9

a all at one go	Graph Y
b in two gulps, half a cup each time	Graph Z
c sip by sip	Graph W
d not at all.	Graph X

10 **a** The baby's weight reduces initially and after 10 days starts to increase again. By day 25 the baby has regained the weight it lost and continues to increase more rapidly.

b 3.0 kg **c** Day 10

d By day 25 **e** 400 g

11 A Z B X C Y

The different shapes of the beakers give different graphs.

Beaker A is widest at the bottom and so initially the height increases slowly. Beaker B has a constant width, so the height increases uniformly with time. Beaker C is narrower at the bottom so initially the height increases more quickly.

C21.3 Rich task

Students might use Autograph or similar software to sketch graphs, this will help them to become familiar with the shapes of different graphs.

Ex C21.3

1 $y = x^2$

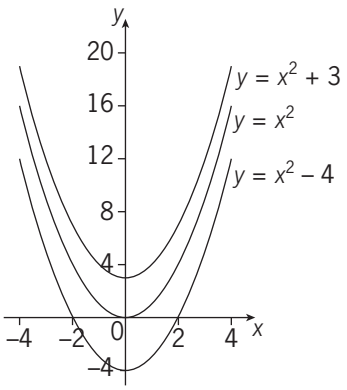
x	−4	−3	−2	−1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

$y = x^2 + 3$

x	−4	−3	−2	−1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
y	19	12	7	4	3	4	7	12	19

$y = x^2 - 4$

x	−4	−3	−2	−1	0	1	2	3	4
x	16	9	4	1	0	1	4	9	16
−4	−4	−4	−4	−4	−4	−4	−4	−4	−4
y	12	5	0	−3	−4	−3	0	5	12



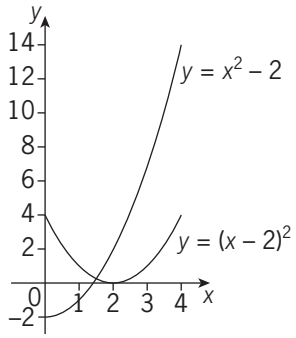
The curves can be transformed by translations: +3 in the y direction, and −4 in the y direction

2 $y = x^2 - 2$

x	0	1	2	3	4
x^2	0	1	4	9	16
-2	−2	−2	−2	−2	−2
y	−2	−1	2	7	14

$y = (x - 2)^2$

x	0	1	2	3	4
$x - 2$	−2	−1	0	1	2
y	4	1	0	1	4

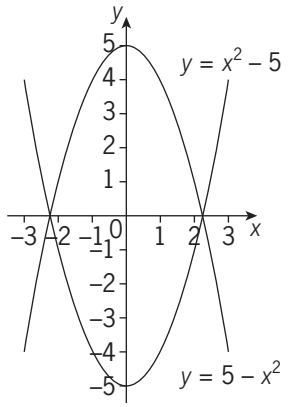


3 $y = x^2 - 5$

x	−3	−2	−1	0	1	2	3
x^2	9	4	1	0	1	4	9
-5	−5	−5	−5	−5	−5	−5	−5
y	4	−1	−4	−5	−4	−1	4

$y = 5 - x^2$

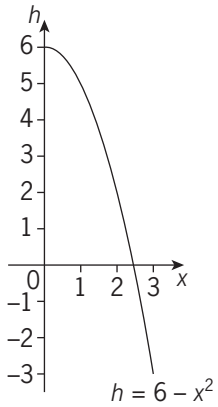
x	−3	−2	−1	0	1	2	3
5	5	5	5	5	5	5	5
$-x^2$	−9	−4	−1	0	−1	−4	−9
y	−4	1	4	5	4	1	−4



a Reflection in the x-axis **b** (2.2, 0) and (−2.2, 0)

4 $h = 6 - x^2$

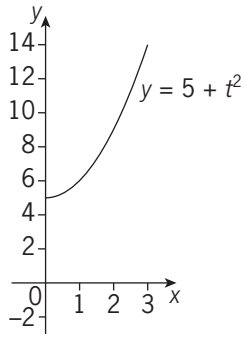
x	0	1	2	3
h	6	5	2	−3



a 6 km **b** 2.4 km **c** 1.4 km

5 $y = 5 + t^2$

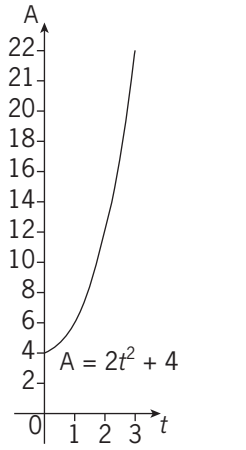
t	0	1	2	3
y	5	6	9	14



a **i** 5°C **ii** 14°C **b** 2.2 hours

6 $A = 2t^2 + 4$

t	0	1	2	3
$2t^2$	0	2	8	18
A	4	6	12	22

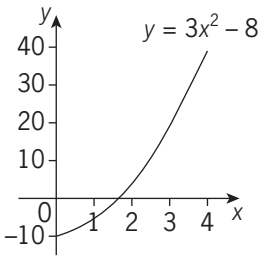


a 4 m² **b** 16.5 m² **c** 1.7 days

d No. This function shows the algae increasing rapidly, without a limit, but the lake must have a finite area.

7 $y = 3x^2 - 8$

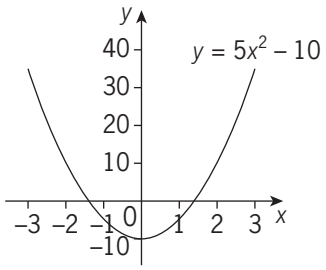
x	0	1	2	3	4
$3x^2$	0	3	12	27	48
-8	−8	−8	−8	−8	−8
y	−8	−5	4	19	40



a 1.6 **b** 2.4 **c** 3.6 **d** $x \leq 1.6$

8 $y = 5x^2 - 10$

x	–3	–2	–1	0	1	2	3
$5x^2$	45	20	5	0	5	20	45
–10	–10	–10	–10	–10	–10	–10	–10
y	35	10	–5	–10	–5	10	35

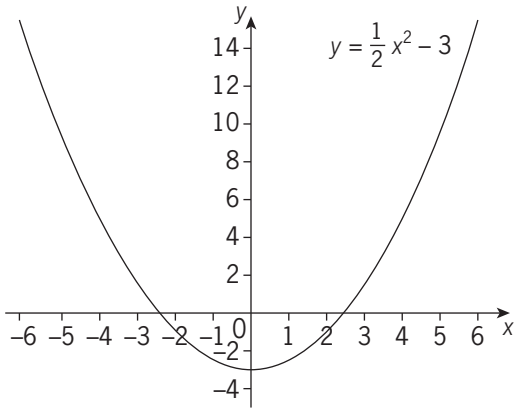


a 1.4, –1.4 **b** 0.6, –0.6 **c** 2.8, –2.8

d $-1.4 \leq x \leq 1.4$

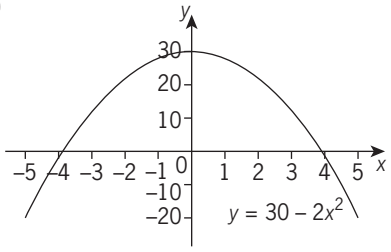
9 $y = \frac{1}{2}x^2 - 3$

x	–6	–4	–2	0	2	4	6
$\frac{1}{2}x^2$	18	8	2	0	2	8	18
–3	–3	–3	–3	–3	–3	–3	–3
y	15	5	–1	–3	–1	5	15



a 2.4, –2.4 **b** 5.1, –5.1

10



a **i** 3.9, –3.9 **ii** 2.2, –2.2 **b** $-2.2 \leq x \leq 2.2$

C21.4 Rich task

Position is fixed using coordinates. OS gives heights using contours.

Ex C21.4

1 A (300, 100, 30) B (200, 100, 50)
C (200, 300, 60) D (300, 200, 20)
E (300, 400, 75) F (100, 0, 40)

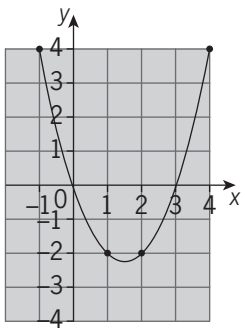
2 A (2, 5, 10) B (2, 3, 10)
C (4, 5, 20) D (5, 1, 20)
E (6, 3, 30) F (7, 4, 40)
G (6, 5.5, 30) H (1, 2, 0)
I (6.5, 1, 30) J (7.5, 3, 30)
K (3, 1, 10) L (4, 3.5, 20)

3 **a** A (20, 20, 50) B (40, 30, 50) C (30, 40, 100)
D (60, 50, 50) E (70, 40, 0) F (40, 15, 0)
G (20, 55, 100) H (60, 20, –50)
I (70, 10, –100) J (20, 40, 150)

b (40, 50, 50) (80, 20, –100) (20, 30, 100) (5, 40, 150) (20, 5, 0) (40, 50, 50) (75, 30, –50) (50, 5, –50)

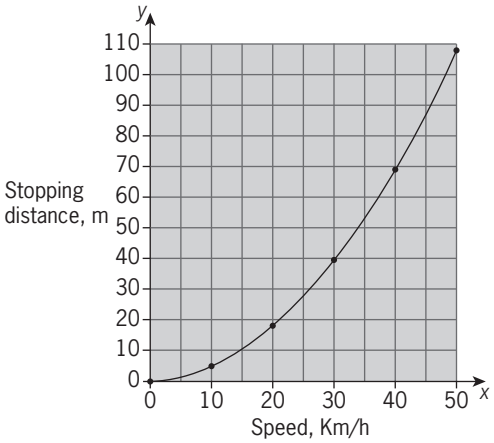
Exam practice

1 **a** 0 **b**



2 **a** Speedy by $\frac{1}{2}$ 1.60 **b** 6 miles

3 **a**



b 53 m,

c Driver's claim is not correct.
Stopping distance of 80 m implies speed of approx 44 km/h from graph, but skid marks are only from application of brakes which does not allow for reaction time. So skid marks of 80 m imply stopping distance greater than 80 m and therefore speed greater than 44 km/h.

Case study: Radio

a 2000Hz = 2kHz

b 2GHz = 2000 MHz

Waves a and c have same volume. Wave b is louder.

Waves a and b have same pitch. Wave c has higher pitch.

a 100.0 FM has frequency:
i 100MHz **ii** 100 000 000 Hz

b Speed = 100 000 × 3 = 300 000 m/s

c Frequency (kHz) = speed (m/s) / wavelength (m)

d Frequency = 300 000 /280 = 1 071.42 856 kHz = 1 071 428.5 Hz

e AM band

The Frequency is 1 071 kHz which is in the AM band and would be shown on the dial by 1071 AM

a 5 mins

b **i** 5 mins **ii** 5 mins **iii** 50 mins

c **i** 3:45pm **ii** 3pm **iii** 3:20pm

C22 Everyday arithmetic and bounds

Check in

1 27

2 **a** 1.3 **b** 0.3

3 **a** 3.56 **b** 8.04 **c** 0.06

Ex C22.1

1 **a** **i** 24°C **ii** 74°F **b** 55 m

2 **a** yes **b** 55 mph

3 **a** **i** 127 miles **ii** 203.2 km **b** 92 miles

4 **a** 26 minutes

b **i** 2312, 2353 **ii** 28 minutes 23 minutes

c arrives earlier

Ex C22.2

1 **a** **i** 4.4°C **ii** 36.9°C **iii** –1.1°C

b **i** 71.6°F **ii** 23°F **iii** 28.4°F

2 John's estimate = 25, true value = 25.9

3 **a** 37 km/h **b** 49 km/h

4 15.8 mg

Ex C22.3

1 £75.96, 3, £25.50

2 £1 = €1.41

3 Paris = £108.70, New York = £102.86, New York

4 9.25 BST next day

5 Students should think about flight times and time differences. If the flight time is less than the time difference then it is theoretically possible.

C22.4 Problem

Togo has roughly 310 people were square mile, Tuvalu has around 991 people per square mile. Tuvalu is more crowded.

Rich task

The one filled with sugar will be heavier.

Ex C22.4

1 **a** **i** 19.2 km/h **ii** 17.1 km/h

b **i** 80 mph **ii** 72 mph

2 **a** 7.43 mph **b** 4.88 mph

3 40.6 km/h

4 4 km

5 **a** 3.1×10^{-5} m^3 or 31 cm^3 **b** 1.84 kg or 1840 g

6 258 per square mile

7 2500 sec = 41 minutes 40 seconds

Ex C22.5

1 **a** 67.5 68.5 **b** 3750 3850 **c** 725 735

d 4945 4955 **e** 17.55 17.65 **f** 23.35 23.45

2 **a** 27.5 – 28.5 **b** 47.5 – 48.5 **c** 111.5 – 112.5

d 555 – 565 **e** 9345 – 9355 **f** 9325 – 9375

g 510 – 530 **h** 415 – 425 **i** 43.55 – 43.65

j 5.15 – 5.25 **k** 7.25 – 7.35 **l** 39.5 – 40.5

m 99.5 – 100.5

3 **a** Yes, could be 154.5 cm **b** 16.65 – 16.75 seconds

c Yes, could be 4.375 m

4 **a** 48.5% **b** 78

Exam practice

1 3min 59.35sec 3min 59.45sec

2 **a** 60mph

b 40miles per gallon (using 4.5 litres = l gallon)

3 2.4 kg +5.10

C23 Trigonometry

Check in

1 **a** 40 cm^2 **b** 30 cm^2

2 **a** 250 **b** 6300 **c** 41 **d** 250 **e** 3500

f 4 **g** 5.6 **h** 40 **i** 4.1 **j** 5.2

C23.1 Rich task

Oatiebisks have the most efficient packaging in terms of surface area of the packaging to the weight of the cereal.

Ex C23.1

1 **a** **i** 12 **ii** 24 **b** **i** 15 **ii** 45

c **i** 30
ii 60 (would overfill the box, but cannot fill exactly)
45 (if cubes can be cut in half)

2 **a** 756 cm^3 **b** 480 cm^3 **c** 306 cm^3

3 **a** 1083 cm^3 **b** 1.083 litres

4 **a** 3 075 000 cm^3 **b** 3075 litres

5 **a** 90 000 cm^3 **b** 90 litres

6 **a** 452 cm^3 **b** 61%

7 **a** 3 cm **b** 6 cm **c** 5 cm **d** 9cm

8 20.4 cm

9 yes, it is 80 cm tall

10

	Length, cm	Width, cm	Height, cm
a	10	10	10
b	10	20	5
c	10	50	2
d	10	100	1
e	20	50	1

The 10 cm × 10 cm × 10 cm pack is suitable.

11 The cube with dimensions 10 cm × 10 cm × 10 cm has the smallest surface area

C23.2 Rich task

The jumbo pack is the most efficient in terms of mass to surface area of packaging.

Ex C23.2

1 **a** 60 cm^3 **b** 75 cm^3

2 102 m^3

3 350 cm^3

4 **a** **i** 80 cm^2 **ii** 640 cm^3
b **i** 72 cm^2 **ii** 216 cm^3
c **i** 92 cm^2 **ii** 184 cm^3

5 **a** 0.96 m^2 **b** 2.4 m^3

6 An infinite number of ways.
The halves are cuboids or prisms with either triangular or trapezium cross-sections

7 **a** 10 cm^2 **b** 7 cm^2 **c** 5 cm^2 **d** 6 cm^2

C23.3 Rich task

By measuring the volume of the water and then pouring it over the metal, subtracting the first measurement from the second, you should be able to find the volume of the metal.

Ex C23.3

1 **a** 314 cm^3 **b** 170 cm^3 **c** 100 cm^3 **d** 126 cm^3

2

Vegetable soup	470 ml
Marmalade	360 ml
Kidney beans	280 ml
Tomatoes	840 ml

3 **a** 78 500 cm^3 **b** 70 650 cm^3

4 no, there is just over 1 litre in the bottle

5 2 litre engine

6 **a** 4 cm **b** 6 cm **c** 2 cm **d** 12 cm

7 **a** 5.6 cm **b** 2.6 cm **c** 4.0 cm **d** 5.9 cm

C23.4 Rich task

Length scale factor = 2

Area scale factor = 4

Volume scale factor = 8

Ex C23.4

1 **a** 3 **b** 9 **c** 27

2 **a** **i** 80 mm **ii** 600 mm^2 **iii** 2000 mm^3
b **i** 3.5 cm **ii** 1.4 cm^2 **iii** 0.5 cm^3

3 **a** 7400 metres **b** 2 100 000 m^2 **c** 0.005 km^3

4 **a** 0.315 km **b** 0.0445 km^2 **c** 0.004 km^3

5 Volume = 0.459 m^3, diameter = 1 m, area = 0.133 m^2

Exam practice

1 **a** 312 cm^3 **b** 113 cm^3

2 125 cm^3

3 **a** 1600 cm^2 **b** 12.5 cm^3

C24 Graphs 2

Check in

1 **a** $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$ **e** 1

2 **a** 1 **b** 1 **c** $\frac{3}{10}$ **d** $\frac{2}{5}$

3 **a** 0.9 **b** 0.4 **c** 0.85

4 **a** 0.01 **b** 0.25 **c** 0.35 **d** 0.05 **e** 0.36

Ex C24.1

1 **a** Unlikely **b** Evens **c** Unlikely **d** Impossible

2 **a** unlikely **b** evens

c **i** unlikely **ii** highly unlikely **d** certain

3 **a** probability that X will happen is 0.6

b probability that Y will happen is $\frac{1}{4}$

c probability that Z will happen is 35% scale

4

Ex C24.2

1 **a** $\frac{7}{25}$ **b** $\frac{18}{25}$ **c** $\frac{9}{25}$

d $\frac{16}{25}$ **e** $\frac{10}{25}$ **f** $\frac{17}{25}$

2 **a** 0 **b** 1 **c** $\frac{18}{30}$ **d** $\frac{12}{30}$

3 0.2

4 **a** 3 is not a factor of 7 **b** 4

5 The second bag, because the fraction of green counters is higher

Ex C24.3

1 **a** 30

b **i** $\frac{7}{30}$ **ii** $\frac{15}{30}$ **iii** $\frac{12}{30}$

iv $\frac{9}{30}$ **v** $\frac{9}{30}$ **vi** $\frac{21}{30}$

2 **a** 0.45 **b** 0.25 **c** 0.3

3 **a** red and blue and orange **b** yellow

c **i** 0.2 **ii** 0.32 **iii** 0.23

4 **a** eg odd and even **b** eg 1 or 2, 3 or 4, 5 or 6

c eg even and prime, 2 is both

C24.4 Rich task

Students answers should address whether the spinner landed on any one particular colour more than you would expect and also whether this is sufficient evidence that the spinner is biased or not, and perhaps should suggest that further tests might be required to see if any noticeable trend continues.

Ex C24.4

1 140

2 120

3 0.625

4 **a** $22 \times 2 = 44$ **b** $\frac{34}{120}$ **c** 50

5 **a** **i** 0.11 **ii** 0.32 **iii** 0.27

iv 0.41 **v** 0.87

b **i** 80 **ii** 120

6 174

Ex C24.5

1 fair; would expect 140

2 fair; all frequencies between 31 and 35

3 **a** 0.4 0.35 0.37 0.33 0.36 0.37 0.37 0.36 0.36 0.35

b 0.35 **c** biased; fair is 0.5

4 **a** $\frac{8}{36}$ $\frac{9}{36}$ $\frac{10}{36}$ $\frac{9}{36}$

b fair; all approximately equal

5 **a** $\frac{11}{30}$

b biased; other relative frequencies are $\frac{5}{30}$ or less

Ex C24.6

1 **a** table **b** **i** $\frac{1}{12}$ **ii** $\frac{3}{12}$

2 **a** table **b** $\frac{1}{4}$

3 **a** table **b** $\frac{7}{12}$

c 2 girls; P(2 boys) = $\frac{2}{12}$, P(2 girls) = $\frac{3}{12}$

4 **a** table

b **i** $\frac{2}{36}$ **ii** 0 **iii** $\frac{6}{36}$

c even; P(even) = $\frac{27}{36}$ P(odd) = $\frac{9}{30}$

5 **a** table **b** **i** $\frac{1}{9}$ **ii** $\frac{3}{9}$

c both have 2 out of 9 outcomes

Exam practice

1 **a** eg. 5, 6, 7, 8, 9

b eg. 1, 3, 5, 7, 9

c eg. 1, 2, 3, 5, 5

2 **a** $\frac{11}{44}$

b $\frac{22}{44}$

3 $\frac{15}{36}$

Case study: Weather

16°C; 2.5°C; -8.9°C

Belfast 7°C

Birmingham 6°C

Cardiff 7°C

Edinburgh 5°C

London 7.5°C

Newcastle 6°C

South West England had the highest temperatures

Scotland had the lowest temperatures

Difference in temperature = 12.5°C

Westerly wind (270°), 15 knots

South Easterly wind (135°), 30 knots

North Westerly wind (305°), 11.7 knots