

# Student book answers

## A1 Integers and decimals

### Check in

- 1 **a** four thousand      **b** four hundred  
**c** four tenths      **d** four thousandths
- 2 **a** check number line      **b**  $-3, -2.4, -1.8, 0, +1.5, +5, +6$

### Ex A1.1

- 1 **a** no      **b** yes      **c** yes
- 2 **a** no      **b** no      **c** yes
- 4 821, 128; 693; 495

Students will find the number is always divisible by 99.

This can be proven algebraically:

Using a place value table:

100 s	10 s	1 s
x	y	z

$$(100x + 10y + z) - (100z + 10y + x) = 99x - 99z = 99(x - z)$$

Students may find something similar using numbers rather than algebra.

### Ex A1.2

- 1 **a**  $-2$       **b** 1      **c**  $-19$       **d** 8  
**e**  $-27$       **f** 20      **g** 4      **h**  $-3$   
**i** 77      **j** 12      **k**  $-5$
- 2 **a** 1.125      **b**  $\frac{11}{12}$       **c** 5.5
- 3 **a** 72,  $-54$       **b**  $3 \times 4 + 4 \times -3 = 0$   
**c** answered 3 questions, all correct; answered 10 questions, 6 correct; answered 17 questions, 9 correct
- 4 **a**  $\pm 1, 0$       **b**  $n < -1, 0 < n < 1$

### Ex A1.3

- 1 **a** 19.37      **b** 19.4      **c** 0.00752  
**d** 0.008      **e** 153.262      **f** 153.26
- 2 **a** 76 000      **b** 76 300      **c** 23 400  
**d** 58 000      **e** 46 000      **f** 23.40

### Ex A1.4

- 1 **a** 61      **b** 23.83      **c** 1.5625      **d** 0.35  
**e**  $1.\dot{3}$       **f** 156.25      **g** 500      **h** 7.5
- 2 **a** 1.736 307 692      1.7      **b** 3.12244898      3.12  
**c**  $-0.191347436$        $-0.19$       **d** 12.68283582      12.7  
**e** 3.839270764      3.84      **f** 2.370021097      2.4

- 3 **a** 0.25      **b** 0.1      **c** 0.4  
**d** 0.27      **e** 5      **f** 6.25  
**g** 2      **h** 4      **i** 2.5
- 4 **a** 1296      **b** 78 125      **c** 100 000  
**d** 262 144      **e** 410.0625      **f** 0.004 096

## Exam practice

- 1 **a** 1.716961 498      **b** 2
- 2  $2\frac{2}{9}$  or  $2.\dot{2}$
- 3 Uranus; she would weigh 60 kg

## A2 Summary statistics

### Check in

- 1 **a** Nicola – primary; Maddy – secondary  
**b** Primary data is data you collect yourself e.g. from questionnaires  
Secondary data is data collected from other sources e.g. published statistics
- 2 **a** **i** Because 4 occurs in both of the first two groupings  
**ii** Because no times below 3 hours have been allowed for in the table  
**b** Using groupings like:  
less than 3 hours; at least 3 hours but less than 4 hours, etc
- 3 **a**  $22\frac{1}{2}$       **b** 21      **c** 6      **d** 21

### Ex A2.1

- 1 various
- 2 **a** 11%      **b** % of Welsh with AB for example
- 3 **a** survey      **b** General practitioners  
**c** various      **d** various

### Ex A2.2

- 1 **a** **i** cont.      **ii** disc      **b** **i** cont.      **ii** disc  
**c** **i** disc      **ii** cont      **d** **i** disc      **ii** cont
- 2 **a** other years different preferences  
**b** **i** List all 1000 names, number 000 to 999, generate 50 random 3-digit numbers on calculator, select students with these numbers  
**b** **ii** List all 1000 names, generate 2-digit random number between 01 and 20, select this student then every following 20<sup>th</sup> name

- 3 a** proportion of boys too high  
**b i** 200 **ii**  $\frac{4}{5}$  **iii** 40 **iv** 10
- 4 a** need to include patients who rarely use antibiotics  
**b** 15–45 years (parents of children) **c** various
- 5** various
- 6** 21 girls under 16  
 31 boys under 16  
 14 girls aged 16–18  
 24 boys aged 16–18

### Ex A2.3

- 1 a i** B 4.0 sec., G 4.2 sec.  
**ii** B 3.8 sec., G bi-modal distribution  
**iii** B 5.3 sec., G 6.3 sec.  
**iv** B 5.18 sec., G 6.15 sec.  
**v** B 2.4 sec., G 2.1 sec.
- b** various
- 2 a**
- | P                 |   | Z                 |
|-------------------|---|-------------------|
| 9 8 8 6           | 0 | 8 9               |
| 9 8 7 7 4 3 2 1 0 | 1 | 1 2 4 5 6 7 7 8 9 |
| 9 7 4 2           | 2 | 1 3 4 5 8 9       |
| 7 1               | 3 | 2 3               |
- Key 1|3|2 means puzzle P 31 mins Z 32 mins
- b i** P 31 mins, Z 25 mins  
**ii** P bi-modal, Z 17 mins  
**iii** P 17.5 mins, Z 19.5 mins  
**iv** P 17 mins, Z 18 mins  
**v** P 14 mins, Z 11 mins
- c** various
- 3** 8.29
- 4** 78.6%

### Ex A2.4

- 1** plot positive skew  
**2** plot positive skew  
**3** plot positive skew  
**4** plot positive skew  
**5** plot symmetrical

### A2.5 Rich task, p24

For: the boys have the fastest recall time; the lower quartile is lower for boys; the longest recall time for boys is shorter than the longest recall time for girls.  
 Against: mean recall for girls is lower; upper quartile for girls is significantly lower.

### Ex A2.5

- 1 a** plot  
**b** Students' answers will vary and should address issues such as the spread of data and the inter quartile range.
- 2 a** plot  
**b** Students' answers will vary but should address issues such as the spread of data and should focus on the inter quartile range.
- 3** eg. more than 75% of older girls spend longer than any of younger girls
- 4** eg. median times same, some girls have longer times than any boys, fastest 25% of boys faster than fastest 25% girls

### A2.6 Rich task, p26

The ranges of the two data sets are the same.  
 Calculate the mean; 16 for 15-year olds and 15 for 11-year olds

### Ex A2.6

- 1 a i** 6 **ii** 6 **iii** 5.8 **iv** 4 **v** 3  
**b i** 5 **ii** 5 **iii** 5 **iv** 4 **v** 2  
**c i** 4 **ii** 6 **iii** 6.6 **iv** 6 **v** 5  
**d i** 6 **ii** 6 **iii** 5.7 **iv** 6 **v** 3
- 2 a** mode: B 18, G 15; median: B 16, G 16; mean: B 16.2, G 16.1; range: B 6, G 6; IQR: B 3.5, G 3  
 various answers
- b** various answers

### Ex A2.7

- 1 a**  $20 < B \leq 30$  **b** £24.20  
**c** It uses all the data available
- 2 a** £88 **b**  $80 < M \leq 120$  **c** No
- 3** Mean: Teachers 11.5 km Office Workers 14.1 km  
 Median class: Teachers  $10 < d \leq 15$  Office  
 Workers  $10 < d \leq 15$   
 Modal class: Teachers  $10 < d \leq 15$  Office  
 Workers  $10 < d \leq 15$   
 The mean supports David's conclusion, although there is no great difference in distances travelled by the 2 groups.

### A2.8 Rich task, p30

What kind of services the council is providing to home owners; local council; question 1 is possible; question 2 should have boxes for "Not very happy" and "Very unhappy"; question 3 should have a box for "More than one

year ago”; yes, in question 2 you can tick both “Yes they are okay” and “Mostly happy”.

### Ex A2.8

- 1 **a** eg. may not listen to show and both boxes are positive
- b** eg. may not listen to radio; boxes do not cover range of answers; 1 hour in two boxes
- c** various
- 2 **a i** eg. boxes do not cover range of answers
- b** various
- 3 various

### Exam practice

- 1 **a** 4.1 mins **b**  $4 < m \leq 5$
- 2 Would need  $\frac{4}{15}$  of each type of official and this does not give whole numbers
- 3 median: B 66, G 62; mode: B 66, G 57; mean: B 64, G 64.7; range: B 44, G 46  
There is no significant difference between Girls and Boys' results.

## A3 Constructions

### Check in

- 1  $a = 35^\circ, b = 105^\circ$
- 2 Check constructions
- 3 Check constructions

### Ex A3.1

- 1 **a i** 6 m **ii** 8 m **iii** 14 m
- b i** 18 feet **ii** 24 feet **iii** 42 feet
- 2 **a** 23 cm **b** 38 cm **c** 46 cm **d** 162 kg
- e** 177 kg **f** 189 kg **g** 640 g **h** 780 g
- i** 850 g **j**  $10.4^\circ\text{C}$  **k**  $11.9^\circ\text{C}$  **l**  $12.6^\circ\text{C}$
- m** 3.26 **n** 3.38 **o** 3.42 **p** 7.25 m
- q** 8.75 m **r** 9.5 m
- 3 1.189 m by 0.841 m
- 4

	Volume		Mass		Height	
	litres	ml	grams	kg	metres	mm
Milk	2.27	2270	2400	2.4	0.32	320
Ice-cream	2.5	2500	1470	1.47	0.15	150

- 5 **a i** 60 cm **ii** 24 km **iii** 2.5 kg
- iv** 27 litres **v** 2 litres
- b i** 3.3 feet **ii** 25 miles **iii** 18 lbs

**iv** 31.5 pints **v** 3.5 pints

- 6 196 lbs and 169 lbs, 27 lbs lost
- 7 **a** 256 km 14 km/litre
- b** 4 gallons 40 miles/gallon
- 8 Every country in the world has adopted the metric system except for Liberia, Myanmar and the US.
- 9 Historically, the metre was defined as one ten-millionth of the distance from the equator to the North Pole. In 1983 the metre was redefined as the distance travelled by light in  $\frac{1}{299\,792\,458}$  of a second, where 299 792 458 m/s is the speed of light.

### Ex A3.2

- 1 **a**  $59^\circ, 51^\circ, 70^\circ$  **b**  $38^\circ, 81^\circ, 61^\circ$
- c**  $83^\circ, 56^\circ, 41^\circ$  **d**  $119^\circ, 32^\circ, 29^\circ$
- 2 AB = 6.6 km  
BC = 7.3 km
- 3 2 km
- 4 no
- 5 due east or due west
- 6 tetrahedron (triangular-based pyramid)
- 7 32 metres
- 9 Take bearings of a point on the opposite bank from two places, P and Q, on this side of the canal. Measure the distance PQ. Make a scale drawing and measure the canal's width.
- 10 no, there are two possible angles for C
- 11 An impossible triangle. Alter either  $57^\circ$  or the length of one side to make it possible.

### Ex A3.3

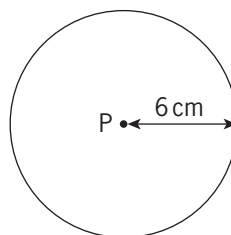
Check students' constructions.

### A3.4 Rich task, p44

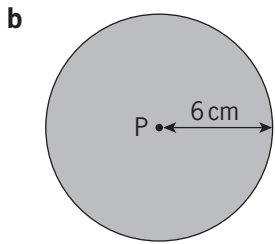
Construct a perpendicular bisector of the line PQ.

### Ex A3.4

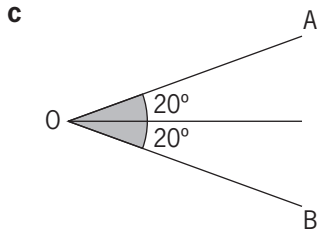
1 a



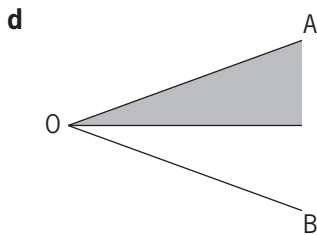
the circle



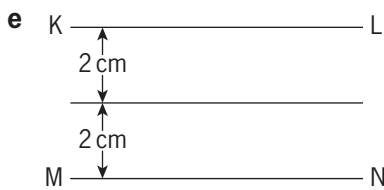
the area inside the circle



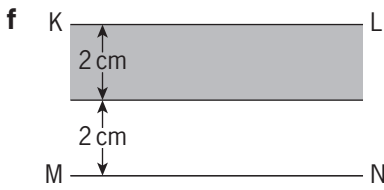
the line bisecting the angle AOB



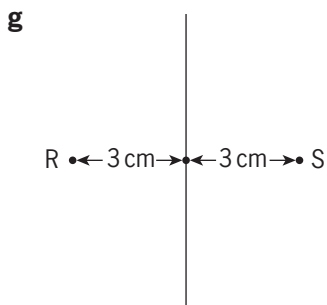
the area between the line OA and the bisector



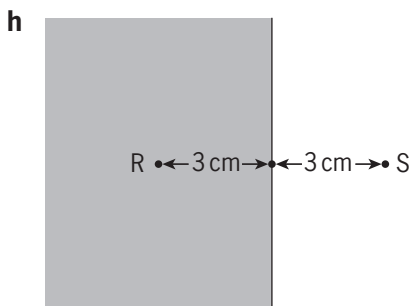
the line parallel to KL and MN and midway between them



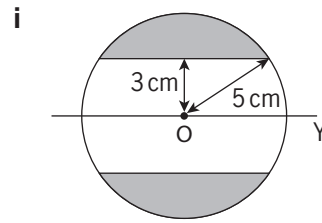
the area between the line KL and the mid-line



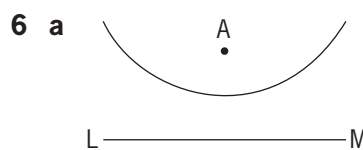
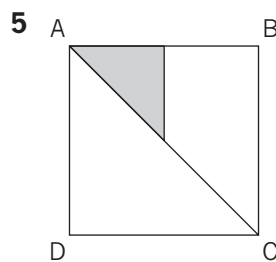
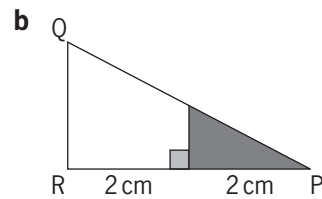
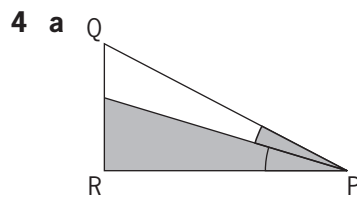
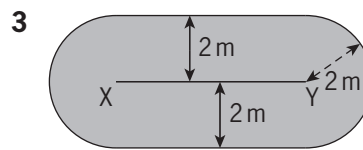
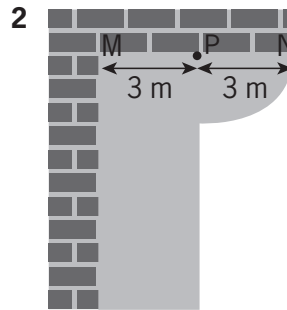
the perpendicular bisector of the line RS



the area on the side of the perpendicular bisector that contains R



the area inside two segments of circle centre O radius 5 cm



**7** A hypocycloid is a curve generated by the trace of a fixed point on a small circle that moves around a larger circle.

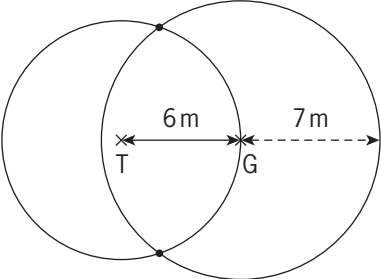
### A3.5 Rich task, p46

060°, 150°, 240°, 330°. It will take around 1 hour 7 minutes to complete the course.

#### Ex A3.5

- i 156 km ii 108 km
  - Dublin to Belfast 108 km, bearing 010°  
Belfast to Liverpool 188 km, bearing 123°
- 270°
  - 045°
  - 225°
- 49 km
  - 70 km
- 685 km on bearing 188°
- $(180 + x)^\circ$
- Bearing of A from B is 240°  
Bearing of A from C is 290°
- 253 km per h on a bearing of 067°

#### Exam practice

- 
- Check students' construction
- 115 km 244°
  - 190 km 260°

#### Case study: Sport

- 0.01s
  - 0.1s

Rank	Lower bound (s)	Time (s)	Upper bound (s)	Athlete
1	9.575	9.58	9.584	Bolt
2	9.685	9.69	9.694	Gay
3	9.715	9.72	9.724	Powell
4	9.785	9.79	9.794	Greene
5	9.835	9.84	9.844	Bailey
	9.835		9.844	Surin
7	9.845	9.85	9.854	Burrell
	9.845		9.854	Gatlin
	9.845		9.854	Fasuba
10	9.855	9.86	9.864	Lewis
	9.855		9.864	Fredericks
	9.855		9.864	Boldon
	9.855		9.864	Obikwelu

### A4 Factors, multiples and ratio

#### Check in

- 1, 2, 3, 4, 6, 12, 24
- 13, 26, 39, 52, 65, 78
- 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
- 45
  - 100

#### Ex A4.1

- $2 \times 2 \times 2 \times 2 \times 3$
  - $3 \times 3 \times 3$
  - $2 \times 3 \times 3 \times 3$
  - $2 \times 2 \times 2 \times 3 \times 5$
  - $2 \times 2 \times 2 \times 3 \times 3$
  - $2 \times 3 \times 7 \times 7$
  - $7 \times 53$
- 432 6
  - 3960 3
  - 3384 2
  - 196 7
  - 390 5
- A prime number squared has exactly three factors because the only factors are: 1, the prime number, and the square itself. (the only nontrivial divisor of the square is the prime number).
  - The numbers that have exactly four factors are either product of two primes (then the only factors will be 1, the product, and the two primes) or a cube of a prime (then the only factors will be 1, the cube, the square of the prime, and the prime itself).

#### A4.2 Rich task, p56

The ratio tends to 1:1.61803 – the golden ratio.

#### A4.2 Rich task 2, p56

A3:  $297 \times 420$  mm  
 A4:  $210 \times 297$  mm  
 A5:  $148 \times 210$  mm  
 A6:  $105 \times 148$  mm  
 Ratio is always 1:1.41

#### Ex A4.2

No questions, only activities

#### A4.3 Rich task, p58

Fred is 2, Marvin is 6.  
 Fred will be 8, Marvin will be 12.

#### Ex A4.3

- £8 : £40
  - £56 : £8
  - £24 : £84
  - £56 : £40
  - £60 : £120 : £240
  - £75 : £150 : £200
- £45.30
- £8.40

- 4 94  
 5 138  
 6 680 g  
 7 a 6:5    b 7:6    c 6 years, 3 years  
 8 b 2:1

## Exam practice

- 1  $2 \times 2 \times 3 \times 7$   
 2 45  
 3 a  $\frac{3}{5}$     b 5:3

## A5 Sequences

### Check in

- 1 a 10, 12    b 70, 64    c 16, 22  
 d -2, -5    e 48, 96    f  $\frac{1}{6}, \frac{1}{7}$   
 2  $4(n-1), 15-n, 2n+7$ , and then  $2n^2$  and  $\frac{9}{n} + 15$  are equal  
 3 Square numbers

### A5.1 Rich task, p64

64 leaves

### Ex A5.1

- 1 a 3, 6, 9, 12, 15    b 5, 9, 13, 17, 21  
 c 4, 11, 18, 25, 32    d 10, 15, 20, 25, 30  
 e -4, -2, 0, 2, 4    f  $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}$   
 g 4, 3, 2, 1, 0    h 90, 81, 72, 63, 54  
 2 3, 6, 9, 12, 15, ... matches to  $3n$   
 5, 11, 17, 23, 29, ... matches to  $6n-1$   
 7, 11, 15, 19, 23, ... matches to  $4n+3$   
 46, 42, 38, 34, 30, ... matches to  $50-4n$   
 3 a 8    b 15    c 21    d 9  
 4 a 18, 20, 22, 48    b 20, 23, 26, 62  
 5 4 people, 6 handshakes 5 people perform 10 handshakes.  
 6 people perform 15 handshakes.  
 Pattern is triangular numbers thus  $n$  people perform  
 $\frac{n(n-1)}{2}$  handshakes.

### A5.2 Rich task, p66

$T(1) = 1$   
 $T(2) = 1 + 4 = 5$   
 $T(3) = 5 + 8 = 13$   
 $T(4) = 13 + 12 = 25$   
 $T(n) = 1 + 4(n-1) + 4(n-2) + 4(n-3) + \dots$

### Ex A5.2

- 1 a  $5^2 = 1 + 3 + 5 + 7 + 9, 6^2 = 1 + 3 + 5 + 7 + 9 + 11,$   
 $7^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13$   
 b 199    c 625  
 2 a  $4^2 - 2^2$     b  $4^2 - 1^2, 8^2 - 7^2$   
 c 8, 12, 16, 20. All in the 4 times table.  
 3 a 55    b 210    c 4950  
 d 20100    e 500500  
 4 Option 1 gives £667.95, Option 2 gives £520.  
 Choose Option 1.

### A5.3 Rich task, p68

Cards should be arranged with  $n$  or  $x^n$  on the left hand side or the terms of the sequence will be constant.

Some possible sequences are:

$T(n) = n / 2$   
 $T(n) = n - 1$   
 $T(n) = 1 - x^n$   
 $T(n) = x^n / n$   
 $T(n) = x^n / n - 1$

If brackets are introduced, more sequences are possible  
 e.g.  $T(n) = x^n / (n-1)$

### Ex A5.3

- 1 Missing line is 3, 6, 9, 12, 15.  
 The  $n$ th term is  $T(n) = 3n - 2$ .  
 2 a  $T(n) = 7n$     b  $T(n) = 3n + 2$   
 c  $T(n) = 4n + 10$     d  $T(n) = 6n - 1$   
 e  $T(n) = 10n - 2$     f  $T(n) = 5n + 20$   
 g  $T(n) = 4n - 10$     h  $T(n) = 0.2n + 2$   
 i  $T(n) = 100 - n$     j  $T(n) = 50 - 5n$   
 k  $T(n) = 5 - 4n$   
 3 Check three sequences given with 3<sup>rd</sup> term of 12. Check correct  $n$ <sup>th</sup> term and  $T(100)$  found.  
 4 a  $T(n) = 3n + 10$ . First five terms are 13, 16, 19, 22, 25.  
 b  $T(n) = 8n - 5$ . First five terms are 3, 11, 19, 27, 35.  
 c  $T(n) = 5n + 3$ . First five terms are 8, 13, 18, 23, 28.  
 5 Both Isla and Flora are correct as expanding the brackets of  $n(n+2)$  gives  $n^2 + 2n$ .

### A5.4 Rich task, p70

Number of sides	1	2	3	4	5	6	7
Number of diagonals	-	-	0	2	5	9	14

$T(n+1) = T(n) + n - 1$

For an octagon this would be  $T(8) = T(7) + 6 = 14 + 6 = 20$

## Ex A5.4

- $m = 4n + 1$ . Each new pentagon requires four matchsticks plus one more is needed to close the first pentagon.
- a**  $w = 2r + 6$     **b** 106  
**c** Each red tile is linked to 2 white tiles, one above, one below. Six more white tiles sit three at either end.
- Each new triangle requires 3 straws (2 on the long side, one on the short) plus two more straws are needed to close the first triangle.
- a**  $H = 2C + 2$   
**b** Octane has the chemical formula  $C_8H_{18}$ .  
**c** Further investigation.
- $t = h(h + 1)$

## Exam practice

- a** 16, 19    **b** Double previous term    **c** 2, 5, 10
- 10 mins after the depth was 28 mm.
- a** 14, 20    **b**  $d = \frac{p(p-3)}{2}$ . As  $p - 3 > 0$ ,  $p > 3$  ie.  $p \geq 4$

## A6 Representing and interpreting data

### Check in

- a** 62            **b** 70            **c** 60  
**d** 45            **e** 102        **f** 108
- a** £50        **b** £70
- 2 days

### Ex A6.1

- a** polygons  
**b** **i**  $T \ 20 < m \leq 30$ , OW  $30 < m \leq 40$ ;  
**ii** T 50 mins OW 50 mins  
**c** on average OW take longer than T
- a** polygons  
**b** **i** D  $40 < m \leq 60$ , J  $40 < m \leq 60$ ;  
**ii** D 100 miles J 80 miles  
**c** Similar journey length were made, but journeys in January were less varied than in December
- a** polygons  
**b** mode: D  $0 < t \leq 5$ , P  $5 < t \leq 10$ ;  
median: D  $10 < t \leq 15$ , P  $10 < t \leq 15$ ; mean:  
D 11.7, P 12.5 Recommend Powerblast.
- 2002–2003, range for 2003–2004 less than 60
- Sell-a-lot; average for Sell-well  $> 80$

## Ex A6.2

- a** **i** 30    **ii** 10    **b** **i** 10    **ii** 65  
**c** **i** 37    **ii** 21
- diagram    **i** 950    **ii** 2hours 17 mins

## A6.3 Rich task, p82

	Lower quartile	Median	Upper quartile
Jodie	83	98	110
Fred	88	96	107

## Ex A6.3

- Dentists: min value = 0 min, Q1 = 23, med = 35, Q3 = 53, max value = 80, box and whisker plot  
Doctors: min value = 0 min, Q1 = 22, med = 29, Q3 = 42, max value = 70 min, box and whisker plot, dentists waiting time longer
- First group: min value = 70, Q1 = 91, med = 96, Q3 = 105, max value = 140, box and whisker plot  
Second group: min value = 70, Q1 = 91, med = 97, Q3 = 105, max value = 130, box and whisker plot, second group slightly better

## Ex A6.4

- a** freq dens 1.2, 2.0, 2.3, 1.45, 1.2, 0.2; histogram; 20  
**b** freq dens 4.0, 5.0, 5.4, 4.4, 0.6; histogram; 39  
**c** freq dens 30, 40, 44, 25, 6; histogram;  
**i** 37            **ii** 18
- a** 10            **b** 4, 18, 53, 20, 10    **c** 105
- a** 7            **b** 3, 7, 27, 20, 8    **c** 65
- a** 8, 6, 5, 8, 11, 16, 12, 6  
**b** vertical scale for histogram is 0.4, 0.8, 1.2, 1.6, 2.0, 2.4 and **fd**; histogram

## A6.5 Rich task, p88

Boys' modal class is 70–80 seconds. Girls' modal class is 80–100 seconds.

## Ex A6.5

Comments need to refer to the diagrams

## A6.5

- Aged 11: modal class 125–135 cm; skewed to the right.  
Aged 16: modal class 170–180 cm; skewed to the left.
- Apples: modal class 150–160 g; skewed to the right.  
Pears: modal class 100–110 g; skewed to the right.
- Girls: modal class 7–8 s; skewed to the left. Boys: modal class 6–6.5 s; skewed to the left.

## Exam practice

- 1 **a** polygon  
**b** eg. most common length is same for both months ( $5 < m \leq 10$ ) longest journey time is longer in August than July
- 2 frequency densities are 3.2, 11.5, 13.5, 20.0, 15.5, 4.4, 1.6.
- 3 **a** diagram  
**b**  $Q1 = 32$ ,  $med = 40$ ,  $Q3 = 45$   $Min = 18$ ,  $Max = 67$

## Case study: Sandwich shop

1

Day	Number of Customers	
	Week 1	Week 2
Monday	50	54
Tuesday	68	60
Wednesday	47	53
Thursday	58	57
Friday	52	56
Saturday	76	70
TOTAL	351	350

- a** Daily average (mean) = 58.4  
**b** Weekly average (mean) = 350.5  
**c** Without Saturdays;  
 weekly average (mean) = 277.5  
**d** With coach trip on second Wednesday;  
 weekly average (mean) = 362.5

From frequency polygon:  
 busiest time 1pm to 2pm;  
 quietest time 9am to 10am

Customer numbers vary from hour to hour.  
 There is a peak around lunchtime.

Customer numbers build at the start of the day and tail off towards the end of the day, with a lull in the early afternoon.

This seems to be quite a suitable pattern for an average day.

- 2 Sales for second week:  
 Ham 91, Cheese 63, Humous 35, Tuna 52.5, Chicken 108.5

Product	Stock (packs)	Portions per pack	Portions left	Stock needed	Amount to order
Bread	6	20	120	351	12
Ham	2.5	10	25	91	7
Cheese	3	10	30	63	4
Humous	2	8	16	35	3
Tuna	1.5	14	21	53	3
Chicken	1	10	10	109	10

Estimate of stock left on Wednesday morning:  
 Bread 3, Ham -6 (will have none left), Cheese 9,  
 Humous 4, Tuna 3, Chicken -27 (will have none left)

The shop will not be able to cater for the coach trip.  
 There would be enough tuna and cheese, but not enough of the other ingredients (including the bread).

The manager could calculate a percentage surplus to order so that unexpected customers can be catered for.  
 This value would need to be calculated so that the resulting waste was minimised.

## A7 Formulae and equations

### Check in

- 1 **a** 45    **b** 52    **c** -26    **d** 196  
**e** 7    **f** 13    **g** -50    **h** 30
- 2 **a**  $= 9 + 6$     **b**  $= 18 - 3$ .  
**c**  $= 5 \times 3$     **d**  $= 27 - 12$
- 3 **a** 3    **b** 4    **c** 10    **d** 6  
**e** 4    **f** 25    **g** 33    **h** 7

### A7.1, Rich task p96

14 faces are red. 10 cubes would have 42 red faces.  
 100 cubes would have 402 red faces. For the tower of three cubes she will see 4 blue faces. For the tower of 10 cubes she will see blue 18 faces. For the tower of 100 cubes she will see 198 blue faces.

### Ex A7.1

- 1 **a**  $V = 850 \text{ mm}^3$     **b**  $V = 192\pi \text{ m}^3$     **c**  $V = 36\pi \text{ cm}^3$
- 2 **a**  $A = l^2$     **b**  $A = b^2$     **c**  $A = \frac{1}{2}\pi r^2$
- 3 **a** Area of Square  $= 2x \times 2x = 4x^2$  and area of circle  $= \pi x^2$ .  
 Thus area of paving  $= 4x^2 - \pi x^2 = x^2(4 - \pi)$   
**b** Area of paving  $= (16 - 4\pi) \text{ m}^2$
- 4 **a** Red glass is circle of radius  $\frac{1}{2}k$  thus area of red glass,  $R = \pi \times \left(\frac{1}{2}k\right)^2 = \frac{1}{4}\pi k^2$ .  
**b** Area of Square  $= k \times k = k^2$  and thus area of yellow glass,  $Y = k^2 - \frac{1}{4}\pi k^2 = k^2\left(1 - \frac{1}{4}\pi\right)$ .  
**c**  $Y = 16\left(1 - \frac{1}{4}\pi\right) = 16 - 4\pi \text{ m}^2 \approx 3.4 \text{ m}^2$ .

### A7.2 Rich task, p98

They are the same at  $-40^\circ\text{C}/^\circ\text{F}$

### Ex A7.2

- 1 **a**  $x = k - t$     **b**  $x = w - y + z$     **c**  $x = 11a$   
**d**  $x = a - bc$     **e**  $x = t - p^2$     **f**  $x = f + gh + k$   
**g**  $x = p - \sqrt{q}$     **h**  $x = b$

2 a  $y = \frac{a}{b}$     b  $y = kt$     c  $y = \frac{p-r}{q}$   
 d  $y = \frac{x-z}{w}$     e  $y = \frac{b-c^2}{a}$     f  $y = k(m+n)$   
 g  $y = t^2(g-h)$     h  $y = \frac{k+t}{mn}$

3 a  $428^\circ\text{F}$     b  $C = \frac{5}{9}(F-32)$   
 c  $177^\circ\text{C} = 180^\circ\text{C}$  (2 sig fig)

4 a  $a = \sqrt[3]{x}$     b  $a = \sqrt[3]{k+t}$     c  $a = \sqrt[3]{\frac{x}{y}}$   
 d  $a = \sqrt[3]{mn}$     e  $a = t^2$     f  $a = (p-q)^2$

g  $a = 4k^2$     h  $a = b^3$   
 5 a  $r = \sqrt[3]{\frac{3V}{4\pi}}$     b  $6.0822\dots = 6.08$  cm (3 sig fig)

- 6 a In line 2,  $+c$  should be  $-c$ . In line 3, needs to divide both sides by 2.  
 b In line 2, deal with the  $k$  first i.e. add  $k$  to both sides. Then in line 3, divide both sides by  $x$ .  
 c In line 2, deal with the  $y$  first. Divide both sides by  $y$  and then square root in line 3.

### A7.3 Rich task, p100

Middle row, right: 49  
 Middle row, left: 46  
 Bottom row, middle: 29

If the numbers on the bottom row are 17, 20, 29; middle row is 37, 49; top row is 86.

If the numbers on the bottom row are 20, 17, 29; middle row is 37, 46; top row is 83.

If the numbers on the bottom row are 20, 29, 17; middle row is 49, 46; top row is 95.

If the numbers on the bottom row are 29, 17, 20; middle row is 46, 37; top row is 83.

If the numbers on the bottom row are 29, 20, 17; middle row is 49, 37; top row is 86.

### Ex A7.3

- 1 a  $x = 4$     b  $y = 7$     c  $t = 3$     d  $k = 0$   
 e  $p = 10$     f  $q = \frac{1}{2}$     g  $g = \frac{2}{3}$     h  $a = \frac{9}{10}$   
 2 a  $x = 1$     b  $y = 3$     c  $a = 5$     d  $b = 7$   
 e  $m = 3$     f  $n = 28$     g  $k = 6$     h  $t = 0$   
 3 a 19 cm by 5 cm    b 8 cm by 4 cm  
 4 a  $x = 1$     b  $k = 3$     c  $t = 5$   
 d  $k = 2$     e  $a = 4$     f  $b = 4$   
 5 a The angles are  $85^\circ$ ,  $125^\circ$ ,  $60^\circ$  and  $90^\circ$ .  
 b Victoria is 8, Jamie is 7 and Jonathan is 4.  
 6 a  $a = 2$     b  $n = 2$     c  $b = 1$     d  $m = 2$   
 e  $q = 6$     f  $x = 2$     g  $p = 20$     h  $x = 1$   
 7 a £7    b 12 km

### A7.4 Rich task, p102

$k = 7$

### Ex A7.4

- 1 a  $x = 7$     b  $k = 2$     c  $n = 1$   
 d  $t = -4$     e  $a = -2$     f  $p = -1$   
 2 a In line 2,  $+4$  should be  $+8$ . Hence the answer is  $x = 1$ .  
 b In line 2,  $2x$  should be  $4x$ . In line 3, 16 should be 8. Hence the answer is  $x = 2$ .  
 c In line 3, 1 should be  $-1$ . The answer is  $x = -\frac{1}{2}$ .  
 3 a  $x = 3$     b  $t = 1$     c  $y = 3$     d  $a = 2$   
 e  $k = 2$     f  $b = 4$     g  $p = 0$     h  $d = 4$   
 i  $q = 5$     j  $m = 6$     k  $r = -2$     l  $n = -1$   
 4 a 1    b 2    c 6  
 5 a Length of one side = 12  
 b Length of the rectangle = 10

### A7.5 Rich task, p104

No, you do not add numerators and denominators independently.

$$1/u + 1/v = v/uv + u/uv = u + v/uv$$

### Ex A7.5

- 1 a  $a = 27$     b  $x = 36$     c  $b = 2\frac{1}{2}$     d  $y = 2\frac{2}{3}$   
 e  $f = 12$     f  $m = 10$     g  $g = -4$     h  $n = -9$   
 i  $p = 9$     j  $t = 2$     k  $q = 4$     l  $k = 3$   
 2 a  $x = 2$     b  $a = 7$     c  $p = -5$     d  $m = 1\frac{2}{5}$   
 e  $y = 4$     f  $b = 2$     g  $q = 3$     h  $n = 1\frac{3}{4}$   
 i  $f = 3$     j  $g = 0$     k  $k = 5$     l  $t = \frac{1}{5}$   
 3 a  $a = 10$     b  $b = 15$     c  $x = 4$     d  $y = 11$   
 e  $q = 3$     f  $p = 5$     g  $m = 4$     h  $n = 3$   
 4 a 9  
 b 6  
 c 5  
 5 Sides on first trapezium are 7 cm and 13 cm.  
 Sides on second trapezium are 5 cm and 7 cm.  
 6 a Interior angle =  $180 - \frac{360}{n}$   
 b i 5    ii 9    iii 15

### A7.6 Rich task, p106

$x = 3.75$  cm

$$A = x^2 + \frac{1}{2}x(3-\frac{x}{2}) + \frac{1}{2}x(3-\frac{x}{2}) + \frac{1}{2}x(10-x)$$

## Ex A7.6

- 1 Both correct.  $\frac{ty}{t} = y$ .
- 2 a  $x = \frac{y}{a+b}$     b  $x = \frac{t}{p} - q$     c  $x = \frac{n^2}{m} + k$   
 d  $x = \frac{w}{k^2 - y}$     e  $x = \frac{p}{\pi} - a$     f  $x = 2a + y$   
 g  $x = 5g - gh$     h  $x = \pi s + k^2$
- 3 a  $y = a - b$     b  $y = q - p^2$     c  $y = ab - t$   
 d  $y = \frac{k-r}{a}$     e  $y = \frac{n-m}{k}$     f  $y = p - \frac{d}{q}$   
 g  $y = x - \frac{k^2}{h}$     h  $y = m - kn$
- 4 a  $u = v - at$     b  $17ms^{-1}$
- 5 a  $k = \frac{a}{b}$     b  $k = \frac{t}{p}$     c  $k = \frac{mn}{t}$   
 d  $k = \frac{d}{rt}$     e  $k = \frac{xy}{w}$     f  $k = \frac{p}{a-q}$   
 g  $k = \frac{h}{y^2 + z}$     h  $k = \frac{m}{p} - n$
- 6 a  $t = \frac{d}{s}$     b 42 minutes
- 7 a  $x = \frac{q-b}{a-p}$     b  $x = \frac{n-z}{m-w}$     c  $x = \frac{a+b}{c+d}$   
 d  $x = \frac{f-g}{g+1}$     e  $x = \frac{rd+kc}{k+r}$     f  $x = \frac{2c+k}{1-c}$
- 8  $t = \frac{x}{a-b}$

## Exam practice

- 1 a 15    b  $2\frac{8}{9}$     c  $1\frac{1}{19}$
- 2  $b = \frac{4}{5}(e-32)$
- 3 a  $2n + 7$   
 b 19

## A8 Pythagoras and trigonometry

### Check in

- 1 a 49    b 52    c 34    d 48  
 e 45    f 80    g 5    h 7
- 2 a  $x = 6y$     b  $x = 5y$     c  $x = 10y$   
 d  $x = \frac{2}{y}$     e  $x = \frac{5}{y}$     f  $x = \frac{8}{y}$

## A8.1 Rich task, p112

Pattern	Small square	Other small square	Large square
A	2	2	4
B	8	8	16
C	4	4	8
D	16	16	32

The total number of tiles in the two small squares is the same as the number of tiles in the big square.

## A8.1 Rich task 2, p112

If they have been drawn and cut accurately, the areas X and Y will fit exactly into area Z.

### Ex A8.1

- 1 a 5 cm    b 17 cm    c 13 cm  
 d 10.3 cm    e 10.8 cm    f 9.9 cm
- 2 a 19.8 cm    b 10 cm    c 9 cm  
 d 7.5 cm    e 5.4 cm    f 7.1 cm
- 3 6.8 metres
- 4 3.5 km
- 5 5.66 metres
- 6 6.5 miles
- 7 6.93 cm
- 8 7.2 cm
- 9 i 5    ii 4.24    iii 13    iv 20  
 v 14.14    vi  $\sqrt{(c-a)^2 + (d-b)^2}$
- 10 3.43 cm
- 11 b  $A = 2x\sqrt{100 - x^2}$   
 Max area when  $x = 7.1$  cm

### Ex A8.2

- 1 Ratios: 1.5; 1.67, 1.4; 0.88  
 Angles:  $56^\circ$ ;  $59^\circ$ ;  $54^\circ$ ;  $40^\circ$ .  
 TanA: 1.48; 1.66; 1.38; 0.84.  
 The tangents of the angles match the ratios to within measuring errors.
- 2 a 2.48    b 1.23    c 0.90    d 0.60

## A8.3 Rich task, p118

The ratio is often said to be “rise over run”. Run is the *horizontal distance* and rise is the *vertical distance* travelled. Cyclists and cars can typically cope with a gradient of no more than 1:10.

The steepest roads are in Rosedale Abbey, N Yorks which is 1:3 and Ffordd Penllech, Harlech, Wales (officially described as not suitable for motor vehicles) which is 1:2.91 at its steepest point.

### Ex A8.3

- 1 a  $p = 8.2$  cm    b  $q = 3.7$  cm    c  $r = 16.8$  cm  
 d  $s = 4.4$  cm    e 3.0 cm    f 6.4 cm  
 g 3.8 cm    h 30.5 cm
- 2 70 metres
- 3 104 metres
- 4 22 metres

- 5 **a**  $63.4^\circ$  **b**  $51.3^\circ$  **c**  $32.7^\circ$  **d**  $38.7^\circ$   
**e**  $17.6^\circ$  **f**  $26.6^\circ$  **g**  $34.9^\circ$  **h**  $45^\circ$
- 6  $72.6^\circ$
- 7  $63.4^\circ$
- 8  $21^\circ$
- 9 183 metres
- 10  $h = 11.7$  cm

### Ex A8.4

- 1  $p = 2.3$  cm  $q = 3.1$  cm  $r = 4.2$  cm  
 $s = 3.8$  cm  $t = 5.7$  cm  $u = 14.4$  cm
- 2 38 metres
- 3 15.2 metres
- 4 3.2 km
- 5 724 metres
- 6 5.2 cm
- 7 **a**  $= 36.9^\circ$  **b**  $= 44.4^\circ$  **c**  $= 60.1^\circ$  **d**  $= 36.9^\circ$   
**e**  $= 40.1$  **f**  $= 27.7^\circ$
- 8  $68^\circ$
- 9  $8.9^\circ$
- 10 3.9 metres
- 11 A reflection in the line  $\theta = 45^\circ$   
 $\sin 30^\circ = \cos 60^\circ$   
 In general,  $\sin x = \cos(90^\circ - x)$

### Ex A8.5

- 1 **a**  $n = 5.35$  cm  $p = 5.95$  cm  
**b**  $q = 5.27$  cm  $\theta = 53^\circ$   
**c**  $s = 4.14$  cm  $r = 1.76$  cm  
**d**  $t = 6.04$  cm  $\theta = 39.0^\circ$   
**e**  $\alpha = 36.9^\circ$   $\beta = 53.1^\circ$   $u = 20$  cm  
**f**  $v = 6.10$  cm  $w = 3.50$  cm  
**g**  $x = 5.14$  cm  $y = 6.13$  cm  $\theta = 40^\circ$   
**h**  $z = 7.21$  cm  $\alpha = 56.3^\circ$   $\beta = 33.7^\circ$
- 2 **a**  $x = 11.1$  cm  $y = 12.7$  cm  $\alpha = 60.4^\circ$   
**b**  $x = 11.5$  cm  $y = 13.2$  cm  $\alpha = 29.5^\circ$   
**c**  $\alpha = 36.2^\circ$   $x = 8.75$  cm  $y = 3.77$  cm
- 3 **a** 13.0 cm **b** 10.4 cm **c** 10.0 cm **d** 24.7 cm
- 4 767 metres
- 5 8.5 cm
- 6  $65.4^\circ$   $65.4^\circ$   $49.2^\circ$
- 7 5 cm  $\sqrt{20}$  cm  $\sqrt{5}$  cm  
 $5^2 = (\sqrt{20})^2 + (\sqrt{5})^2$

- 8 0.62 cm
- 9 10 km 146.9°
- 10 7.30 metres
- 11 **a** 35.4 cm **b** 22.6 cm **c** 17.3 cm
- 12 239 metres
- 13 3.9 metres
- 14 7.8 metres
- 15 **a** 77.5 cm 63.2 cm

**b**  $AB = 10\sqrt{x}$  cm and  $BC = 10\sqrt{y}$  cm provided  $x$  and  $y$  are measured in cm and  $x + y = 100$

- 16  $\sqrt{40}$  cm
- 17  $4 \text{ m} \times 4 \text{ m}$   $6 \text{ m} \times 2 \text{ m}$

### Exam practice

- 1 **a** 6.1 cm **b**  $22.5^\circ$
- 3 55.9 m

### B9 Fractions, decimals and percentages

#### Check in

- 1 **a**  $\frac{1}{2}$  **b**  $\frac{3}{4}$  **c**  $\frac{4}{5}$  **d**  $\frac{19}{20}$  **e**  $\frac{3}{4}$
- 2 **a** 0.75 **b** 0.4 **c** 0.7 **d** 0.45 **e** 0.17
- 3 **a**  $\frac{1}{2}$  **b**  $\frac{1}{4}$  **c**  $\frac{3}{10}$  **d**  $\frac{4}{5}$  **e**  $\frac{9}{20}$
- 4 **a**  $20 \times 30$  **b**  $360 \div 60$   
**c**  $1200 - 800$  **d**  $7000 + 6000$

#### Ex B9.1

- 1 **a** 20 250 **b** 96 **c** 2496  
**d** 3.3 **e**  $86.1\bar{6}$  **f** 36.25
- 2 **a** **i** 13.2 **ii** 0.87  
**b** **i** 21.4 **ii** 19.902  
**c** **i** 7600 **ii** 0.045  
**d** **i** 22.5 **ii** 7.2
- 3 **a** 3 **b** 100 **c** 2 **d** 8  
**e** 8 **f** 0.5 **g** 12 **h**  $\frac{1}{14}$   
**i** 2 **j** 15 000 **k** 1.3 **l** 150
- 4 **a** 7  
**b** 12
- 5 40

#### Ex B9.2

- 1  $\frac{15}{32}$
- 2 Both £187.50
- 3 **a** 16 **b** 12 **c** 66 **d** 12.4

- 4 a 0.06      b 0.48      c 0.015  
 d 0.06      e 0.006      f 3  
 g 52      h 272.5      i 100

5 a  $0.08 \times 0.02 = 0.0016$    b  $0.2 \div 0.02 = 10$

6 25% of 60 = 15, 20% of 70 = 14

7 50% of 30 = 15, 40% of 40 = 16

8 a i 25      ii 25

9 a 400      b 350

### Ex B9.3

- 1 a  $\frac{1}{2}$       b  $\frac{11}{16}$       c  $\frac{27}{70}$       d  $\frac{59}{60}$   
 2 a  $\frac{1}{4}$       b  $\frac{16}{27}$       c  $\frac{1}{10}$       d  $\frac{5}{12}$   
 3 a  $\frac{1}{6}$       b  $\frac{3}{35}$       c  $\frac{24}{35}$       d  $\frac{1}{6}$   
 4 a £22.50   b £21      c £10      d £16  
 5  $\frac{3}{10}$   
 6 a  $\frac{1}{10}$       b  $\frac{1}{4}$       c  $\frac{3}{20}$       d  $\frac{3}{20}$   
 e  $\frac{2}{5}$       f  $\frac{8}{9}$       g  $\frac{15}{16}$       h  $\frac{3}{4}$   
 7 a i  $\frac{3}{8} \times \frac{1}{10} = \frac{3}{80}$       ii  $\frac{1}{2} + \frac{2}{5} + \frac{1}{10} = 1$   
 b i  $\frac{1}{5} \times \frac{2}{7} = \frac{2}{35}$       ii  $\frac{1}{2} + \frac{1}{5} + \frac{3}{10} = 1$

### Ex B9.4

- 1 a  $1\frac{3}{5}$       b  $7\frac{1}{2}$       c  $7\frac{1}{2}$       d  $\frac{3}{4}$   
 e  $9\frac{1}{6}$       f  $2\frac{2}{3}$       g  $12\frac{1}{2}$       h  $\frac{6}{7}$   
 2 a  $3\frac{31}{40}$       b  $11\frac{3}{4}$       c  $8\frac{1}{12}$       d  $5\frac{2}{5}$   
 3 a  $2\frac{5}{26}$       b  $1\frac{13}{20}$       c  $-\frac{5}{12}$       d  $-\frac{11}{12}$   
 4 a  $1\frac{7}{20}$       b  $\frac{2}{3}$       c  $\frac{5}{6}$   
 5 215 cm<sup>2</sup>  
 6 a  $\frac{14}{45}$       b  $\frac{15}{38}$       c  $1\frac{1}{2}$       d  $\frac{40}{51}$   
 e  $3\frac{1}{6}$       f  $1\frac{17}{78}$

#### Rich Task

- a decreases      b increases

### B9.5 Rich task, p140

$2 = 2$   
 $5 = 5$   
 $10 = 2 \times 5$   
 $25 = 5 \times 5$   
 $40 = 2 \times 2 \times 2 \times 5$   
 $8 = 2 \times 2 \times 2$

They can all be written as a product of 2 and 5 i.e. the fractions can all be written as products of  $\frac{1}{2}$  and  $\frac{1}{5}$ , which are terminating fractions.

### Ex B9.5

- 1 a  $0.\dot{1}, 0.\dot{2}, 0.\dot{3}$  etc  
 b recurring digit is same as numerator  
 2 a  $0.\dot{0}\dot{9}, 0.\dot{1}\dot{8}, 0.\dot{2}\dot{7}$  etc  
 b repeated number is 9 times numerator  
 c  $\frac{1}{2} - \frac{4}{11} = \frac{3}{22}$ ,  $\frac{1}{2} - \frac{7}{11} = \frac{-3}{22}$  equally close  
 3 a  $\frac{3}{35}$  When in simplest forms, other fractions have denominators with no prime number factors other than 2 and 5  
 b  $\frac{6}{30} = \frac{1}{5}$ , a fraction whose denominator is multiple of 5  
 4 a i  $0.\dot{2}$       ii  $0.0\dot{2}$   
 iii  $0.00\dot{2}$       iv  $0.000\dot{2}$   
 b denominators are  $9 \times 10^n$  and the  $n$  tells you the number of zeros before the recurring.  
 5 a  $\frac{34}{90} = \frac{17}{45}$       b  $\frac{17}{90}$       c  $\frac{65}{99}$       d  $\frac{21}{37}$   
 e  $\frac{1234}{9999}$       f  $\frac{69}{9000} = \frac{23}{3000}$       g  $\frac{103}{9990}$   
 6  $22/7 = 3.142857\ 142857\ 142857 \dots$   
 The series tends to pi.

### Exam practice

- 1  $2\frac{7}{9}$   
 2 a  $\frac{1}{18}$       b  $\frac{31}{99}$   
 3 Yes

### Case study: Weather

16°C; 2.5°C; -8.9°C

England:

Range = 38.5 -- 26.1 = 64.6°C

Wales:

Range = 35.2 -- 23.3 = 58.5°C

Scotland:

Range: 32.9 -- 27.2 = 60.1°C

N.Ireland:

Range = 30.8 -- 17.5 = 48.3°C

### B10 Circles, angles and lines

#### Check in

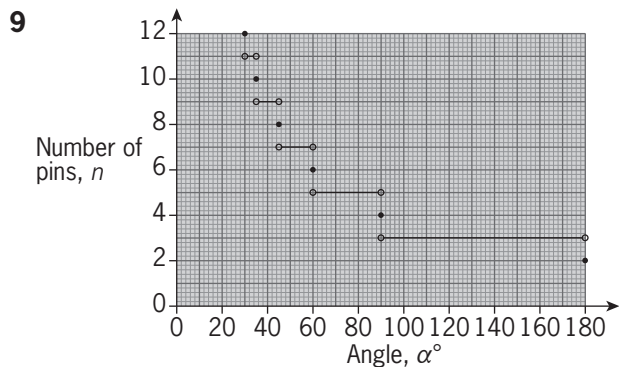
- 1 a 43°      b 139°  
 2 a 102°      b 29°

### B10.1 Rich task, p148

32 minutes and 44 seconds

### Ex B10.1

- 1  $c = 150^\circ$      $d = 30^\circ$      $e = 30^\circ$
- 2  $f = 120^\circ$      $g = 120^\circ$
- 3  $j = 100^\circ$      $k = 50^\circ$      $l = 30^\circ$
- 4  $m = 40^\circ$      $n = 70^\circ$      $p = 110^\circ$
- 5  $r = 45^\circ$
- 6  $u = 95^\circ$
- 7  $v = 70^\circ$      $w = 50^\circ$
- 8  $x = 30^\circ$



### B10.2 Rich task, p150

$$y^\circ = 180^\circ - x^\circ$$

The only realistic possibilities for the baby lie in the region  $0^\circ < x < 90^\circ$  and therefore  $90^\circ < y < 180^\circ$ .

### Ex 10.2

- 1  $a = 35^\circ$
- 2  $m = 20^\circ$      $n = 20^\circ$      $p = 70^\circ$      $q = 70^\circ$
- 3  $e = 20^\circ$      $f = 80^\circ$      $g = 20^\circ$
- 4  $x = 50^\circ$
- 5  $v = 60^\circ$
- 6  $x = y = 38^\circ$
- 7  $u = 30^\circ$      $v = 60^\circ$      $w = 60^\circ$
- 8  $a = 60^\circ$      $b = 30^\circ$
- 9  $c = 90^\circ$
- 10  $43\frac{1}{3}^\circ$
- 11  $24^\circ$      $48^\circ$      $96^\circ$      $192^\circ$
- 12  $60^\circ$      $130^\circ$
- 13  $x = 50^\circ$
- 14  $y = 60^\circ$
- 15  $z = 30^\circ$
- 16  $a = 36^\circ$
- 17  $p = 70^\circ$      $q = 40^\circ$
- 18  $b = 10^\circ$      $c = 55^\circ$
- 19  $w = 115^\circ$      $x = 35^\circ$   
 $y = 30^\circ$      $z = 115^\circ$

- 20  $a = 65^\circ$
- 21  $x = 50^\circ$
- 22  $83\frac{1}{3}^\circ$ ,  $83\frac{1}{3}^\circ$ ,  $90^\circ$ ,  $103\frac{1}{3}^\circ$

### B10.3 Rich task, p153

The general rule for a polygon with  $n$  sides is  $(n - 2) \times 180^\circ$ .

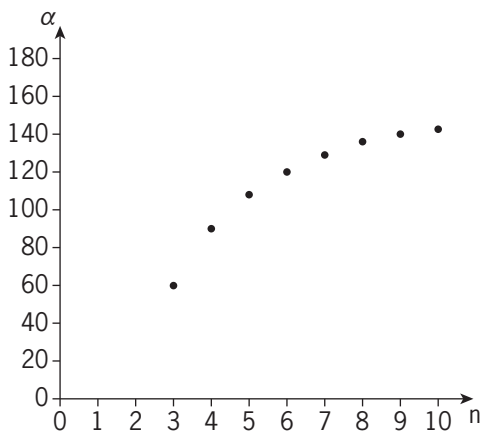
### Ex B10.3

1

Regular polygon	Number of sides	Each interior angle	Each exterior angle	Sum of all interior angles
Triangle	3	$60^\circ$	$120^\circ$	$180^\circ$
Quadrilateral	4	$90^\circ$	$90^\circ$	$360^\circ$
Pentagon	5	$108^\circ$	$72^\circ$	$540^\circ$
Hexagon	6	$120^\circ$	$60^\circ$	$720^\circ$
Octagon	8	$135^\circ$	$45^\circ$	$1080^\circ$
Decagon	10	$144^\circ$	$36^\circ$	$1440^\circ$

- 2 **a**  $a = 120^\circ$      $b = 60^\circ$      $c = 60^\circ$   
**b**  $x = 140^\circ$      $y = 110^\circ$      $z = 40^\circ$   
**c**  $s = 30^\circ$      $t = 120^\circ$      $u = 150^\circ$
- 3 **a**  $103^\circ$ ,  $103^\circ$ ,  $103^\circ$ ,  $132^\circ$ ,  $93^\circ$ ,  $186^\circ$   
**b**  $102^\circ$ ,  $102^\circ$ ,  $112^\circ$ ,  $112^\circ$ ,  $112^\circ$
- 4  $40^\circ$ ,  $40^\circ$ ,  $40^\circ$ ,  $30^\circ$ ,  $30^\circ$
- 5 **a** **i**  $30^\circ$     **ii**  $10^\circ$     **iii**  $18^\circ$     **iv**  $20^\circ$   
**b** No. of sides    Exterior angle    Interior angle  
**i**    9     $40^\circ$      $140^\circ$   
**ii**    15     $24^\circ$      $156^\circ$   
**iii**    24     $15^\circ$      $165^\circ$   
**iv**    30     $12^\circ$      $168^\circ$
- 6 **a**  $180^\circ n - 360^\circ$   
**b**  $180^\circ - \frac{360^\circ}{n}$ ,  $\frac{360^\circ}{n}$      $162^\circ$ ,  $18^\circ$
- 7  $m = 67.5^\circ$      $n = 92.5^\circ$
- 8  $x = 120^\circ$
- 9

No. of sides, $n$	3	4	5	6	7	8	9	10	...	20	...	100
Interior angle, $a$	$60^\circ$	$90^\circ$	$108^\circ$	$120^\circ$	$128\frac{4}{7}^\circ$	$135^\circ$	$140^\circ$	$144^\circ$	...	$162^\circ$	...	$176.4^\circ$



As  $n \rightarrow \infty$ ,  $\alpha \rightarrow 180^\circ$ , the polygon is tending to a circle

### Ex B10.4

- 1  $a = 130^\circ$
- 2  $b = 40^\circ$
- 3  $c = 50^\circ$
- 4  $d = 100^\circ$
- 5  $e = 110^\circ$
- 6  $f = 55^\circ$
- 7  $g = 25^\circ$
- 8  $h = 250^\circ$
- 9  $i = 115^\circ$
- 10  $j = 40^\circ$
- 11  $l = 65^\circ$
- 12 1130 km (3 SF)
- 13 9.2 cm

### Ex B10.5

- 1 a  $a = 30^\circ$      $b = 50^\circ$   
 b  $c = 25^\circ$      $d = 155^\circ$   
 c  $e = 70^\circ$   
 d  $f = 25^\circ$   
 e  $g = 100^\circ$   
 f  $h = 300^\circ$   
 g  $i = 40^\circ$   
 h  $j = 20^\circ$      $k = 40^\circ$   
 i  $l = 60^\circ$      $m = 30^\circ$   
 j  $n = 35^\circ$      $p = 55^\circ$      $q = 35^\circ$   
 k  $r = 20^\circ$   
 l  $s = 40^\circ$      $t = 65^\circ$
- 2 a  $x = 110^\circ$     b  $y = 210^\circ$     c  $z = 40^\circ$   
 d  $p = 140^\circ$     e  $x = 40^\circ$     f  $z = 55^\circ$
- 3 When  $\theta = \frac{180^\circ}{n}$  where  $n$  is integer, the ball describes an  $n$ -sided regular polygon over and over again. For other values of  $\theta$ , the path of the ball is more complicated.

## Exam practice

- 1  $x = 139^\circ$  (vertically opposite),  $y = 57^\circ$  (alternating)
- 2 a  $a = 35^\circ$  (angles in same segment),  $b = 145^\circ$  (cyclic quadrilateral)  
 b Because  $62^\circ \neq \frac{1}{2}$  of  $122^\circ$
- 3  $138^\circ$

## B11 Straight lines

### Check in

- 1 Check graphs
- 2 a  $6\frac{1}{2}$     b  $5\frac{1}{4}$     c  $4\frac{5}{7}$     d  $-9\frac{1}{11}$   
 e  $\frac{7}{3}$     f  $\frac{15}{2}$     g  $\frac{23}{4}$     h  $-\frac{35}{8}$
- 3 a 2    b 1    c  $\frac{1}{2}$

### B11.1 Rich task, p166

Comments should include  $y$ -intercept and steepness, i.e. relation to the general equation of a straight line ( $y = mx + c$ ).

### Ex B11.1

- 1 a  $y = 0$     b  $x = 3, x = 10$   
 c  $y = 3x - 5, y = 2x, y = x$   
 d  $y = x^3, y = x^2 + 2x$
- 2 a i 

x	0	1	2
y	2	2	2

    ii 

x	0	1	2
y	0	3	6

  
 iii 

x	0	1	2
y	-1	1	3

    iv 

x	0	1	2
y	3	1	-1
- b Graphs of  $y = 2, y = 3x, y = 2x - 1$  and  $y = 3 - 2x$  plotted on separate axes from  $-8$  to  $8$ .
- 3 a Square    b  $x = 1, x = 4, y = 2, y = 5$
- 4 a Any point on the graph of  $y = 3x - 2$   
 b 16    c 4
- 5 a i, ii and iii  
 Graphs of  $y = 2x - 3, y = 2x$  and  $y = 2x + 5$  plotted on same set of axes.  
 b The lines are parallel.  
 c Any line of the form  $y = 3x + c$  where  $c$  is a constant.
- 6 a i, ii and iii  
 Graphs of  $y = \frac{1}{2}x + 1, y = 2x + 1$  and  $y = 5x + 1$  plotted on same set of axes.  
 b The lines all cut the  $y$ -axis at the point  $(0, 1)$ . Each graph is steeper than the last.  
 c Any line of the form  $y = ax + c$  where  $a > 3$  and  $c$  is a constant.

## B11.2 Rich task, p168

Left:  $-2$

Right:  $3$

Bottom:  $0.5$

### Ex B11.2

- 1 **a** Gradient =  $2$ ,  $y$ -intercept =  $(0, 2)$   
**b** Gradient =  $\frac{1}{2}$ ,  $y$ -intercept =  $(0, \frac{1}{2})$   
**c** Gradient =  $-1$ ,  $y$ -intercept =  $(0, 0)$   
**d** Gradient =  $-3$ ,  $y$ -intercept =  $(0, 3)$   
**e** Gradient =  $0$ ,  $y$ -intercept =  $(0, -3)$
- 2 **a**  $2$       **b**  $4$       **c**  $-1$       **d**  $\frac{1}{2}$   
**e**  $-2$       **f**  $\frac{1}{3}$       **g**  $-\frac{1}{2}$       **h**  $0$

The line is horizontal and hence parallel to the  $x$ -axis.

- 3 **i** Graphs of  $y = 3x + 5$ ,  $y = 2x - 3$ ,  $y = \frac{1}{2}x + 1$  and  $y = 3 - x$ , plotted on separate axes.  
**ii** **a** Gradient =  $3$ ,  $y$ -intercept =  $(0, 5)$   
**b** Gradient =  $2$ ,  $y$ -intercept =  $(0, -3)$   
**c** Gradient =  $\frac{1}{2}$ ,  $y$ -intercept =  $(0, 1)$   
**d** Gradient =  $-1$ ,  $y$ -intercept =  $(0, 3)$   
**iii** The gradient is the coefficient of  $x$  and the  $y$ -intercept can be read from the value of the constant term.
- 4 **a** Graph of  $y = 2x + 1$  plotted on axes from  $-5$  to  $5$ .  
**b** Graph of  $y = \frac{1}{2}x - 1$  plotted on axes from  $-5$  to  $5$ .  
**c** Graph of  $y = 4 - 3x$  plotted on axes from  $-5$  to  $5$ .
- 5 **a**  $t = 6$  and hence the coordinates are  $(5, 6)$  and  $(8, 18)$ .  
**b**  $k = 2$  and hence the coordinates are  $(2, 8)$  and  $(6, 20)$ .

## B11.3 Rich task, p170

Parallel lines always have the same gradient.

If the gradient of a line is  $m$ , the perpendicular line has a gradient of  $-\frac{1}{m}$ .

### Ex B11.3

Equation	Gradient	Coordinates of $y$ -intercept
<b>a</b> $y = 2x + 5$	$2$	$(0, 5)$
<b>b</b> $y = 7x - 1$	$7$	$(0, -1)$
<b>c</b> $y = 3x$	$3$	$(0, 0)$
<b>d</b> $y = \frac{1}{3}x + 2$	$\frac{1}{3}$	$(0, 2)$
<b>e</b> $y = 4(x - 3)$	$4$	$(0, -12)$
<b>f</b> $2y = x - 3$	$\frac{1}{2}$	$(0, -\frac{1}{2})$
<b>g</b> $x + 4y = 12$	$-\frac{1}{4}$	$(0, 3)$
<b>h</b> $3x + 4y = 8$	$-\frac{3}{4}$	$(0, 2)$

- 2  $y = \frac{1}{3}x - 1$  matches with line  $c$ .  
 $y = 3$  matches with line  $a$ .

$y = -x$  matches with line  $e$ .

$y = 1 - 3x$  matches with line  $d$ .

$y = 2x + 1$  matches with line  $b$ .

- 3 **a** Line A has equation  $y = 3x + 1$ .  
**b** Line B has equation  $y = x - 5$ .  
**c** Line C has equation  $y = \frac{1}{2}x - 3$ .  
**d** Line D has equation  $y = 3x - 2$ .
- 4 **a** False. The lines are perpendicular.  
**b** True. The lines are parallel as they both have a gradient of  $5$ .  
**c** True. The lines are perpendicular because  $2 \times -\frac{1}{2} = -1$ .  
**d** False. The gradients of the lines are  $2$  and  $\frac{1}{2}$ ; their product is not  $-1$ .
- 5 **a** Graph with the points  $(1, 3.5)$ ,  $(3, 6.5)$ ,  $(3.5, 7.25)$ ,  $(4.2, 8.3)$  and  $(5.6, 10.4)$  plotted.  
**b** Gradient is  $1.5$ .  
**c** Equation is  $t = \frac{3}{2}k + 2$ .

## B11.4 Rich task, p172

- a**  $y = -x + k$ ;  $y = -x - k$ ; in this case  $k = 3$ .  
**b**  $y = -mx + c$ ;  $y = -mx + d$ ; in this case  $c = 1$  and  $d = 5$ .

### Ex B11.4

- 1 Ticks underneath  $y = x - 3$ ,  $x + y = 1$  and  $y = \frac{3}{2}x - 4$ .  
2 Line A has the equation  $y = 3x + 2$ .  
Line B has the equation  $y = 5 - 2x$ .  
Line C has the equation  $y = 2x - 5$ .  
Line D has the equation  $y = \frac{1}{4}x - 3$ .  
3 Line  $a$  has the equation  $x = 2$ .  
Line  $b$  has the equation  $y = x + 1$ .  
Line  $c$  has the equation  $y = 2x - 1$ .  
Line  $d$  has the equation  $y = -3x$ .  
Line  $e$  has the equation  $y = -\frac{1}{2}x + 2$ .
- 4 **a**  $y = 2x + 1$       **b**  $y = 5x - 2$   
**c**  $y = \frac{1}{2}x + 1$       **d**  $y = \frac{1}{5}x - 2$   
**e**  $y = 4 - x$       **f**  $y = 10 - 3x$   
**g**  $5x + 2y = 2$       **h**  $y = 2 - \frac{1}{3}x$

## Exam practice

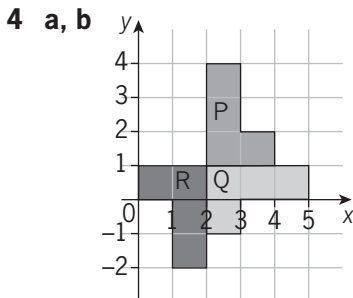
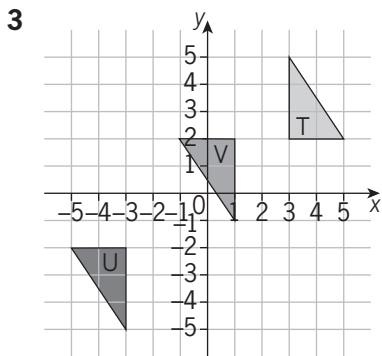
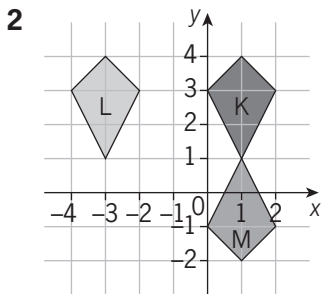
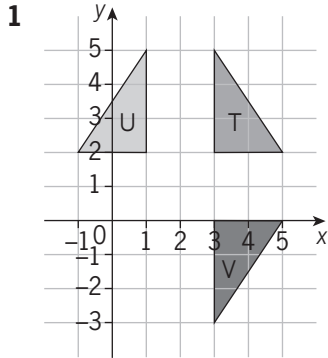
- 1 **a**  $y = 3x + 2$   
**b** Second line has gradient  $-\frac{1}{3}$  and  $3 \times -\frac{1}{3} = -1$ , the lines are perpendicular
- 2 The first line has gradient  $2$  and the third gradient  $-\frac{1}{2}$ .  
As these two lines are perpendicular (since  $2 \times -\frac{1}{2} = -1$ ), therefore the triangle is right-angled.
- 3 **a**  $-5$       **b**  $5y = x + 16$

# B12 Transformations

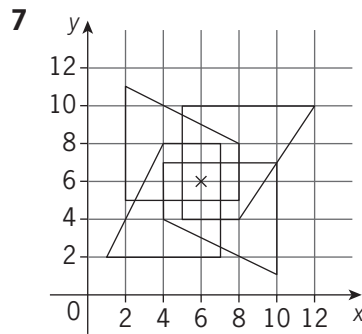
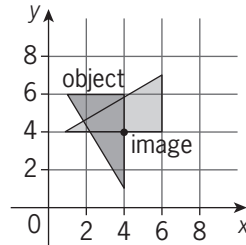
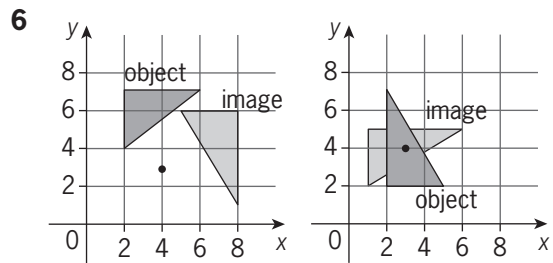
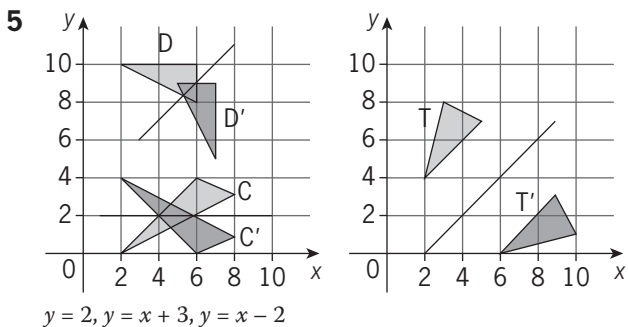
## Check in

- 1 A (3, 2), B (5, 5), C (4, -1), D (-2, -3), E (1, 6)  
 2 a  $x = 3$       b  $x = 5$       c  $y = x$   
 d  $y = 2$       e  $y = -1$

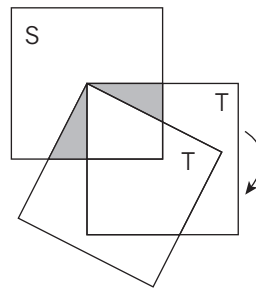
## Ex B12.1



c P gets rotated onto R because two quarter-turns about (2, 1) give a half-turn



- 8 (6, 4), 180°  
 (6, 1), 90° clockwise  
 (6, 1), 270° anticlockwise  
 (6, 7), 90° anticlockwise  
 (6, 7), 270° clockwise
- 9 The area of overlap is a quarter of the square.  
 As T rotates, the area gained by the leading side equals the area lost by the following side.



- 10 a 3 : 1  
 When cog B has completed a full turn, cog A has turned 120° in the opposite direction.
- b Increase the number of teeth on the rear gear.  
 On flat ground, the ratio is 4 : 1. Uphill, the ratio could be 1 : 1

## B12.2 Rich task, p181

- a  $F' = F + 12$  cm      b  $F' = F + a + x$  cm

## Ex B12.2

- 1 a 8 km due east, then 16 km due north, then 4 km due west  
 b 14 km due east, then 12 km due south, then 8 km due west

2 a  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$     b  $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$     c  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$     d  $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

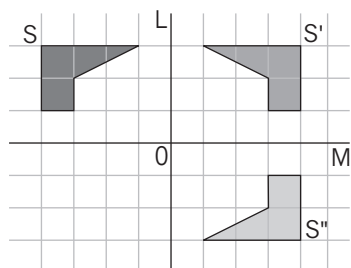
3 The order does not matter  $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ .

4 a Will tessellate                      b Will tessellate  
c Will not tessellate                  d Will tessellate

5  $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ ,  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ ,  $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ ,  $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ,  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

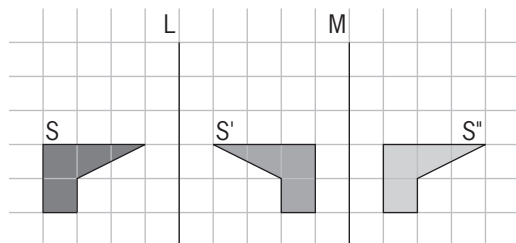
### Ex B12.3

1 a



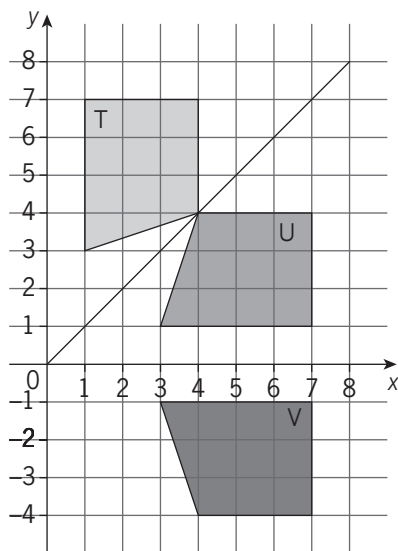
Rotation of  $180^\circ$  about origin, O

b



Translation of  $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

2



Rotation  $90^\circ$  clockwise with centre (0, 0)

3 a Translation of  $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

b Rotation of  $180^\circ$  about (3, 5)

c Translation of  $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$     d Translation of  $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$

e Rotation  $90^\circ$  anticlockwise about (3, 7)

4 a Reflection in the line  $x = 5$   
Rotation of  $180^\circ$  about (5, 6)

Translation of  $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$

b Reflection in the line  $x = 5$   
Rotation of  $180^\circ$  about (5, 2)

Rotation of  $90^\circ$  clockwise about (5, 0)  
Rotation of  $90^\circ$  anticlockwise about (5, 4)

Translation of  $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

5 Translation to the right of distance (2AD + 2AB)

6  $\begin{pmatrix} -5.0 \\ 0.6 \end{pmatrix}$  and  $\begin{pmatrix} 2.0 \\ -4.6 \end{pmatrix}$

7 The maximum distance that a knight can travel in 2 moves is 3 squares across and 3 squares up, because

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

So, when the starting square is included, the knight can travel in 2 moves along the diagonal of a  $4 \times 4$  chess board.

In  $m$  moves, the maximum diagonal travel is  $\frac{1}{2}m \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ .

So, for values of  $n$  which are one more than a multiple of 3 (i.e.  $n = 4, 7, 10, \dots$ ), the smallest number of moves,  $m$ , to travel between opposite corners on

an  $n \times n$  chess board is given by  $\frac{3m}{2} + 1 = n$ ,

i.e.  $m = \frac{2(n-1)}{3}$ . Other values of  $n$  are more complicated to deal with.

### Ex B12.4

1

Object and image are.....	Reflections	Rotations	Translations	Enlargements
.... congruent	yes	yes	yes	no
.... similar	yes	yes	yes	yes

2 a similar

b not similar

3 a congruent

b similar

c different

d congruent

4 a  $x = 50^\circ$   $y = 50^\circ$

$z = 60^\circ$  similar

b  $x = 40^\circ$   $y = 60^\circ$

$z = 80^\circ$  similar

c  $x = 100^\circ$   $z = 20^\circ$

$y = 60^\circ$  similar

5 120 cm

6 a 20 cm

b 25 cm, 100 cm

7 a 7.5 metres

b 4 metres

8  $180 \times 130$  mm

9  $12 \times 8$  inches and  $18 \times 12$  inches are enlargements  
 $10 \times 8$  inches and  $7 \times 5$  inches will need to be cropped

10 Measure the length of the shadow of the tree or building and also measure the length of the shadow of the metre rule. 'Angle of elevation' is the angle that the line from the sun, through the top of the tree to the top of the shadow makes with the ground. The tangent of this angle is the ratio of the height of the metre rule (and tree) to the length of the shadow.

## Rich task B12.5, p190

An enlargement scale factor of  $\frac{1}{2}$  means that the length is multiplied by 0.5 (i.e. is halved). Other fractional scale factors reduce the length in a similar way.

### Ex B12.5

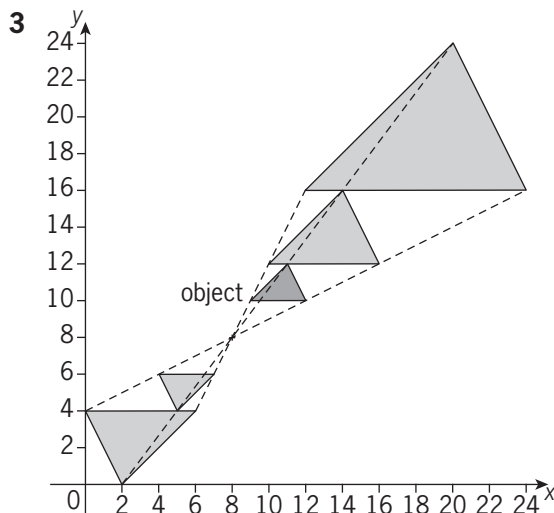
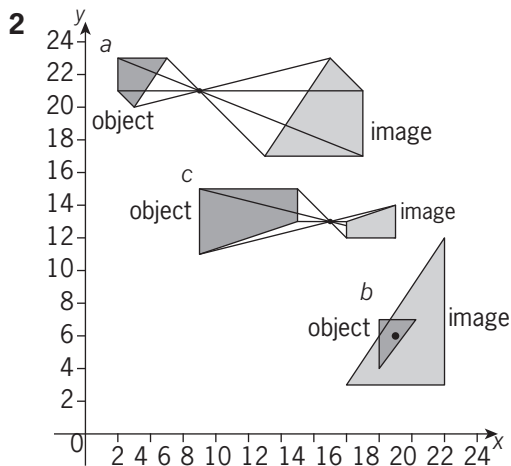
- Vertices of images are
  - (4, 6) (8, 6) (4, 12)    **b** (18, 0) (24, 6) (15, 9)
  - (3, 16) (11, 16) (11, 20) (3, 22)
  - (18, 17) (22, 17) (20, 15)
  - (5, 3) (9, 3) (6, 6)
- i** (0, 3)    **ii** 2    **b i** (12, 1)    **ii** 3
  - i** (5, 7)    **ii** 4    **d** Not an enlargement
- $\frac{1}{2}$     **b**  $\frac{1}{3}$     **c**  $\frac{1}{4}$
- (1, 7),  $\frac{1}{2}$     **b** (4, 8),  $\frac{1}{4}$
- 200
- A4 to A3 has a scale factor 2  
An enlargement (scale factor 2) is used for each successive paper size.

## B12.6 Rich task, p192

$$h = \frac{wz}{x}$$

### Ex B12.6

- Check students' diagrams



- Scale factor  $-\frac{1}{2}$

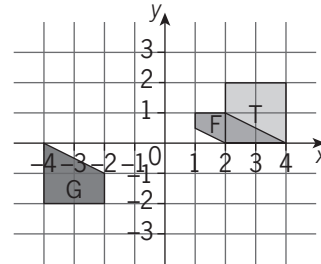
The centre of the enlargement is the intersection of the medians (the line joining each vertex with the midpoint of the opposite side). It is the centre of gravity because it is the intersection of the medians.

## Exam practice

- reflection in the  $x$ -axis
    - rotation about the origin by  $+90^\circ$

- reflection in  $y = x$

2



- Enlargement centre the origin with scale factor  $-2$

- $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$      $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$      $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$     **b**  $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

## Case study: Recycling

Chart text, from DEFRA:

There has been a change in the composition of recycled waste over time. In 1997/8 paper and card was the largest component, making up 37% of the total, followed by compost (20 per cent) and glass (18 per cent). In 2007/08 compost was the largest component (36.1 per cent of the total) with the next largest being paper and card (18.1 per cent) followed by co-mingled (17.7 per cent).

Co-mingled collections - the collection of a number of recyclable materials in the same box or bin, for example paper, cans, plastics - have become more widespread in recent years.'

3 Largest components:

1997/98      Paper and card, Compost & Glass.

2007/08      Compost, Paper and card &  
Co-mingled waste.

25.3 million tonnes of household waste was collected in England in 2007/08;

Total amount of waste collected for re-use, recycling and composting: 8.84 million tonnes

34.9% of this waste was collected for re-use, recycling or composting.

The amount of household waste not re-used, recycled or composted was 16.46 million tonnes, a decrease of 7.0 % from 2006/07. The amount of household waste not re-used, recycled or composted in 2006/07 was 17.7 million tonnes making the total amount of household waste collected in 2006/07 25.76 million tonnes.

This decrease equates to 324kg per person of residual household waste and shows progress towards the 2010 target, in the Waste Strategy 2007, of reducing this amount to 15.8 million tonnes.

New can weighs 55.8g (to 3 s.f)

Weight has decreased by approximately 50%

Volume of container = 1296 cm<sup>3</sup>

Volume of six tomatoes = 679cm<sup>3</sup>

48% of the available volume is empty.

## B13 Bivariate data and time series

### Check in

- check coordinates
- a 10    b 25    c 20    d 12.5    e 27.5

### B13.1 Rich task, p200

Yes, altitude and lung capacity are related. There is a strong positive correlation.

#### Ex B13.1

- scatter diagram
- scatter diagram
- More alcohol associated with longer reaction times  
More time on games associated with shorter reaction times. All subjects took longer at 3.0 am than mid-morning. Also the quicker your reaction time at 3 am, the quicker it is mid-morning.

#### Ex B13.2

- positive correlation. People living at higher altitudes have larger lung capacity
- a positive correlation. Students tend to be good at both subjects or weak at both. Stats marks are higher than maths  
b positive correlation. Taller people have longer arms than shorter people.
- a diagram  
b positive  
c Quick writers are quick with both hands. All but one student took longer with left than right-left-hand.
- diagram. No correlation. Other factors, eg population density, modernity of buildings, affect death toll.

#### Ex B13.3

- a £108  
b £97
- a 4.9 litres  
b i data only collected to 1200 m  
ii data only collected for men
- a answers from graph  
b correlation not very strong; horse data extreme values;

best to use for lifespans between 3 and 18 years.

### Ex B13.4

- No pattern; highs in June and Dec, lows in March and August
- Highest in winter quarters, Oct–Dec and Jan–Mar. Each quarter in 2005 higher than same in 2004, probably price per unit increased
- Rises steadily Mar–Aug. Steep fall in September., maybe back to school. Rise in Dec.
- Same pattern each year with high in May–Aug and low in Jan–Apr. Amount for same period rises each year.
- Same pattern over 2 years. High in Apr–Jun, low in Oct–Dec. Slightly lower in 2005 compared with same period in 2004

### Exam practice

- graph. Temperatures vary, though no trend is evident.
- graph. Number of weddings rises from Jan to Aug, then decreases to Dec. Same pattern both years. Months with 5 Saturdays have slightly more than expected.
- Data can be compared through trends shown by scatter graph, also mean, median, mode, range, IQR or box-and-whisker plot. Mean waiting times are A: 4 mins B: 3.3 mins.

## B14 Simultaneous equations and inequalities

### Check in

- $A = lw$                       Area of rectangle  
 $A = \pi r^2$                       Area of circle  
 $a^2 + b^2 = c^2$                       Length of sides in a right angled triangle  
 $V = lwh$                       Volume of a cuboid  
 $A = \frac{1}{2}bh$                       Area of a triangle
- a  $5\frac{2}{3}$                       b  $2\frac{1}{2}$                       c  $\pm 5$   
d 16                      e 2                      f 2
- a 

x	1	2	3
y	5	8	11

                      b 

x	-1	0	6
y	14	12	0

  
Check graph                      Check graph

### B14.1 Rich task, p212

Top: 3  
Bottom left: 7  
Bottom right: 5  
Method is to add all the numbers in the squares and divide by 2, then to share that number out between the circles in the correct proportions.

#### Ex B14.1

- a  $x = 1, y = 3$                       b  $a = 1, b = 2$                       c  $p = 2, q = 3$   
d  $m = 2, n = 5$                       e  $c = 4, d = 2$                       f  $k = 5, t = 2$   
g  $x = -1, y = 2$                       h  $a = 2, b = -3$

- 2 a  $x = 4, y = 1$     b  $p = 2, q = 1$     c  $c = 6, d = 2$   
 d  $m = 3, n = 1$     e  $a = 2, b = 2$     f  $k = 2, t = 3$   
 g  $x = 2, y = -2$     h  $p = -1, q = -5$
- 3 a  $a = 3, b = 5$     b  $p = 1, q = 6$     c  $c = 5, d = -2$   
 d  $k = 1, t = \frac{1}{2}$     e  $m = 4, n = 2$     f  $x = 2, y = \frac{1}{3}$   
 g  $a = -3, b = 2$     h  $p = 4, q = -1$
- 4 An Americano costs £1.60 and a Cappuccino costs £1.95.

## B14.2 Rich task, p214

Red: 12  
 Green: 5  
 Blue: 10  
 Missing total: 32

### Ex B14.2

- 1 a  $x = 2, y = 3$     b  $a = 5, b = 1$     c  $p = 5, q = 2$   
 d  $m = 2, n = 1$     e  $m = 3, n = 4$     f  $t = 3, k = 2$   
 g  $x = -2, y = 2$     h  $a = 5, b = -2$
- 2 a  $x = 4, y = 2$     b  $p = 2, q = 5$     c  $a = 7, b = 3$   
 d  $m = 3, n = -1$     e  $k = -3, t = 4$     f  $p = -1, q = -2$   
 g  $a = 5, b = \frac{1}{2}$     h  $x = -1, y = 1\frac{1}{2}$
- 3 a Cherries are worth 9 and grapes are worth 11.  
 b Apples are worth 8 and bananas are worth 15.
- 4 a Edward sells 43 badges.  
 b Jacob needs 5 minicabs and 4 black cabs.  
 c i  $a = -3$  and  $b = 2$ .  
 ii Gradient =  $1\frac{1}{2}$  and  $y$ -intercept =  $(0, 2)$

## B14.3 Rich task, p216

There are 6 intersections. The lines that intersect in the first quadrant are  $x + 3y = 7$ ,  $7y - x = 13$  and  $2y - x = -2$ .

### Ex B14.3

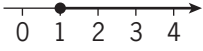
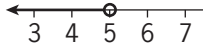
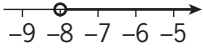
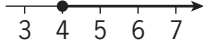
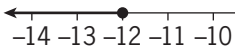
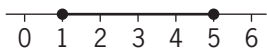
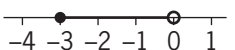
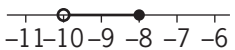
- 1 a i  $x = 2, y = 5$     ii  $x = 1, y = 2$   
 iii  $x = 6, y = 1$     iv  $x = 3, y = 0$
- b These lines are parallel.
- 2 a i  $x + 2y = 5$  and  $2x + y = 4$     ii  $x - y = 2$  and  $2x + y = 4$   
 iii  $x + y = 5$  and  $2x + y = 4$     iv  $x - y = 2$  and  $x + 2y = 5$   
 v  $x + 2y = 5$  and  $x + y = 5$     vi  $x + y = 5$  and  $x - y = 2$
- b Check working shown to solve simultaneous equations algebraically.
- 3 a Graphs of  $x + y = 2$  and  $3x - y = 2$  drawn and solution given as  $x = 1, y = 1$ .  
 b Graphs of  $x + y = 7$  and  $x - 2y = 1$  drawn and solution given as  $x = 5, y = 2$ .  
 c Graphs of  $x - y = -5$  and  $2x + y = 8$  drawn and solution given as  $x = 1, y = 6$ .  
 d Graphs of  $2x - y = 7$  and  $3x + y = 8$  drawn and solution given as  $x = 3, y = -1$ .

- 4 a The two numbers are 2 and 5.  
 b Alexander is 7 and William is 4.
- 5 Check pair of simultaneous equations with more than one solution given, e.g. 1 linear, 1 quadratic.

## B14.4 Rich task, p218

The rectangles are  $4 \times 4$  (area = perimeter = 16) and  $3 \times 6$  (area = perimeter = 18). The algebraic proof involves rearranging  $w \times l = 2w + 2l$  to make  $w$  the subject, and separately rearranging to make  $l$  the subject. Substitute one into the other and solve.

### Ex B14.4

- 1 a  $a \geq 6$  where  $a$  is the age of a child in years.  
 b  $h \geq 1.2$  where  $h$  is the height of a person in metres.  
 c  $s \leq 5$  where  $s$  is the speed of a vehicle in mph.  
 d  $d < 5$  where  $d$  is the age of a child in years.
- 2 a  $x \geq 2$     b  $x < 1$     c  $x > -4$   
 d  $x \leq 0$     e  $-1 < x \leq 2$     f  $-3 \leq x < -1$
- 3 a     b   
 c     d   
 e     f   
 g     h 
- 4 a 2, 3, 4, 5    b 1, 2, 3, 4, 5    c -6, -5, -4  
 d -1, 0, 1    e None    f 1, 2  
 g -2, -1, 0    h -4, -3, -2, -1
- 5 a  $x \leq 3$     b  $y > 5$     c  $a < -3$   
 d  $q \geq 21$     e  $n > -12$     f  $t < 3$   
 g  $p \geq 12$     h  $b < 4$     i  $k \leq 3$   
 j  $y \leq 6$     k  $x > 4$     l  $m \geq 12$
- 6 a  $x \leq 4$     b  $y \geq 4$     c  $n > 1$   
 d  $b < 2$     e  $t \geq 6$     f  $k > 5$   
 g  $m \geq 8$     h  $a < -2$
- 7 a  $-1 < x \leq 3$     b  $1.5 \leq y < 5$
- 8 a  $4 \leq x \leq 7$     b  $\frac{1}{2} < a \leq 4$   
 c  $-\frac{1}{2} < y \leq 2$     d  $-2 \leq b < 4$

## B14.5 Rich task, p220

The mean of the squares of two numbers is always larger than the square of the means.

The mean of the squares is  $\frac{(x^2 + y^2)}{2}$ . The square of the

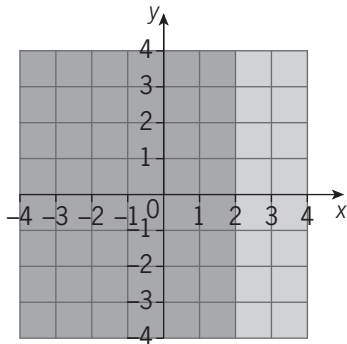
$$\text{means is } \left(\frac{(x+y)}{2}\right)^2 = \frac{(x^2 + y^2 + 2xy)}{4} = \frac{(x^2 + y^2)}{2} - \frac{(x^2 + y^2)}{4} + \frac{2xy}{4}$$

$$= \frac{(x^2 + y^2)}{2} - \frac{(x-y)^2}{4} > \frac{(x^2 + y^2)}{2} - \frac{(x-y)^2}{4}$$

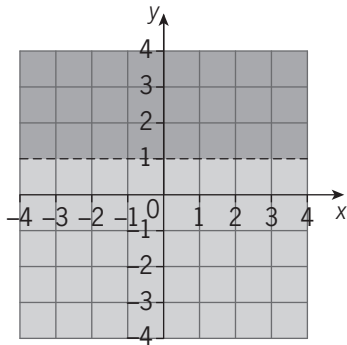
## Ex B14.5

- 1 **a**  $x \geq 2$                       **b**  $y > 1$   
**c**  $x < -1$                         **d**  $x \geq -2$  and  $y \geq -1$   
**e**  $-3 < x \leq 1$                   **f**  $y > 0$  and  $y \leq -3$

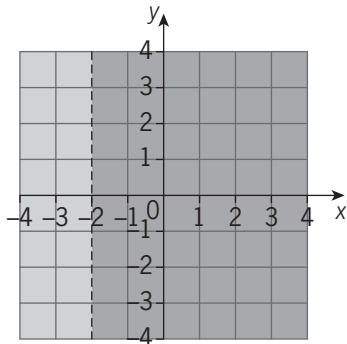
2 **a**



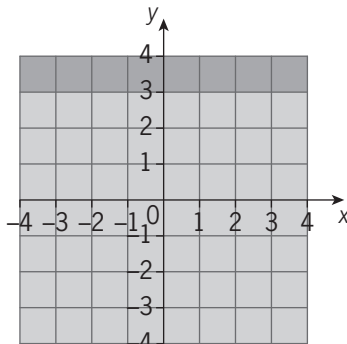
**b**



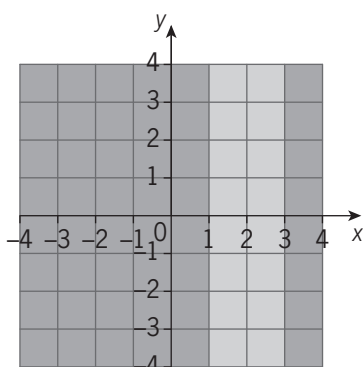
**c**



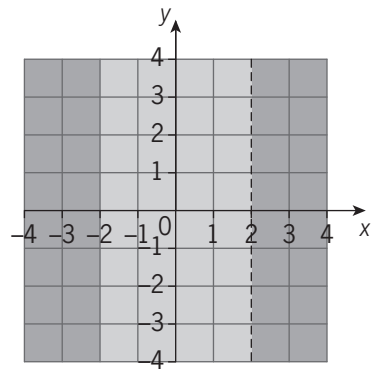
**d**



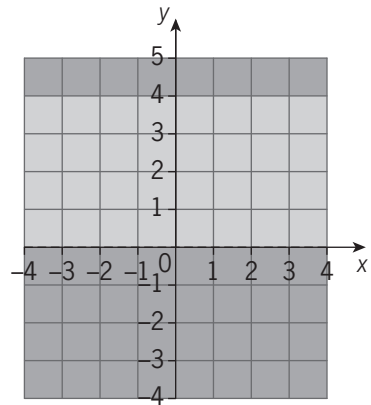
**e**



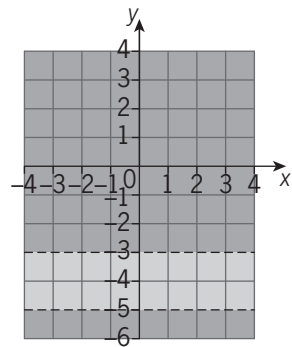
**f**



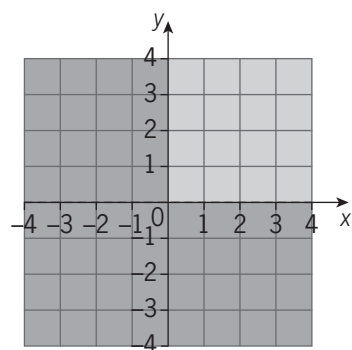
**g**



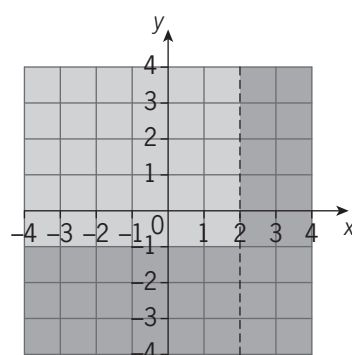
**h**



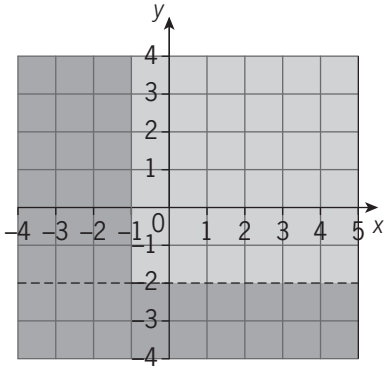
**i**



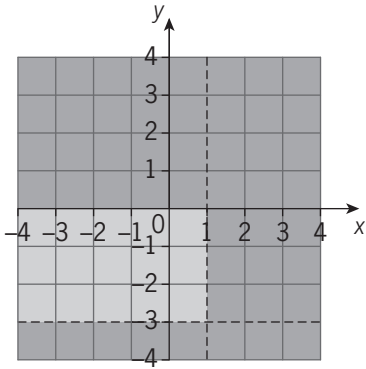
**j**



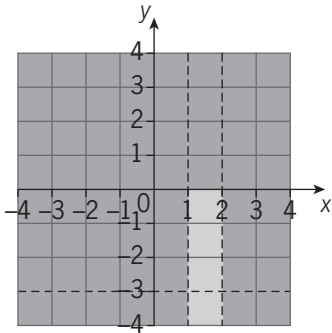
3 a



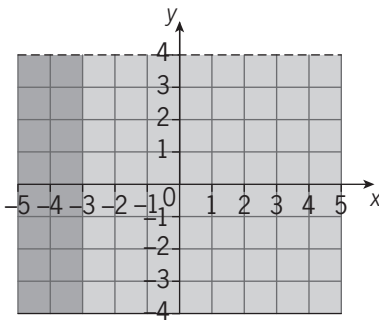
b



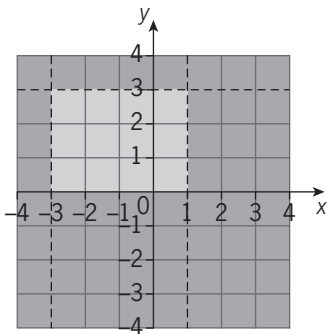
c



d



4



The integer pairs are (0, 0), (0, 1), (0, 2), (-1, 0), (-1, 1), (-1, 2), (-2, 0), (-2, 1), (-2, 2).

## B14.6 Rich task, p222

Many possibilities, only one for each listed.

a  $x > 0, x < 2, y > 0, y < 2$

b  $x > 0, x < 2, y > 0, y < 6$

c  $x > 0, y > 0, x + y < 4$

d  $y > 0, x + y < 6, y - x < 0$

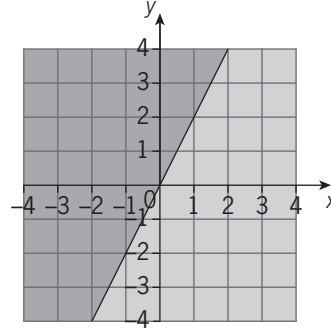
## Ex B14.6

1 a  $y > x$

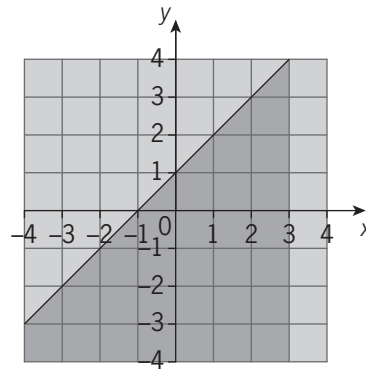
b  $x + y \leq 2$

c  $y > 2x - 2$

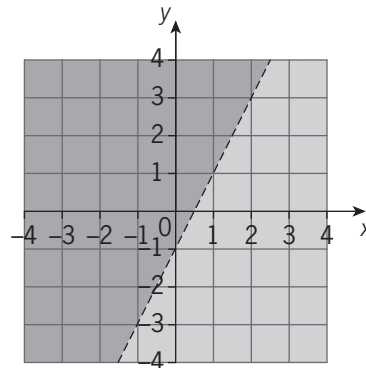
2 a



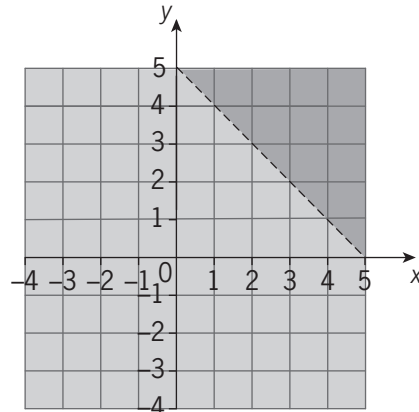
b

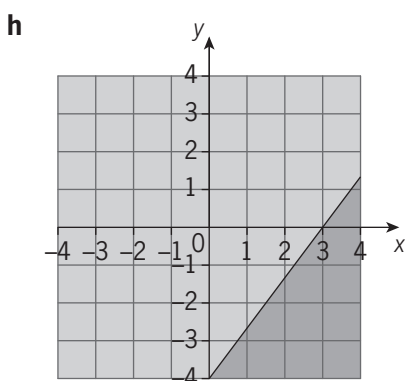
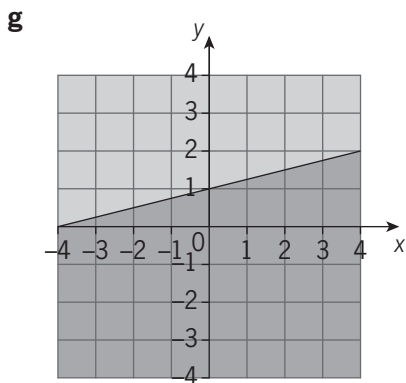
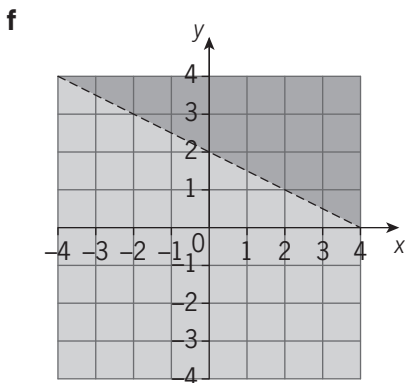
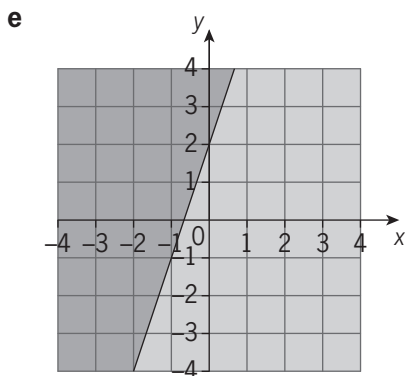


c



d





- 3 a**  $x < 3, y \leq 1, x + y \geq 1$     **b**  $-1 < x \leq 1, y \geq -1, y < x + 2$   
**c**  $-3 \leq y \leq 2, y < 4 - 2x, y > -4 - 2x$
- 4 a** The inequalities  $x \geq 0, y \geq 0$  and  $2x + y \leq 4$  plotted correctly. Shape is triangle.  
**b** The inequalities  $y > -1, y < x + 2$  and  $x + y < 6$  plotted correctly. Shape is isosceles triangle.  
**c** The inequalities  $-3 < y < 1, x > -4$  and  $y < 1 - 2x$  plotted correctly. Shape is trapezium.  
**d** The inequalities  $-2 \leq x < 3, 2y \leq x + 2$  and  $2y \geq x - 6$  plotted correctly. Shape is parallelogram.
- 5 a**  $6x + 10y \leq 100$  represents the cost of the CDs and the fact that this cannot exceed £100 thus  $3x + 5y \leq 50$ .  
 $x + y \geq 10$  represents the fact that the total must be at least 10 CDs.

**Student book answers**

$x \leq y$  represents the fact that he wants to buy no more bargain basement CDs than chart CDs.

- b** Correctly drawn inequalities.  
**c** Points are (0, 10), (1, 9), (2, 8), (3, 8), (3, 7), (4, 6), (4, 7), (5, 5), (5, 6), (5, 7), (6, 6).  
**d** Harry should buy 5 bargain basement CDs and 7 chart CDs or just buy 10 chart CDs.

## Exam practice

**1 a**  $x = \frac{4}{3}, y = \frac{11}{3}$     **b**  $x = \frac{1}{2}, y = -2$

**2 a i**  $x \geq 7\frac{1}{2}$     **ii**

**b** 3, 4, 5, 6, 7

**3** £5.79

## B15 Indices and surds

### Check in

- 1 a**  $3^2$     **b**  $4^5$     **c**  $6^3$     **d**  $5^4$   
**2 a**  $3 \times 3 \times 3$     **b**  $6 \times 6$   
**c**  $4 \times 4 \times 4 \times 4 \times 4$     **d**  $8 \times 8 \times 8$   
**e**  $7 \times 7 \times 7 \times 7$     **f**  $5 \times 5 \times 5 \times 5 \times 5$
- 3 a** 16    **b** 27    **c** 16  
**d** 125    **e** 128    **f** 7

### B15.1 Rich task, p228

- a i** 12    **ii**  $2^5$     **b i** 81    **ii**  $3^4$   
**c** Multiplying powers makes the powers themselves add; dividing powers makes one power subtract from the other.

### B15.1 Rich task 2, p228

- i** 1    **ii**  $5^0$   
Any number to the power 0 gives 1.

### Ex B15.1

- 1 a i**  $5^3$     **ii**  $5^3$     **iii**  $2^6, 4^3$   
**b i**  $16^3$     **ii** all squares    **iii**  $4^6, 16^3$
- 2 a**  $2^4 \times 3$     **b**  $2 \times 3 \times 7$     **c**  $2^3 \times 3 \times 5^2$   
**d**  $2^8 \times 5$     **e**  $3 \times 5 \times 7^2$
- 3 a**  $7^9$     **b**  $3^8$     **c**  $8^9$     **d**  $15^8$     **e**  $4^{12}$   
**f**  $3^6$     **g** 8    **h**  $9^5$     **i**  $6^4$
- 4 a** 2, 4    **b i** 6    **ii** 4
- 5 a** 729    **b** 65 536    **c i** 2048    **ii** 8192  
**d** 10 000 000 000
- 6 a**  $2^{17}$     **b**  $3^{11}$     **c**  $4^7$
- 7 a** 1    **b** 1
- 8 a**  $6^2 \times 6^3 = 6^5$     **b**  $6^6 \div 6^2 = 6^4 = 1296$

## B15.2 Rich task, p230

- a  $16^1$       b 16  
 c Because  $16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = (16^{\frac{1}{2}})^2 \sqrt{16} \times \sqrt{16} = (\sqrt{16})^2 = 16$ .  
 This is true for any number.

## B15.2 Rich task 2, p230

- a  $5^{-1}$       b  $\frac{1}{5}$   
 c Because  $5^{-1} \times 5^1 = 5^0 = \frac{1}{5} \times 5 = \frac{5}{5} = 1$ . This is true for any number.

### Ex B15.2

- 1 a 8      b 4      c 13      d 3  
 e 15      f 3      g 3      h 9  
 i 2      j 2      k  $\frac{1}{16}$       l  $\frac{1}{94}$   
 m  $\frac{1}{16}$       n  $\frac{1}{125}$       o  $\frac{1}{100000}$       p  $\frac{1}{3}$   
 q  $\frac{1}{3}$       r  $\frac{1}{2}$       s 10      t  $\frac{1}{10}$   
 2 a  $7^6$       b  $17^{15}$       c  $6^{14}$       d  $12^{12}$       e  $5^{-6}$   
 3 a  $\frac{1}{4}$       b 2      c  $\frac{1}{625}$       d  $\frac{1}{27}$       e  $\frac{1}{32}$   
 4 a  $\frac{5}{36}$       b  $\frac{1}{343}$       c  $\frac{3}{16}$       d  $\frac{2}{25}$       e  $\frac{25}{7}$   
 5 a  $9^2$       b  $3^4$   
 6 a  $2^{3p}$       b  $\frac{2}{3}$   
 7 a 13 g      b i  $133 = 5 + 2^t$       ii 7

## B15.3 Rich task, p232

$\sqrt{10}$

## B15.3 Rich task 2, p232

- a i 9      ii 9  
 b i 10      ii 10  
 $\sqrt{r} + \sqrt{s} = \sqrt{(r+s)}$

### Ex B15.3

- 1 a 10      b 7      c 4  
 d  $\frac{1}{2}$       e 9      f 80  
 2 a  $3\sqrt{11}$       b  $2\sqrt{15}$       c  $5\sqrt{3}$   
 d  $10\sqrt{2}$       e  $5\sqrt{5}$       f  $8\sqrt{2}$   
 3 a 15      b 14      c 6      d  $6\sqrt{5}$   
 4 a 8      b 2      c 16  
 5 a  $3\sqrt{2}$       b  $18\sqrt{2}$       c  $\frac{1}{18}$   
 6 b i  $\frac{\sqrt{3}}{2}$       ii  $\frac{1}{\sqrt{3}}$       iii  $\sqrt{3}$   
 c i  $\frac{1}{\sqrt{2}}$       ii  $\frac{1}{\sqrt{2}}$

## B15.4 Rich task, p235

- i In this method you know that  $2^2 = 4$  and  $3^2 = 9$  so 2 is the nearest integer to  $\sqrt{5}$ . The  $\frac{1}{2}$  is arbitrarily chosen; squaring both sides twice reduces the size of the fraction on the left hand side to  $\frac{1}{16}$  and so the 'error' in the calculation is reduced by a factor of  $\frac{1}{8}$ . Rearranging to make  $\sqrt{5}$  the subject allows an approximate equality to be constructed.  
 ii The approximate value would become more precise.  
 iii  $\frac{17}{12}$       iv  $\frac{28}{16}$

### Ex B15.4

- 1 a 3      b 2      c 5      d 4  
 e 4      f 7      g  $\frac{3}{2}$       h  $\frac{4}{3}$   
 2 a  $4\sqrt{3}$       b  $7\sqrt{3}$       c  $3\sqrt{5}$       d  $18\sqrt{2}$   
 e  $5\sqrt{7}$       f  $10\sqrt{5}$   
 3 a 46      b -1      c 32  
 d 12      e  $31 - 10\sqrt{6}$       f  $11 - 4\sqrt{7}$   
 g  $11 - 6\sqrt{2}$       h  $\frac{1}{3}(4\sqrt{3} + \sqrt{15})$       i  $\frac{3\sqrt{7}}{7} + 1$   
 5  $4\sqrt{3}$   
 6 a  $\frac{4\sqrt{3}-6}{3} = \frac{4}{3}\sqrt{3} - 2$  so  $p = -2$  and  $q = \frac{4}{3}$   
 b AC is negative ( $\sqrt{5} - 5$ ) so has no real root

### Exam practice

- 1 a  $\frac{5}{16}$       b 1      c 18  
 2  $\frac{1}{2}$   
 3  $\frac{841\sqrt{2}}{2}$ ,  $\frac{841}{2}$

## Case study: Business

Original table:

	January (£)	February (£)	March (£)
Quantity of standard	18	16	24
Quantity of deluxe	12	14	9
Standard card sales	45.9	40.8	61.2
Deluxe card sales	43.2	50.4	32.4
TOTAL INCOME	89.1	91.2	93.6
Materials used	7.8	7.6	9
Wages	42	44	42
Craft fair fees	10	10	10
Advertising	5	5	5
TOTAL EXPENDITURE	64.8	66.6	66
NET CASH SURPLUS/DEFICIT	24.3	24.6	27.6
CASH BALANCE BROUGHT FORWARD		24.3	48.9
CASH BALANCE TO CARRY FORWARD	24.3	48.9	76.5

Fair fees £15:

	January (£)	February (£)	March (£)
Quantity of standard	18	16	24
Quantity of deluxe	12	14	9
Standard card sales	45.9	40.8	61.2
Deluxe card sales	43.2	50.4	32.4
TOTAL INCOME	89.1	91.2	93.6
Materials used	7.8	7.6	9
Wages	42	44	42
Craft fair fees	15	15	15
Advertising	5	5	5
TOTAL EXPENDITURE	69.8	71.6	71
NET CASH SURPLUS/DEFICIT	19.3	19.6	22.6
CASH BALANCE BROUGHT FORWARD		19.3	38.9
CASH BALANCE TO CARRY FORWARD	19.3	38.9	61.5

Material costs increase by 40%:

	January (£)	February (£)	March (£)
Quantity of standard	18	16	24
Quantity of deluxe	12	14	9
Standard card sales	45.9	40.8	61.2
Deluxe card sales	43.2	50.4	32.4
TOTAL INCOME	89.1	91.2	93.6
Materials used	10.92	10.64	12.6
Wages	42	44	42
Craft fair fees	10	10	10
Advertising	5	5	5
TOTAL EXPENDITURE	67.92	69.64	69.6
NET CASH SURPLUS/DEFICIT	21.18	21.56	24
CASH BALANCE BROUGHT FORWARD		21.18	42.74
CASH BALANCE TO CARRY FORWARD	21.18	42.74	66.74
Materials cost standard	0.42		
materials cost deluxe	0.28		

20% discount:

	January (£)	February (£)	March (£)
Quantity of standard	18	16	24
Quantity of deluxe	12	14	9
Standard card sales	36.72	32.64	48.96
Deluxe card sales	34.56	40.32	25.92
TOTAL INCOME	71.28	72.96	74.88

Materials used	7.8	7.6	9
Wages	42	44	42
Craft fair fees	10	10	10
Advertising	5	5	5
TOTAL EXPENDITURE	64.8	66.6	66
NET CASH SURPLUS/DEFICIT	6.48	6.36	8.88
CASH BALANCE BROUGHT FORWARD		6.48	12.84
CASH BALANCE TO CARRY FORWARD	6.48	12.84	21.72
discounted price standard	2.04		
discounted price deluxe	2.88		

Scenario with £8.00 for materials and £40 for wages.

Constraints:

$$0.3s + 0.2d \leq 8$$

$$s + 2d \leq 40$$

The aim is to maximise profit  $£P$ , where

$$P = 1.25s + 1.40d - 15$$

Gives maximum profit £24 when  $s = 20$  and  $d = 10$

## B16 Vectors

### Check in

1 Check own definitions

2 a 3 lines      b 6 lines      c 8 lines

### Ex B16.1

1 a i 2a    ii 2b    iii -2b    iv 2a + 2b

v b - 2a    vi -a - 2b

b i OX    ii OT    iii OD    iv OQ

v OG    vi OF

2 a true    b false    c true    d true

e false    f true    g true    h true

i false    j true    k true    l true

m false    n true

3 b = 2.5a    c = 3a    d = -2a    e =  $\frac{1}{2}$ a

f =  $-\frac{3}{2}$ a    g =  $-\frac{1}{2}$ a    h = -a

4 a OV = b + a    OW = b + 2a    WO = -2a - b

b PR = b + 2a    QR = b + a

c LN = a + b    NM = b - a

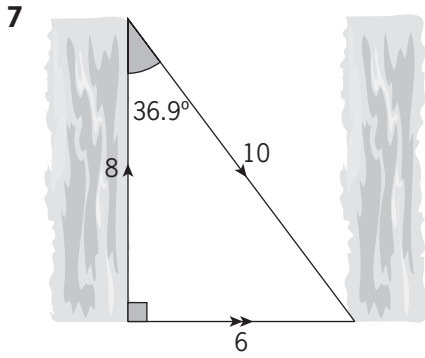
5 a a    b 2a    c 2b    d 3b

e 2a - b    f 3b - a    g 3b - 2a    h 2a - 3b

6 Ground speed is 123 mph

a Greatest ground speed occurs when the wind blows in the same direction as the aircraft.

b Least ground speed occurs when the wind blows in the opposite direction to the aircraft.



The man swims at an angle of  $36.9^\circ$  to the bank. His resultant speed is 6 metres per minute.

### Ex B16.2

1 a  $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$  b  $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$  c  $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$  d  $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$

2  $\begin{pmatrix} 15 \\ 4 \end{pmatrix}$

3  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$  and  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ;  $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ \frac{1}{2} \end{pmatrix}$ ;  $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$  and  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

4 a  $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$  b  $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$  c  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

5 a  $p = 5, q = 3$  b  $m = 5, n = 4$   
c  $c = 5, d = -1$  d  $u = 3, v = 9$

6 a  $x = 4, y = -1$  b  $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

7 Several possibilities

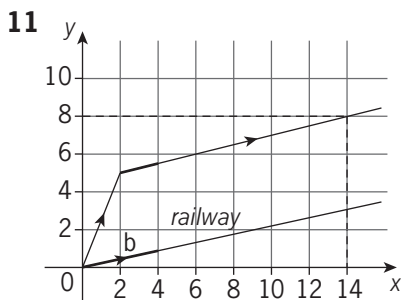
8 a  $\begin{pmatrix} 7 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \end{pmatrix}$  b  $(7, 4), (4, -1)$

9 a  $\begin{pmatrix} 20 \\ 4 \end{pmatrix}$  b  $\begin{pmatrix} 10 \\ 2 \end{pmatrix}$

The diagonals of a parallelogram bisect each other.

10 a  $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$  b  $\begin{pmatrix} 4 \\ \frac{1}{2} \end{pmatrix}$  c  $\begin{pmatrix} 1 \\ -4\frac{1}{2} \end{pmatrix}$  d  $\begin{pmatrix} 2 \\ -9 \end{pmatrix}$

MN and EF are parallel and the length of MN is half that of EF



$x = 14$

12  $t = 3\frac{1}{3}$  hours,  $y = 10$

13 a e b  $a + e$  c  $2a + e$

d  $2e + a$  e  $a + 2e$

AC and OD are parallel and equal in length.

14 a  $p + q$  b  $2(p + q + r)$  c  $p + q$   
ST and VU are parallel and equal in length.  
STUV is a parallelogram.

15 The three medians intersect at a point which is one third of the way along each median.  
This point of intersection is the centre of gravity of the triangular lamina.

### Exam practice

1 a  $-a + 2b$  b  $-\frac{1}{2}a + b$   
c AB and XY are parallel and AB is twice the length of XY

2 Rhombus (Parallelogram with 4 equal sides)

3 a  $b + 1.5a$  b  $-2b + 1.5a$   
c  $-3.5a - b$  d  $0.5a + 2b$

### C17 Percentages and proportional change

#### Check in

1 Gill £400, Paul £100

2  $33\frac{1}{3}\%$

#### C17.1 Problem, p252

The trader made a £12.50 profit.

#### Ex C17.1

1 a £374.40 b £48.95

2 35 p

3 £36 016.80

4 i 164 p for 250 g  $\rightarrow$  0.656 p per gram

ii 123 p for 200 g  $\rightarrow$  0.615 p per gram  
so reducing price is better for customer.

5 i equivalent to a 1% decrease

ii same as i

iii equivalent to a 21% increase

iv equivalent to a 19% decrease

6 a Increasing contents gives 5.5 ml per p and decreasing price gives 5.6 (1dp) ml per p so decreasing price is better for customer

b i Increasing contents gives 6 ml per penny; decreasing cost gives 6.25 ml per p.

ii Increasing contents gives 7 ml per penny; decreasing cost gives 8.3 ml per p.

iii Increasing contents gives 7.5 ml per penny; decreasing cost gives 10 ml per p.

c Decreasing price is always better.

d 20% decrease on price

## C17.2 Problem, p254

It would have cost \$498.58.

### Ex C17.2

- 1 £32.00
- 2 £85.00
- 3 \$147.20
- 4 360
- 5 £902.00
- 6 £45.00
- 7 £850.00
- 8 650 cm<sup>3</sup>
- 9 £515 000

### C17.3 Rich task, p256

- iv You usually can't fold a piece of paper more than 7 times, but it's a myth that this is the limit for any piece of paper of any size.
- v 0.162 mm
- vi 24 folds

### Ex C17.3

- 1 a 5.4 m      b 4
- 2 a i 2 360 279      ii 3 316 022  
b 13 years
- 3 a i 23 699      ii 14 370  
b 21 years
- 4 a i £21 732.77      ii £26 586.28  
b 11 years
- 5 £3 099.31
- 6 4.6% paid annually is best. 4.5% paid monthly is equivalent to 4.5939825% paid annually.

### Ex C17.4

- 1 a  $y = 6x$       b 42
- 2 a  $y = 1.5x$       b 7.5
- 3 a  $e = 1.25 m$       b i 1.6 kg      ii 6.25 cm
- 4 a  $y = 0.5x^3$       b 13.5
- 5 a  $E = 15e^2$       b i 960      ii 5
- 6 a  $E = 1.6v^2$       b i 25.6      ii 10
- 7  $\pi$
- 8  $\frac{4\pi}{3}$

### Ex C17.5

- 1 a  $y = \frac{20}{x}$       b  $\frac{2}{3}$

- 2 a  $y = \frac{1.2}{x}$       b 0.75
- 3 a 20      b 13 hours 20 mins
- 4 a  $V = \frac{800}{P}$       b  $3.2 m^3$       c  $166\frac{2}{3} N/m^2$
- 5 a  $y = \frac{15}{\sqrt{x}}$       b i 7.5      ii 100
- 6 a  $y = \frac{1}{x^2}$       b i  $\frac{1}{36}$       ii 0.2
- 7 a  $F = \frac{12}{d^2}$       b i 48      ii 4

### Exam practice

- 1 £102.00
- 2 a 228 g      b 6
- 3 a  $s = 0.375 d$       b 112 cm

## C18 Area and perimeter

### Check in

- 1 i a 6 cm by 2 cm, perim = 16 cm  
b 4.5 cm by 4.5 cm, perim = 18 cm  
c 4 cm by 7 cm by 8.1 cm, perim = 19.1 cm  
ii a 12 cm<sup>2</sup>      b 20.25 cm<sup>2</sup>      c 14 cm<sup>2</sup>
- 2 a check net      b 54 cm<sup>2</sup>      c 27 cm<sup>3</sup>

### Ex C18.1

- 1 a i 32 cm      ii 56 cm<sup>2</sup>  
b i 34 cm      ii 38 cm<sup>2</sup>  
c i 27.8 cm      ii 23.5 cm<sup>2</sup>  
d i 48.8 cm      ii 72 cm<sup>2</sup>
- 2 £534.60
- 3  $x = 6$  cm
- 4  $x = 2$  m

### C18.2 Rich task, p268

You need  $\pi$  (3.14...) squares to fill the circle.

### Ex C18.2

- 1 a 16 cm      b 25 cm      c 19 cm      d 63 cm
- 2 a 125.6 cm      b 1592
- 3 452 m<sup>2</sup>
- 4 16.6 m<sup>2</sup>
- 5 a 615 m<sup>2</sup>      b £52300
- 6 a 1.3 cm, 0.8 cm      b 2.01 cm<sup>2</sup>      c 3.30 cm<sup>2</sup>
- 7 15.9 cm
- 8 63.7 cm
- 9 1.4 cm
- 10 2.39 metres
- 11 5 cm

## Ex C18.3

- 1 area, cm<sup>2</sup>      arc length, cm  
a 22.6      7.5  
b 75.4      18.8  
c 191      42.4  
d 24.4      4.9
- 2 a 10.5 cm    b 4.7 cm    c 30 cm    d 28 cm
- 3 a  $\frac{5}{13}$       b 20.9 cm<sup>2</sup>
- 4 51.3 cm<sup>2</sup>
- 5 a 31.4 cm<sup>2</sup>    b 3.9 cm    c 13.7 cm<sup>2</sup>
- 6 245 cm<sup>2</sup>
- 7 a 94.2 cm<sup>2</sup>    b 6 cm      c 9.8 cm
- 8 28.6 cm
- 9 60.1°

## Exam practice

- 1  $\frac{p}{8}, \frac{p\sqrt{2}}{16}, \frac{p\sqrt{2}}{16}$  cm  
2  $21\pi$  cm<sup>2</sup>  
3 7.816 cm<sup>2</sup>

## Case study: Holiday

- 1 £32.63 (assuming 13 weeks)
- 2 £4 per hour  
£52 (assuming 13 weeks)
- 3 CDs £2.50 each, DVDs £3.50 each  
£88.50
- 4 £1 : 1.31 Eur  
90.78 euros for £70 and 1% commission  
£1 : 10.37 FRF  
£1 : 3.09 DEM  
°C =  $\frac{2}{5}F - 16/5$

## C19 Algebraic manipulation

### Check in

- 1 a  $7x$     b  $5a + 16b$     c  $6y + 4$     d  $26x + 2x^2$
- 2 a  $3x + 6$     b  $5y - 40$     c  $x^2 + 5x$     d  $6p^2 + 12p$
- 3 a  $5^{10}$     b  $2^2$     c  $4^4$   
d  $6^{14}$     e  $7^6$     f  $5^6$

## C19.1 Rich task, p278

The numbers above and below always add to twice the number in the middle (this only works for numbers in the same month).  
Call the middle number  $x$ , then the number above is  $x - 7$  and the number below is  $x + 7$ .  $(x - 7) + (x + 7) = 2x$ .

## Ex C19.1

- |   |          |         |             |                |
|---|----------|---------|-------------|----------------|
| 1 | $9a - b$ | $2m$    | $5p + 4$    | $9c$           |
|   | $12mn$   | $2x^2$  | $2xy$       | $4xy + 8x$     |
|   | $15mn$   | $6b^2$  | $4b$        | $2x$           |
|   | $40k^3$  | $5xy^2$ | $5x + 8x^2$ | $\frac{rh}{2}$ |
- 2 a  $x^5$     b  $x^2$     c  $x^4$     d  $x^4$   
e  $y^{13}$     f  $k^3$     g  $n^{12}$     h  $g^3$     i  $b^{-6}$   
j  $g^{-12}$     k  $j^{-6}$     l  $t^{10}$     m  $n^{-2}$
- 3 a  $12h^{11}$     b  $3p^2$     c  $4m^{16}$     d  $36r^{-1}$   
e  $27h^{-9}$     f  $3b^8$     g  $2m^{19}$     h 2
- 5  $2x^5$  Height > 40 cm
- 6 False

## C19.2 Rich task, p280

The number is always divisible by 9.  
The answer can be constructed algebraically then the first number will be  $10x + y$  and the second number will be  $10y + x$ .  
Subtracting the second from the first you get  $9x - 9y$ , which factorises to  $9(x - y)$ , a number divisible by 9.

## Ex C19.2

- 1 a  $20x + 32$       b  $12p^2 - 21p$   
c  $10m - 4m^2$       d  $17y + 30$   
e  $15x^2 + 10xy - 45x$     f  $59 - t$   
g  $3h + 16$       h  $3x^3 + x^4$
- 2 a Perimeter is two widths and two lengths added together.  $2x + 4$   
b  $5x - 15$
- 3 a  $3(x + 2y + 3z)$     b  $5(2p - 3)$       c  $x(5y + 7)$   
d  $3m(2n + 3t)$     e  $4x(4x - 3y)$     f  $3p(1 + 3q)$   
g  $7x(y - 8x)$       h  $3x(x + 4x^2 - 2)$   
i  $(m + n)(3 + m + n)$   
j  $(p - q)(4 + (p - q)^2)$       k  $(a + b)(x + y)$
- 5 a  $3x$       b  $9x$       c  $x + 2$   
d  $\frac{x-2}{3}$       e  $x + 4$       f  $\frac{y^2 - 3y}{2}$
- 6 The answer is always 2

## C19.3 Rich task, p282

$(a + b)^2$   
 $\frac{1}{2}ab$   
 $c^2 = (a + b)^2 - 2ab = a^2 + b^2$   
This is Pythagoras' theorem and can be achieved in a variety of ways.

### Ex C19.3

- 1 a  $x^2 + 7x + 10$     b  $y^2 + 15y + 44$     c  $x^2 + 2x - 15$   
d  $p^2 - 2p - 63$     e  $m^2 + 16m + 64$     f  $y^2 - 49$
- 3 a  $2x^2 + 14x + 24$     b  $12p^2 + 23p + 10$   
c  $6m^2 - 32m + 42$     d  $10y^2 + 17y - 63$   
e  $9t^2 - 12t + 4$     f  $2x^2 + 4x - 15$
- 4  $2x^2 + 10x + 8$
- 5 a  $-3\frac{4}{5}$     b 4
- 6 15 metres
- 7 a  $y^3 - 9y^2 + 27y - 27$   
b  $y^3 - 9y^2 + 27y + 27$   
c True

### C19.4 Rich task, p284

The difference is always the sum of the two consecutive numbers.

The difference between the squares of two numbers that differ by 2 is twice their sum.

The difference between the squares of two numbers that differ by 3 is three times their sum, and so on.

### Ex C19.4

- 1 a  $(x+2)(x+5)$     b  $(x+3)(x+5)$     c  $(x+2)(x+6)$   
d  $(x+5)(x+7)$     e  $(x+2)(x-5)$     f  $(x+5)(x-7)$   
g  $(x-3)(x-5)$     h  $(x+4)(x-5)$     i  $(x+12)(x-20)$   
j  $(x-9)(x+12)$     k  $(x-5)^2$     l  $(x+2)(x-3)$
- 2 a  $(2x+3)(x+1)$     b  $(3x+2)(x+2)$   
c  $(2x+5)(x+1)$     d  $(2x+3)(x+4)$   
e  $(3x+1)(x+2)$     f  $(2x+1)(x+3)$   
g  $(2x+7)(x-3)$     h  $(3x+1)(x-2)$   
i  $(4x-3)(x-5)$     j  $(6x-1)(x-3)$   
k  $(4x+5)(3x+2)$     l  $(4x+1)(2x-3)$   
m  $(2x-5)(3x-6)$  or  $3(2x-5)(x-2)$   
n  $(2x+3)(3x-1)$     o  $(3x+4)(6x-1)$
- 3 a  $(x-10)(x+10)$     b  $(y-4)(y+4)$   
c  $(p-12)(p+12)$     d  $(k-8)(k+8)$   
e  $(x-\frac{1}{2})(x+\frac{1}{2})$     f  $(y-\frac{4}{5})(y+\frac{4}{5})$   
g  $(n-50)(n+50)$     h  $(7-t)(7+t)$   
i  $(2x-5)(2x+5)$     j  $(3y-11)(3y+11)$   
k  $(4m-\frac{1}{2})(4m+\frac{1}{2})$     l  $(20p-13)(20p+13)$
- 4 a  $(4a-3b)(4a+3b)$     b  $(x-4)(x-7)$   
c  $(2x-3)(x+7)$     d  $x(x-3)(x+6)$   
e  $5ab(1+2ab)$     f  $-(x-2)(x+5)$

- g  $10(1-x)(1+x)$     h  $(y-7)(y+9)$   
i  $2x(x^2-66)$     j  $(2x-3)(3x-2)$   
k  $(x^2+y^2)(x-y)(x+y)$
- 5 a  $x+2$     b  $\frac{x-3}{2}$     c  $x-3$   
d  $\frac{x+4}{2}$     e  $\frac{x+4}{x-4}$     f  $\frac{x+1}{3x-1}$
- 6 a 400    b 240000    c 199
- 7 Challenge  $\sqrt{1200}$  or  $20\sqrt{3}$

### C19.5 Rich task, p286

Alex is 6.

Students might try to find other numbers using trial and error, or use Autograph to see where the graphs of  $x+n$  and  $(x-n)^2$  intersect.

### Ex C19.5

- 1 a  $x=0$  and  $x=5$     b  $x=0$  and  $x=-9$   
c  $x=0$  and  $x=3\frac{1}{2}$     d  $x=0$  and  $x=2$   
e  $x=0$  and  $x=9$     f  $x=0$  and  $x=7$   
g  $x=0$  and  $x=12$     h  $x=0$  and  $x=6$
- 2 a  $x=-3$  and  $x=-4$     b  $x=-2$  and  $x=-6$   
c  $x=-6$     d  $x=-5$  and  $x=3$   
e  $x=-7$  and  $x=2$     f  $x=-1$  and  $x=5$   
g  $x=2$  and  $x=3$     h  $x=5$   
i  $x=-\frac{1}{2}$  and  $x=-3$     j  $x=-\frac{1}{3}$  and  $x=-2$   
k  $y=-4$  and  $y=-1\frac{1}{2}$     l  $x=-\frac{2}{3}$  and  $x=-\frac{1}{2}$   
m  $x=2$  and  $x=6$     n  $x=-5$  and  $x=1\frac{1}{2}$   
o  $y=2$  and  $y=3$     p  $x=-3$  and  $x=-7$
- 3 a  $x=-7$  and  $x=7$     b  $y=-8$  and  $y=8$   
c  $x=-\frac{2}{3}$  and  $x=\frac{2}{3}$     d  $x=-\frac{1}{2}$  and  $x=\frac{1}{2}$   
e  $x=-12$  and  $x=12$     f  $y=-2\frac{1}{2}$  and  $y=2\frac{1}{2}$
- 4 a  $x=0$  and  $x=\frac{1}{3}$     b  $x=-3$  and  $x=5$   
c  $x=\frac{2}{3}$  and  $x=3$     d  $y=-1\frac{2}{3}$  and  $y=1\frac{2}{3}$   
e  $x=-1\frac{1}{3}$  and  $x=1\frac{1}{3}$     f  $x=\frac{3}{5}$  and  $x=-\frac{1}{4}$
- 5 a  $x=-6$  and  $x=5$     b  $x=\frac{3}{5}$  and  $x=-\frac{1}{2}$
- 6 10 m by 24 m
- 7 a Any value between 0 and 1  
b  $x=-3, x=3, x=-2$  and  $x=2$

### C19.6 Rich task, p288

The expression described factorizes to  $(x-1)^2+1$ , which can never be negative.

## Ex C19.6

- 1 **a**  $(x+2)^2 + 2$       **b**  $(y+4)^2 - 1$   
**c**  $(x+3)^2$       **d**  $(x+5)^2 + 2$   
**e**  $(y+2)^2 - 4$       **f**  $(x+6)^2 - 26$   
**g**  $(x+7)^2 - 19$       **h**  $(x+2)^2 - 9$   
**i**  $(y+4)^2 - 19$       **j**  $(x+1\frac{1}{2})^2 + 1\frac{3}{4}$
- 2 **a**  $(x+10)^2 - 50$       **b**  $(x+8)^2 - 69$   
**c**  $(x-4\frac{1}{2})^2 - 31\frac{1}{4}$
- 3 **a**  $x=2$  and  $x=10$       **b**  $y=-5$  and  $y=3$   
**c**  $x=-1$  and  $x=5$       **d**  $x=-1$   
**e**  $m=-9$  and  $m=7$       **f**  $x=7$   
**g**  $y=0$  and  $y=8$       **h**  $x=\pm\sqrt{37}-6$   
**i**  $x=-11$  and  $x=8$       **j**  $x=3$
- 4 **a** Minimum value of 1 when  $x=-3$   
**b** Minimum value of  $-2$  when  $x=4$   
**c** Minimum value of  $-40$  when  $x=6$   
**d** Minimum value of  $-5\frac{1}{4}$  when  $x=-2\frac{1}{2}$
- 5 **a** Minimum value is 6  
**b** Equation becomes  $(x+4)^2 = -1$  and a square cannot be negative
- 6 6 by 7 and 3 by 7
- 7 Maximum is when the rectangle is a square with sides of 250 metres

## C19.7 Rich task, p290

4 and 5; 5 and 6. No two consecutive numbers multiply to give 40.

## Ex C19.7

- 1 **a**  $x=-0.785$  and  $x=-2.55$   
**b**  $x=1$  and  $x=0.2$   
**c**  $x=-1\frac{1}{2}$  and  $x=5$   
**d**  $x=-0.268$  and  $x=-3.73$   
**e**  $x=-3.16$  and  $x=0.158$   
**f**  $y=-0.377$  and  $y=2.21$   
**g**  $x=\frac{1}{3}$  and  $x=3$       **h**  $x=0.158$  and  $x=-3.16$
- 2 **a**  $x=-5.61$  and  $x=1.61$       **b**  $x=-2.46$  and  $x=4.46$   
**c**  $x=-0.284$  and  $x=1.17$
- 3 **a**  $(x+7)(x-2)$  or  $x^2 + 5x - 14$   
**c** 1.84 cm by 10.84 cm
- 4 **a**  $x=0.145$  and  $x=11.5$       **b**  $x=-0.535$  and  $x=1.87$   
**c**  $x=-1$  and  $x=1.75$       **d**  $x=-3.79$  and  $x=3.29$
- 5 Will factorise if  $b^2 - 4ac$  is a perfect square

## C19.8 Rich task, p292

The equation produces a graph which is a circle of radius  $r$ .

## Ex C19.8

- 1 **a**  $x=7$  and  $y=6$  or  $x=-7$  and  $y=6$   
**b**  $x=7$  and  $y=-5$  or  $x=7$  and  $y=5$   
**c**  $x=-10$  and  $y=9$  or  $x=10$  and  $y=9$   
**d**  $x=7$  and  $y=-2$  or  $x=7$  and  $y=2$
- 2 **a**  $x=-1$  and  $y=3$  or  $x=4$  and  $y=8$   
**b**  $x=1$  and  $y=0$   
**c**  $x=\frac{1}{2}$  and  $y=\frac{1}{2}$  or  $x=32$  and  $y=4$   
**d**  $x=0$  and  $y=1$  or  $x=\frac{4}{5}$  and  $y=-\frac{3}{5}$
- 3 **a**  $x=3$  and  $y=7$  or  $x=5$  and  $y=17$   
**b**  $p=-2$  and  $q=-1$  or  $p=2$  and  $q=1$
- 4 **a**  $(-1, -5)$  and  $(6, 16)$
- 5 The small square measures 3 metres by 3 metres and the larger 4 metres by 4 metres
- 6  $x=-5$  and  $y=-3$  or  $x=5$  and  $y=3$

## Exam practice

- 1 **a i**  $x=10$  or  $-3$       **ii**  $x=4.24$  or  $-0.24$   
**b i**  $a=3, b=-7$       **ii**  $x=-3 \pm \sqrt{5}$
- 2  $(5,5)$  Line is a tangent to the circle
- 3 **b** 1.32 m

## C20 Surface area and volume

### Check in

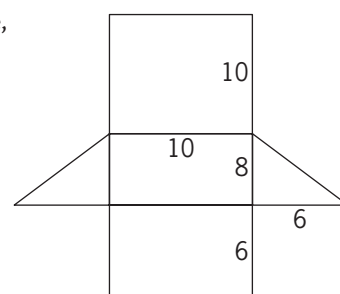
- 1 **a**  $40 \text{ cm}^2$       **b**  $30 \text{ cm}^2$
- 2 **a** 250      **b** 6300      **c** 41      **d** 250      **e** 3500  
**f** 4      **g** 5.6      **h** 40      **i** 4.1      **j** 5.2

## C20.1 Rich task, p298

10 cm  $\times$  10 cm  $\times$  10 cm cube

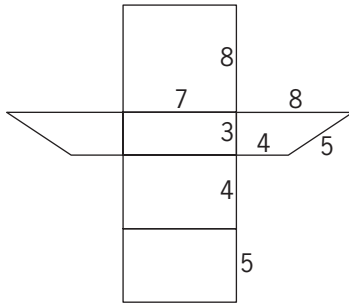
### Ex C20.1

- 1  $168 \text{ cm}^2$
- 2 **a**  $534 \text{ cm}^2$       **b**  $1032 \text{ cm}^2$
- 3 For example,



surface area  $288 \text{ cm}^2$

For example,



surface area  $176 \text{ cm}^2$

4 a  $2.16 \text{ m}^2$     b  $\text{£}26.89$     c  $\text{£}6.50$

5  $12.7 \text{ cm}$  and  $7.7 \text{ cm}$

6 a  $0.14 \text{ m}$     b  $\frac{\sqrt{1+\frac{2}{\pi}}-1}{2}$

7 a  $400 \text{ cm}^2$ ;  $25 \text{ cm} \times 20 \text{ cm}$     b  $20\%$

8 The least surface area is that of a cube of edge  $10 \text{ cm}$ .

### Ex C20.2

1  $64 \text{ cm}^2$

2 a  $153.9 \text{ cm}^2$     b  $266.9 \text{ cm}^2$     c  $282.6 \text{ cm}^2$

3  $15.5 \text{ cm}$

4  $70.0 \text{ cm}$

5 a  $10.9 \text{ cm}$     b  $13.1 \text{ cm}$

6  $12 \text{ cm}$

7  $208.1 \text{ m}$

8 The most economical arrangement depends on the relative height and diameter of the tin

### C20.3 Rich task, p302

i 2

ii 4

iii 8

### Ex C20.3

1 a  $(2, 0)$   $2.4$

b  $(1, 1)$   $3.9$

2  $135 \text{ cm}^2$

3 a 3

b 9

c 27

4 a 3

b 27

c 810 litres

5 Yes,  $2.5$ ,  $6.25$

### Ex C20.4

1 a i  $80 \text{ mm}$     ii  $600 \text{ mm}^2$     iii  $2000 \text{ mm}^3$

b i  $3.5 \text{ cm}$     ii  $1.4 \text{ cm}^2$     iii  $0.5 \text{ cm}^3$

2 a i  $800 \text{ cm}$     ii  $30000 \text{ cm}^2$     iii  $5000000 \text{ cm}^3$

b i  $7.5 \text{ m}$     ii  $0.85 \text{ m}^2$     iii  $0.412 \text{ m}^3$

3 a i  $46 \text{ mm}$     ii  $130 \text{ mm}^2$     iii  $350 \text{ mm}^3$

b i  $32 \text{ cm}$     ii  $250 \text{ cm}^2$     iii  $750000 \text{ cm}^3$

c i  $3000 \text{ m}$     ii  $2000000 \text{ m}^2$     iii  $0.0159 \text{ km}^3$

4 a 7400 metres    b  $2100000 \text{ m}^2$     c  $0.005 \text{ km}^3$

5 a  $0.315 \text{ km}$     b  $0.0445 \text{ km}^2$     c  $0.004 \text{ km}^3$

### Ex C20.5

1 3075 litres

2 No, it contains 1.002 litres

3 61%

4 250 seconds or 4 minutes 10 seconds

5 length    width    height

10    10    10

10    20    5

10    50    2

10    100    1

20    50    1

The  $10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$  pack is suitable

6 The volume of the pyramid is  $\frac{1}{3}$  of the volume of the cube with the same base and height measurements. Both have volumes =  $\frac{1}{3}$  of base area  $\times$  height.

7 a  $8143 \text{ cm}^3$     b  $804 \text{ cm}^3$     c  $599 \text{ cm}^3$

d  $1.4 \text{ cm}^3$     e  $384 \text{ cm}^3$     f  $0.022 \text{ cm}^3$

8  $17.7 \text{ cm}$

9 Yes, it is  $80 \text{ cm}$  high

10  $5.8 \text{ litres}$

11 Pour the water over the metal, so that the metal is submerged, and measure the height of the water,  $h$ .  
Volume =  $16\pi(h - 7)$

12  $5.3 \text{ cm}$

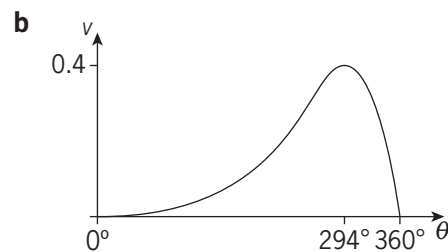
13 a  $7.94 \text{ cm}$     b  $2.3 \text{ cm}$

14  $35 \text{ metres}$

15  $144 \text{ metres high}$ ,  $210 \text{ metres sloping edge}$

16  $20.4 \text{ cm}$

17 a  $V = \frac{\pi}{3} R^3 \left(\frac{\theta}{360}\right)^2 \sqrt{1 - \left(\frac{\theta}{360}\right)^2}$



Maximum  $V$  of  $0.40$  when  $\theta = 294^\circ$

### Ex C20.6

1 a  $432 + 216\pi \text{ cm}^3$      $432 + 72\pi \text{ cm}^2$

b  $750\pi \text{ cm}^3$      $325\pi \text{ cm}^2$

b  $2160\pi \text{ cm}^3$      $588\pi \text{ cm}^2$

2  $0.25 \text{ cm}$

3 a  $420 \text{ cm}^3$     b  $716 \text{ cm}^3$     c  $70 \text{ cm}^3$

4 a 21.3 litres    b 7.8 litres

5  $779 \text{ m}^3$   $760 \text{ m}^2$

### Exam practice

1  $63.3 \text{ cm}^3$

2  $12.5 \text{ cm}^3$

3 3.22 cm

### C21 Graphs

#### Check in

1 a 8, 8    b 18, 18    c -3, 21    d 14, 8

2 Check own graphs

3 Check own graphs

### C21.1 Rich task, p316

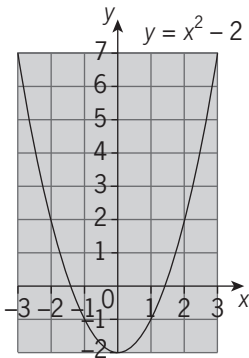
225m; 400m

Each 10 m improvement increases the height by  $\left(\frac{x}{2}\right)^2$  where  $x$  is the total length of the throw.

#### Ex C21.1

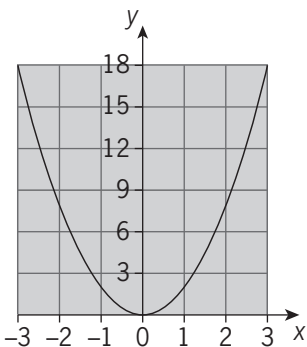
1 a

x	-3	-2	-1	0	1	2	3
$x^2$	9	4	1	0	1	4	9
-2	-2	-2	-2	-2	-2	-2	-2
y	7	2	-1	-2	-1	2	7



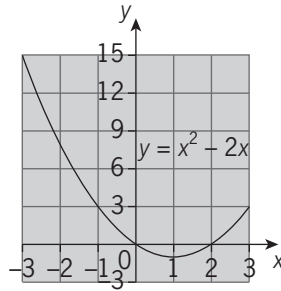
b

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18



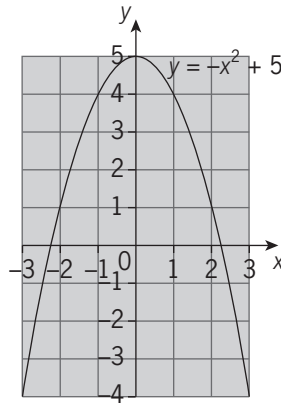
c

x	-3	-2	-1	0	1	2	3
$x^2$	9	4	1	0	1	4	9
-2x	6	4	2	0	-2	-4	-6
y	15	8	3	0	-1	0	3



d

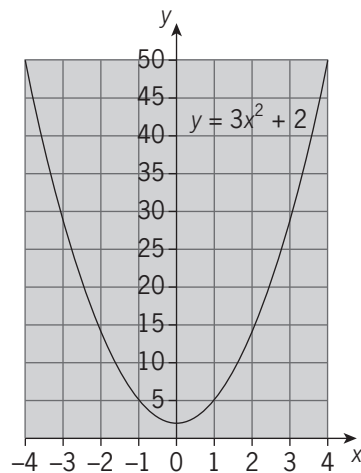
x	-3	-2	-1	0	1	2	3
5	5	5	5	5	5	5	5
$-x^2$	-9	-4	-1	0	-1	-4	-9
y	-4	1	4	5	4	1	-4



2 a i

x	-4	-3	-2	-1	0	1	2	3	4
$3x^2$	48	27	12	3	0	3	12	27	48
+2	2	2	2	2	2	2	2	2	2
y	50	29	14	5	2	5	14	29	50

ii

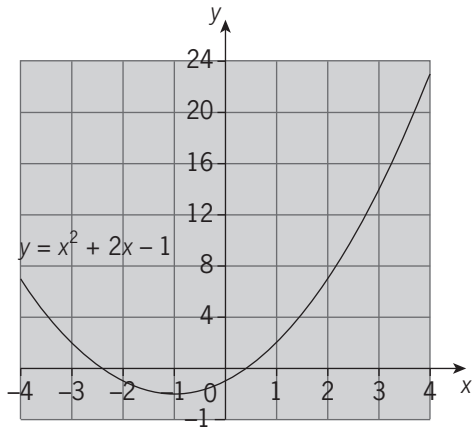


iii Minimum (0, 2) and  $x = 0$  is line of symmetry

b i

$x$	-4	-3	-2	-1	0	1	2	3	4
$x^2$	16	9	4	1	0	1	4	9	16
$2x$	-8	-6	-4	-2	0	2	4	6	8
$-1$	-1	-1	-1	-1	-1	-1	-1	-1	-1
$y$	7	2	-1	-2	-1	2	7	14	23

ii

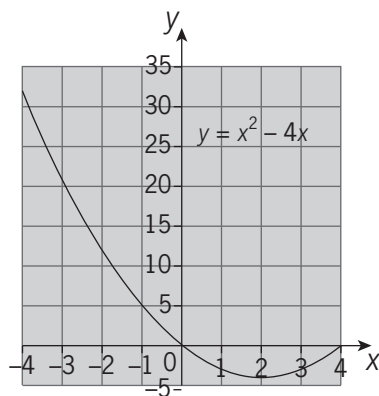


iii Minimum (-1, -2) and  $x = -1$  is line of symmetry

c i

$x$	-4	-3	-2	-1	0	1	2	3	4
$x^2$	16	9	4	1	0	1	4	9	16
$-4x$	16	12	8	4	0	-4	-8	-12	-16
$y$	32	21	12	5	0	-3	-4	-3	0

ii

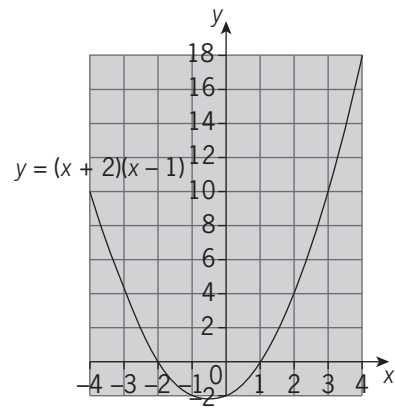


iii Minimum (2, -4) and  $x = 2$  is line of symmetry

d i

$x$	-4	-3	-2	-1	0	1	2	3	4
$x + 2$	-2	-1	0	1	2	3	4	5	6
$x - 1$	-5	-4	-3	-2	-1	0	1	2	3
$y$	10	4	0	-2	-2	0	4	10	18

ii



iii Minimum  $(-\frac{1}{2}, -2\frac{1}{4})$  and  $x = -\frac{1}{2}$  is line of symmetry

3 Height reached is 4 metres. 4 seconds to return to ground

4 Challenge

The graphs intersect the  $x$ -axis at the solutions.

5 Rich Task

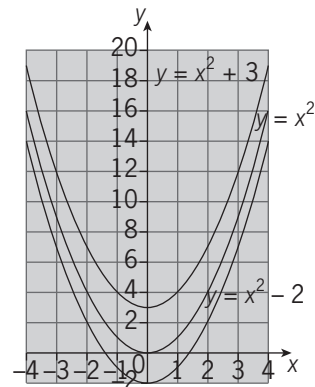
If there is a positive number in front of  $x^2$ , the graph has a minimum point – trough shaped. A negative number in front of  $x^2$  means the graph will have a maximum point (dome shaped).

## C21.2 Rich task, p318

$y = f(x + a)$  moves the graph of  $f(x)$  in the  $x$  direction by  $-a$  units.

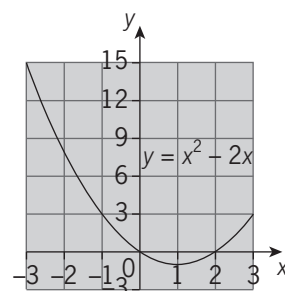
### Ex C21.2

1 a



b Translation of  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$  and translation of  $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$  respectively

2 a



**b** Translation of  $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$  and translation of  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$  respectively

**3** Missing values are 5 (or  $-5$ ), 1 and  $-4$

**4 a** 17    **b** 37    **c** 2    **d** 1 or  $-5$

**5** A is  $y = (x + 4)^2$

B is missing and is  $y = (x + 3)^2 - 1$

C is  $y = x^2 - 4$

D is  $y = x^2 + 2$

E is  $y = (x - 1)^2 + 2$

F is  $y = (x - 2)^2$

**6** A is  $f(x + 4)$  and is a translation of  $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

B is  $f(x + 3) - 1$  and is a translation of  $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$

C is  $f(x) - 4$  and is a translation of  $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

D is  $f(x) + 2$  and is a translation of  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

E is  $f(x - 1) + 2$  and is a translation of  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

F is  $f(x - 2)$  and is a translation of  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

**7**  $y = af(x)$  is a stretch in the  $y$ -direction of  $f(x)$  by a scale factor  $a$ .

$y = f(ax)$  is a stretch in the  $x$ -direction of  $f(x)$  by a scale factor  $\frac{1}{a}$ .

### C21.3 Rich task, p320

A straight line can intersect with a parabola (quadratic graph) 0, 1 or 2 times.

### Ex C21.3

**1 a**  $x = -1.4$  and  $x = 3.4$     **b**  $x = 0$  and  $x = 2$

**c**  $x = -1$  and  $x = 3$     **d**  $x = -1$  and  $x = 4$

**2 a**

$x$	-3	-2	-1	0	1	2	3	4
$x^2$	9	4	1	0	1	4	9	16
$-x$	3	2	1	0	-1	-2	-3	-4
$-6$	-6	-6	-6	-6	-6	-6	-6	-6
$y$	6	0	-4	-6	-6	-4	0	6

**b i** Add  $y = 0$     Solutions are  $-2$  and  $3$

**ii** Add  $y = 3$     Solutions are  $-2.5$  and  $3.5$

**iii** Add  $y = -4$     Solutions are  $-1$  and  $2$

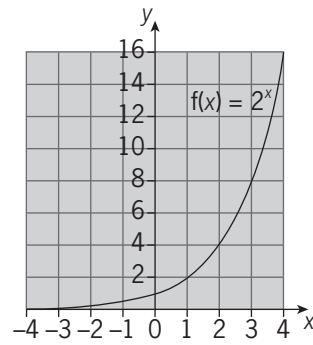
**iv** Add  $y = 0$     Solutions are  $-2$  and  $3$

**3 a**  $x = -3$  and  $x = 1$     **b**  $x = -2.6$  and  $x = 1.6$

**c**  $x = -3.4$  and  $x = 1.4$     **d**  $x = -1$  and  $x = 0$

**4 a** Add  $y = 0$     **b** Add  $y = x + 2$     **c** Add  $y = x - 1$

**5**



**a**  $x = 1.6$     **b**  $x = 2$  or  $x = -0.8$     **c**  $x = 0.2$

### C21.4 Rich task, p322

Width 100 m; height 50 m.

$$-\frac{x^2}{50} + 2x + 100 \text{ or } -\frac{(x-50)^2}{50} + 50$$

### Ex C21.4

**1 a**

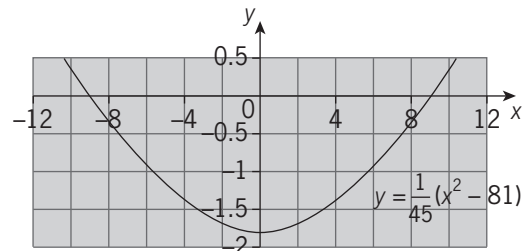
$x$	0	1	2	3	4	5
$25x$	0	25	50	75	100	125
$-5x^2$	0	-5	-20	-45	-80	-125
$y$	0	20	30	30	20	0

**b**  $(2\frac{1}{2}, 31\frac{1}{4})$  The highest point reached by the ball is  $31\frac{1}{4}$  metres after  $2\frac{1}{2}$  seconds

**c** After 5 seconds

**d** The ball is below ground after 10 seconds. This is impossible since the ball lands on the ground and stops at five seconds.

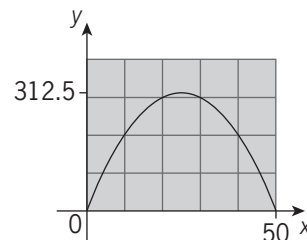
**2 a**



**b** 2.7 metres

**3 a**  $\frac{1}{2}(50 - x)$     **b**  $y = \frac{1}{2}x(50 - x)$

**c**



**d** Largest area is 312.5 metres<sup>2</sup>. A homeowner may want lots of space to fit in furniture and enjoy living in.

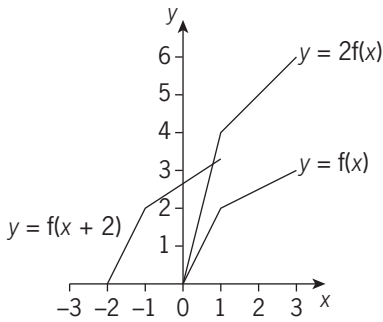
### 4 Challenge

$$y = x^2 - 1$$

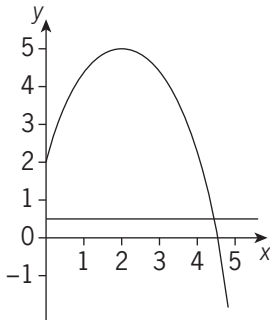
$y$  is the depth of the umbrella's shade and  $x$  is its horizontal distance from the umbrella pole

## Exam practice

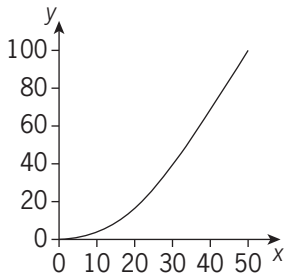
1



- 2 a Graph of  $y = 2 + 3x - \frac{3}{4}x^2$  for  $0 \leq x \leq 5$  (shown)  
b around 4.5 m



3



- b 53.2 m  
c He is right. He was travelling slightly slower.

## Case study: Art

- 1 a 30                      b 4.44m  
c length = 40 hands; height = 29.6 hands  
d 30cm squares    e area scale factor 900
- 2 a i  $B = (2, 60)$     ii  $A = (1, 210)$     iii  $C = (3, 300)$   
b  $D = (1.5, 0)$
- 3 a Circle radius = 4                      b  $a = 4$   
c 9 units                      d  $k = 0.025$
- 5 Description of the crop circle:  
a The central part of the crop circle has been formed in the shape of a circle.  
b A spiral shape then extends outside the inner circle shape. The spiral shape begins when its increasing radius is equal to the radius of the circle and continues to extend as its radius becomes greater than the radius of the circle.

## C22 Everyday arithmetic and bounds

### Check in

- 1 Many different methods  
2 a 4438    b 1977    c 857    d 14225  
3 Many different methods  
4 a 11700    b 78408    c 205    d 67564

### C22.1 Rich task, p330

The 6 year old child's dose is 10 mg using the first formula and 14.4 mg using the second formula.

The 10 year old child's dose is 13.6 mg using the first formula and 24 mg using the second formula.

There is no difference for a child of 6 months.

### Ex C22.1

- 1 a i 4.4    ii 36.9    iii -1.1  
b i 71.6    ii 23    iii 28.4
- 2 true 25.9, estimate 25
- 3 a 15.8 mg    b  $BSA = 0.7666$ ; dose = 15.8mg; no

### Ex C22.2

- 1 £75.96, 3, £25.50  
2 £1 = €1.41  
3 £108.70 Paris, £102.86 NY, New York  
4 09:25 BST Wed  
5 Students' own answers, they should think about flight times. If the flight time is less than the time difference, it should be possible.

### C22.3 Rich task, p334

The box of sugar will weigh more.

### ExC22.3

- 1 a i 19.2 km/h    ii 17.1 km/h  
b i 80 mph    ii 72 mph
- 2 a 7.43 mph    b 4.88 mph
- 3 40.6 km/h
- 4 4 km
- 5 a 11580 kg  
b 1.84 kg
- 6 258 per square mile
- 7 41 min 40 sec

### Ex C22.4

- 1 a, c, e, f

- 2 a 4700      b 230000000      c 0.000079  
 d 12500      e 205000      f 0.0000049  
 g 0.00902      h 0.000007
- 3 a  $5.6 \times 10^4$       b  $8.7 \times 10^6$       c  $5.503 \times 10^3$   
 d  $1 \times 10^5$       e  $4 \times 10^{-3}$       f  $5.6 \times 10^{-3}$   
 g  $2.04 \times 10^{-6}$       h  $5.00006 \times 10^5$

4  $3 \times 10^8$  m/s

5 865 000 miles

6 a  $3.6 \times 10^3$

b  $8.64 \times 10^4$

7 a  $2 \times 10^{-5}$       b  $2.5 \times 10^{-4}$       c  $4 \times 10^{-5}$       d  $1 \times 10^{-7}$

8 1000

### Ex C22.5

1 a  $8 \times 10^8$       b  $3.2 \times 10^8$       c  $1.5 \times 10^{11}$

d  $1.95 \times 10^{-3}$       e  $3.5 \times 10^0$       f  $5.4 \times 10^{-10}$

2 a  $2 \times 10^2$       b  $2 \times 10^{-5}$       c  $1.6 \times 10^8$

d  $3 \times 10^{-1}$       e  $5 \times 10$       f  $4 \times 10^{-15}$

3 a  $3 \times 10^4$       b  $2 \times 10^3$

c  $5 \times 10^4$       d  $3 \times 10^{-4}$

4 a  $4.4 \times 10^5$       b  $5.72 \times 10^5$       c  $4.4998 \times 10^6$

d  $4.32 \times 10^5$       e  $4.0002 \times 10^2$       f  $2.5 \times 10^5$

5 a  $2.16 \times 10^{-20}$       b  $4 \times 10^{27}$

6 a  $6.02 \times 10^{27}$       b  $3.01 \times 10^{23}$

7 a  $1.4 \times 10^6$  km      b 3.71

### Ex C22.6

1 a 27.5–28.5 cm      b 47.5–48.5 g

c 111.5–112.5 g      d 555–565 cm

e 9345–9355 km      f 9325–9375 km

g 510–530 miles      h 415–425 m

i 43.55–43.65 s      j 5.15–5.25 g

k 7.25–7.35 cm      l 39.5–40.5 m

2 a Yes, could be 1.545 m

b 16.65–16.75 sec

c Yes, could be 4.375 m

3 a 172–180 g

b 2350–2450 g

c 14.7–15.3 kg

4 No; max weight is 351.5 kg

5 27.8 cm

### Ex C22.7

1 51

2 4.47 m/s

3 a 0.425 m      b 31

4 19.2 mm

5 28.5 cm/s

6 29.7 mm

### Exam practice

1 2.4 kg, £5.10

2  $1.586 \times 10^{-5}$

3 69

### C23 Trigonometry

#### Check in

1 a 8.1      b 9.8      c 8.5      d 13.7

2 a  $29.7^\circ$       b  $27.0^\circ$       c  $20.6^\circ$       d  $54.0^\circ$

#### Ex C23.1

1  $25.2^\circ$

2  $62.6^\circ$

3 a 64.0 metres      b  $58^\circ$

4 a  $32.9^\circ$       b  $30.0^\circ$

5  $\frac{x}{\sqrt{2}}$

#### C23.2 Rich task, p350

$h = c \sin A$  and  $h = a \sin C$

$a/\sin A = c/\sin C$

$a/\sin A = b/\sin B$

#### Ex C23.2

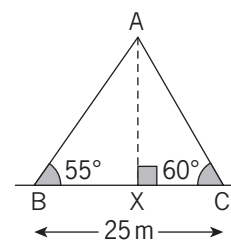
1 a 10.2 cm      b 10.5 cm      c 8.2 cm

d 8.2 cm      e  $46.8^\circ$       f 10.3 cm

g 16.0 cm

2 36.8 metres

3



Use the sine rule in triangle ABC to get the length AC. In the right-angled triangle AXC use the sine of angle ACX to find the length AX.

### C23.3 Rich task, p352

$$c^2 = h^2 + c^2 \cos^2 A$$

$$a^2 = h^2 + b^2 - 2bc \cos A + c^2 \cos^2 A$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

#### Ex C23.3

1 a 7.4    b 12.3    c 7.4    d 10.3

e 29.7°    f 44.4°    g 120°    h 96.3°

2 125°

3 I = 61.2°    J = 46.9°    K = 71.9°

4 a 60° 60° 60°    b 81.9° 37.8° 60.3°

#### Ex C23.4

1 a 6.8 cm    18.7 cm<sup>2</sup>    b 47.2°    44.0 cm<sup>2</sup>

c 63.5°    19.8 cm<sup>2</sup> or 116.5°    9.8 cm<sup>2</sup>  
(2 triangles possible)

2 a 55° 7.4 cm 7.1 cm 24.1 cm<sup>2</sup>

b 81° 6.7 cm 7.2 cm 23.6 cm<sup>2</sup>

c 94.1° 46.5° 39.4° 27.9 cm<sup>2</sup>

3 12.2 km 9.0 km 25.9 km<sup>2</sup>

4 54.7°  $\frac{a^2}{2\sqrt{2}}$

#### Exam practice

1 35.5 cm<sup>2</sup>

2 a 58.3°    b 121.7° (180° - the acute angle)

3 27.1 m

### C24 Graphs 2

#### Check in

1 a  $\frac{1}{3}$     b 4    c  $1\frac{1}{3}$     d  $\frac{2}{5}$

2 a 30    b 8    c 51    d  $\frac{1}{3}$

3 a -30    b  $\frac{1}{8}$     c -51    d  $-\frac{1}{3}$

4 a  $x = -4$  and  $y = -3$  and  $x = 3$  and  $y = 4$

b  $x = -4.27$  and  $y = 4.14$  and  $x = 3.27$  and  $y = 0.363$   
(to 3 s.f.)

#### Ex C24.1

1 a  $x = 8$  (or  $x = -9$ )    b  $x = 6$

c  $x = 25$  (or  $x = -27$ )    d  $x = 7$

2 a  $x(x + 4) = 192$  12 cm by 16 cm

b  $x(x + 1)(x + 2) = 9240$  The numbers are 20, 21 and 22

c  $x(x + 1)(x + 2) = 990$  9 cm by 10 cm by 11 cm

3 -

4 a  $x = 8.2$  (to 1d.p.) (or  $-12.2$ )

b  $x = 8.9$  (to 1d.p.)    c  $x = 4.0$  (to 1d.p.) (or  $-4.6$ )

d  $y = 15.9$  (to 1d.p.) (or  $-13.9$ )

5 4.4 (to 1d.p.)

### C24.2 Rich task, p362

The graphs cross at  $\sim 0.4$  and  $\sim 4.6$ .

There are either 0, 1 or 2 possible solutions for a given value of  $a$ .

#### Ex C24.2

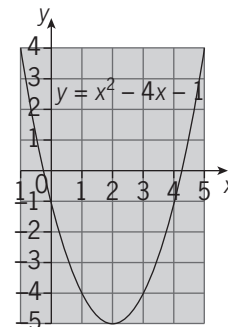
1 a  $x = -1, y = -2$  and  $x = 4, y = 3$

b  $x = -2, y = 3$  and  $x = 2, y = -5$

c  $x = -0.4, y = -3.9$  and  $x = 4.4, y = 5.7$

2 a

x	-1	0	1	2	3	4	5
x <sup>2</sup>	1	0	1	4	9	16	25
-4x	4	0	-4	-8	-12	-16	-20
-1	-1	-1	-1	-1	-1	-1	-1
y	4	-1	-4	-5	-4	-1	4



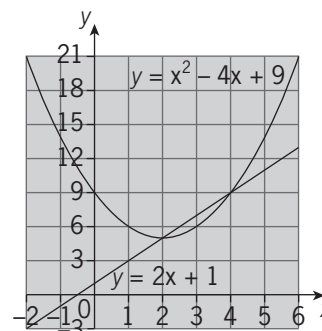
b i Add  $y = x - 2$ . Solutions are  $x = 0.2, y = -1.8$  and  $x = 4.8, y = 2.8$

ii Add  $y = \frac{1}{2}(x - 4)$ . Solutions are  $x = 0.2, y = -1.9$  and  $x = 4.3, y = 0.2$

iii Add  $x + 2y = 4$ . Solutions are  $x = 4.2, y = -0.1$  and  $x = -0.7, y = 2.4$

c Solutions are  $x = 0.2, y = -1.8$  and  $x = 4.8, y = 2.8$

3 a



Solutions are  $x = 2, y = 5$  and  $x = 4, y = 9$

- 5 Challenge: There are many possible answers provided the equation reduces to  $x^2 + 2x = 0$ , for example  $y = x^2 + 3x + 1$  and  $y = x + 1$

## C24.3 Rich task, 364

### Part 1

$x^\circ$	0	45	90	135	180	225	270	315	360
$\sin x^\circ$	0	0.71	1	0.71	0	-0.71	-1	-0.71	0

### Part 2

$x^\circ$	0	45	90	135	180	225	270	315	360
$\cos x^\circ$	1	0.71	0	-0.71	0	-0.71	0	0.71	1

The graph in Part 2 is a translation of the graph in Part 1 of  $90^\circ$  in the negative  $x$ -direction.  
 $\cos x^\circ = \sin(x + 90)^\circ$

## Ex C24.3

- Check on calculator
- Check on calculator
- a  $130^\circ$                       b  $118^\circ$

c  $40^\circ$                          d  $323^\circ$

e  $278^\circ$                       f  $82^\circ$
- a  $330^\circ$                       b  $294^\circ$

c  $210^\circ$                       d  $123^\circ$

e  $196^\circ$                       f  $282^\circ$
- graph repeats itself
- $\begin{pmatrix} -90^\circ \\ 0 \end{pmatrix}$

## Ex C24.4

- $y = x^2 + 2$  & upright parabola

$y = 2$  & horizontal line

$y = x + 2$  & diagonal line

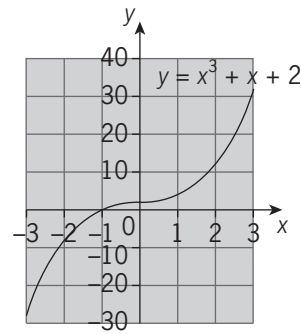
$y = 2 - x^2$  & upside down parabola

$y = x^3 + 2$  and "S" shaped curve

$x = 2$  and vertical line

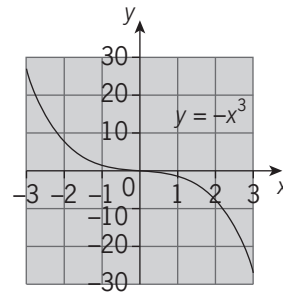
2 a

$x$	-3	-2	-1	0	1	2	3
$x^2$	-27	-8	-1	0	1	8	27
$+x$	-3	-2	-1	0	1	2	3
$+2$	2	2	2	2	2	2	2
$y$	-28	-8	0	2	4	12	32



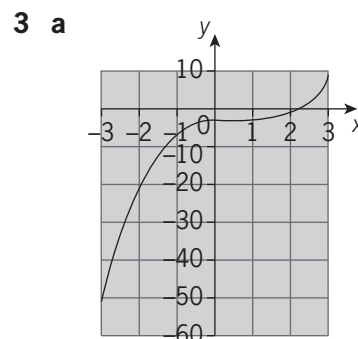
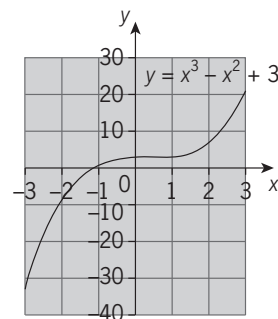
b

$x$	-3	-2	-1	0	1	2	3
$y$	27	8	1	0	-1	-8	-27



c

$x$	-3	-2	-1	0	1	2	3
$x^3$	-27	-8	-1	0	1	8	27
$-x^2$	-9	-4	-1	0	-1	-4	-9
$+3$	3	3	3	3	3	3	3
$y$	-33	-9	1	3	3	7	21



- b i  $x = -1.9$             ii  $-1.8$

4 a  $y = 2x^2$                 b  $y = x^3 - 2$  and  $y = 2x^2$

c  $y = 2(x + 1)$  &  $y = x^3 - 2$             d  $x = 4$

## 5 Challenge

$y = (x - 1)(x + 1)(x + 2)$  intersects the  $x$ -axis at  $(1, 0)$ ,  $(-1, 0)$  and  $(-2, 0)$

The required graph crosses at  $(10, 0)$ ,  $(5, 0)$  and  $(-12, 0)$

## C24.5 Rich task, p368

$1 \times 24$ ;  $2 \times 12$ ;  $3 \times 8$ ;  $4 \times 6$ ;  $-1 \times -24$ ;  $-2 \times -12$ ;  $-3 \times -8$ ;  $-4 \times -6$ .

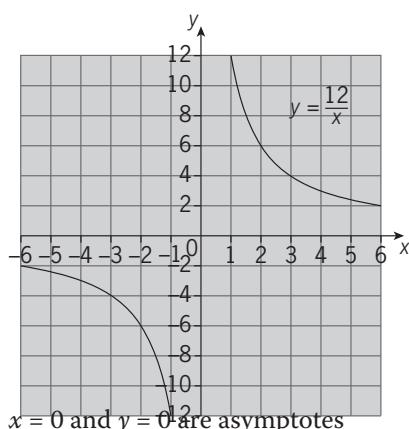
The line described by these points is the line  $y = \frac{24}{x}$

### Ex C24.5

1  $f(x) = \frac{2}{x}$ ,  $y = \frac{3}{x} - 1$  and  $y = \frac{1}{x+1}$

2 a

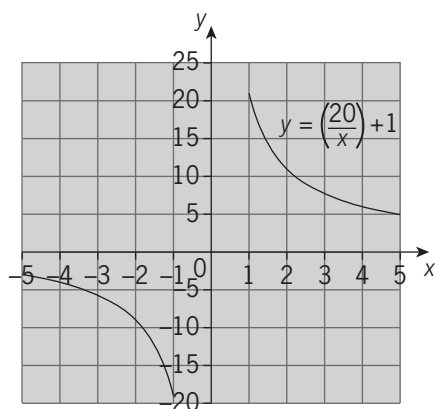
x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-2	-2.4	-3	-4	-6	-12	-	12	6	4	3	2.4	2



$x = 0$  and  $y = 0$  are asymptotes

b

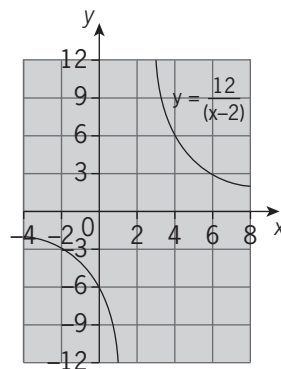
x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-3	-4	$-5\frac{2}{3}$	-9	-19	-	21	11	$7\frac{2}{3}$	6	5



$x = 0$  and  $y = 1$  are asymptotes

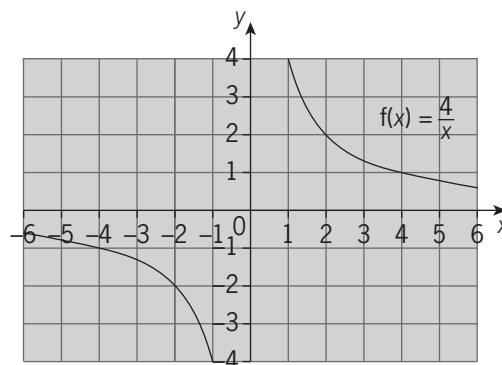
c

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
$x - 2$	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	-2	-2.4	-3	-4	-6	-12	-	12	6	4	3	2.4	2



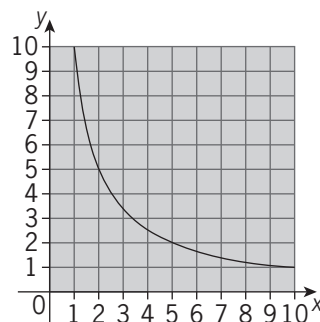
$x = 2$  and  $y = 0$  are asymptotes

3



$x = 2.1$

4



$y = \frac{10}{x}$

5 Challenge  $x = -4, y = -2$  &  $x = 2, y = 4$

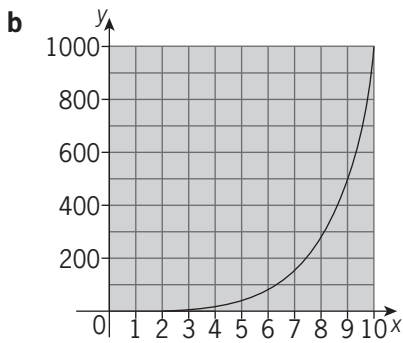
## C24.6 Rich task

The exponential decay is so slow that even after ten half-lives (57 300 years) have gone by there will still be almost 0.5 grams of carbon-14 remaining, which is quite a large number of carbon-14 atoms. As time goes on this number will continue to diminish but will not reduce to zero.

### Ex C24.6

1 a

Time ( $\times$ hrs)	0	1	2	3	4	5	6	7	8	9	10
Bacteria (y)	1	2	4	8	16	32	64	128	256	512	1024

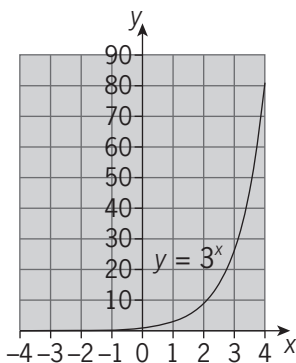


**c** 9.9 hours    **d**  $y = 2^x$

**e** Would be the graph of  $y = 3^x$ . This increases from (0, 1) far more steeply

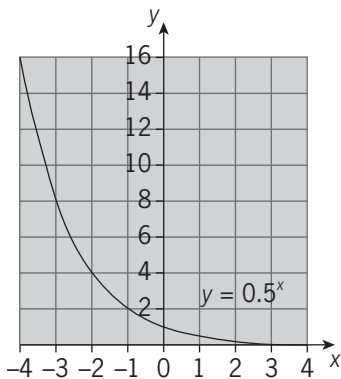
**2 a**

<b>x</b>	-4	-3	-2	-1	0	1	2	3	4
<b>y</b>	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81

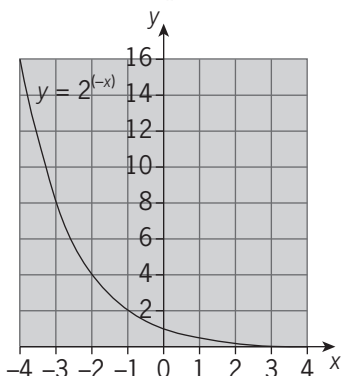


**b**

<b>x</b>	-4	-3	-2	-1	0	1	2	3	4
<b>y</b>	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

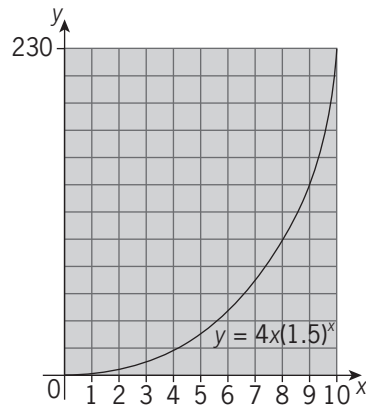


**c** Table same as part b



**3 a** 4    **b i** 6    **ii** 9

**c** Let  $y = P$  and  $x = t$  years. Then graph will be:



7.9 years

### C24.7 Rich task, p372

$y = f(x) + a$  is a translation of  $f(x)$  by  $a$  units in the  $y$ -direction.

$y = f(x + a)$  is a translation of  $f(x)$  by  $-a$  units in the  $x$ -direction.

### Ex C24.7

**1** Check that graph  $f(x)$  is translated as follow:

**a**  $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$     **b**  $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$     **c**  $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$     **d**  $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

**2 a** A    **b** C    **c** D    **d** B

**3** by  $\begin{pmatrix} 90^\circ \\ 0 \end{pmatrix}$

### C24.8 Rich task, p374

$y = af(x)$  is a stretch in the  $y$ -direction of  $f(x)$  by a scale factor  $a$ .

$y = f(ax)$  is a stretch in the  $x$ -direction of  $f(x)$  by a scale factor  $\frac{1}{a}$ .

### Ex C24.8

**1 a** The graph will have twice as many cycles cutting the  $x$ -axis at  $90^\circ, 180^\circ, 270^\circ, 360^\circ$

**b** The graph will have amplitude 2, but otherwise be the same

**c** The graph will be translated by  $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

**2 a** The graph will have only half a cycle, cutting  $x$ -axis at  $180^\circ$  only

**b** The graph will have amplitude  $\frac{1}{2}$  but otherwise be the same

**c** The graph will be translated by  $\begin{pmatrix} 60^\circ \\ 0 \end{pmatrix}$

- 3 a  $(-1, 0), (0, 10), (2, 6)$  b  $(-4, 0), (0, 10), (8, 6)$   
 c  $(-\frac{1}{2}, 0), (0, 10), (1, 6)$  d  $(2, 0), (0, 10), (-4, 6)$   
 e  $(-1, 5), (0, 15), (2, 11)$

- b  $(40, 50, 50)$   $(80, 20, -100)$   
 $(20, 30, 100)$   $(5, 40, 150)$   
 $(20, 5, 0)$   $(40, 50, 50)$   
 $(75, 30, -50)$   $(50, 5, -50)$

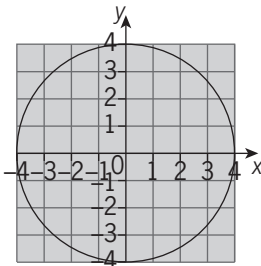
### C24.9 Rich task, p376

The equation of a circle with centre  $(a, b)$  and radius  $r$  is  $(x - a)^2 + (y - b)^2 = r^2$ .

#### Ex C24.9

- 1 a  $x^2 + y^2 = 16$  b  $x^2 + y^2 = 36$  c  $x^2 + y^2 = \frac{1}{4}$   
 d  $x^2 + y^2 = 0.16$  e  $x^2 + y^2 = 169$  f  $x^2 + y^2 = 7$
- 2 a Centre  $(0, 0)$  and radius 7  
 b Centre  $(0, 0)$  and radius 12  
 c Centre  $(0, 0)$  and radius 15  
 d Centre  $(0, 0)$  and radius  $\sqrt{10}$   
 e Centre  $(0, 0)$  and radius  $\frac{4}{5}$   
 f Centre  $(0, 0)$  and radius 0.3
- 3  $x^2 + y^2 = 4, x^2 + y^2 = 36$  and  $x^2 + y^2 = 64$  from smallest to largest
- 4 They do not intersect since they are concentric

5 a



- b Twice  
 c i Twice ii Not at all iii Twice
- 6 a  $3^2 + 4^2 = 25$   
 b  $x^2 + y^2 = 16$  and  $(2, \sqrt{12}), x^2 + y^2 = 169$  and  $(5, 12)$   
 $x^2 + y^2 = 10$  and  $(1, 3), x^2 + y^2 = 100$  and  $(6, 8)$

### C24.10 Rich task, p378

Position is fixed using coordinates. OS gives heights using contours.

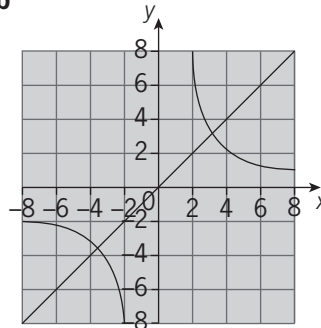
#### C24.10

- 1 A  $(300, 100, 30)$ ; B  $(200, 100, 50)$ ; C  $(200, 350, 60)$ ; D  $(300, 200, 20)$ ; E  $(300, 400, 75)$ ; F  $(100, 0, 40)$
- 2 A  $(2, 5, 10)$ ; B  $(2, 3, 10)$ ; C  $(4, 5, 20)$ ; D  $(5, 1, 20)$ ; E  $(6, 3, 30)$ ; F  $(7, 4, 40)$ ; G  $(6, 4.5, 30)$ ; H  $(1, 2, 0)$ ; I  $(6.5, 1, 30)$ ; J  $(8, 3, 30)$ ; K  $(3, 1, 10)$ ; L  $(4, 3.5, 20)$
- 3 a A  $(20, 20, 50)$ ; B  $(40, 30, 50)$ ; C  $(30, 40, 100)$ ; D  $(60, 50, 50)$ ; E  $(70, 40, 0)$ ; F  $(40, 15, 0)$ ; G  $(20, 55, 100)$ ; H  $(60, 20, -50)$ ; I  $(70, 10, -100)$ ; J  $(20, 40, 150)$

### Exam practice

- 1 a  $0.7, -1, -2.7$  b  $y = 2x + 1$

2 a,b



- c when  $x = \frac{8}{x}, x^2 = 8$ , and  $x = \sqrt[3]{8}$

3 7.7

### Case study: Radio

- 1 a  $35 \text{ GHz} = 35,000,000,000 = 35,000,000 \text{ kHz} = 3.5 \times 10^7 \text{ kHz}$   
 b  $300 \text{ Hz} = 0.000300 \text{ MHz} = 3.0 \times 10^{-4} \text{ MHz}$
- 2 Wave **b** has the greatest volume because it has the greatest amplitude and wave **c** has the smallest volume because it has the smallest amplitude. The volume of wave **a** is between that of waves **b** and **c**.

Wave **c** has the highest pitch because it has the greatest frequency and wave **b** has the lowest pitch because it has the lowest frequency. The pitch of wave **a** is between that of waves **b** and **c**.

- 3 a i  $93.2 \text{ MHz}$  ii  $93,200,000 \text{ Hz} = 9.32 \times 10^7 \text{ Hz}$   
 iii  $0.0932 \text{ GHz} = 9.32 \times 10^{-2} \text{ GHz}$

b For Maths FM:

$$\begin{aligned} \text{wave speed} &= \text{frequency} \times \text{wavelength} \\ &= 9.32 \times 10^7 \times 3.22 \\ &= 300,104,000 \text{ m/s} \end{aligned}$$

For Maths AM:

$$\begin{aligned} \text{Wavelength} &= \text{wave speed} / \text{frequency} \\ &= 300,104,000 / 930,000 \\ &= 322.7 \text{ m} \end{aligned}$$

- c frequency (kHz)  $\times$  wavelength (m) = 300,104

$$\text{so frequency} = \text{wavelength} = \sqrt{300,104} = 547.8 \text{ kHz}$$

Radio Alpha is an AM radio station as its frequency is within the AM band.

- 4 a i 40 mins ii 20 mins  
 b Yes. A third of the show (33%) is speech based.

## C25 Study of chance

### Check in

- 1 a 0.55    b 0.04    c 0.72    d 0.625  
 e 0.6    f 0.34    g 0.9    h 0.75  
 i 0.18    j 0.17    k 0.192    l 0.12
- 2 a  $\frac{1}{6}$     b  $\frac{4}{5}$     c  $\frac{2}{9}$     d  $\frac{3}{4}$   
 e  $\frac{13}{15}$     f  $\frac{11}{12}$     g  $\frac{5}{9}$     h  $\frac{8}{45}$

### C25.1 Rich task, p386

The chances of him receiving more money than he paid is 52%

#### Ex C25.1

- 1 0.66
- 2 a  $\frac{9}{25}$     b  $\frac{16}{25}$     c  $\frac{8}{25}$   
 d  $\frac{17}{25}$     e  $\frac{10}{25}$     f  $\frac{19}{25}$
- 3 a Town twice as likely to win  
 b 0.15
- 4 a
- |      |   | 1 | 2  | 3  | 4  | 5  | 6  |
|------|---|---|----|----|----|----|----|
| BLUE | 1 | 1 | 2  | 3  | 4  | 5  | 6  |
|      | 2 | 2 | 4  | 6  | 8  | 10 | 12 |
|      | 3 | 3 | 6  | 9  | 12 | 15 | 18 |
|      | 4 | 4 | 8  | 12 | 16 | 20 | 24 |
|      | 5 | 5 | 10 | 15 | 20 | 25 | 30 |
|      | 6 | 6 | 12 | 18 | 24 | 30 | 36 |
- b i  $\frac{2}{36}$     ii 0    iii  $\frac{6}{36}$
- c Even;  $P(\text{odd}) = \frac{19}{20}$ ,  $P(\text{even}) = \frac{27}{36}$  or odd score only when both factors are odd

5 a

		FIRST DICE			DICE		
		1	2	3	4	5	6
SECOND DICE	1	2	-1	4	-3	6	7
	2	3	0	5	-2	7	8
	3	-2	-5	0	-7	2	3
	4	5	2	7	0	9	10
	5	6	3	8	1	10	11
	6	7	4	9	2	11	12

b  $\frac{6}{36}$

#### Ex C25.2

- 1 a  $\frac{7}{20}$     b  $\frac{13}{20}$     c  $\frac{1}{20}$     d  $\frac{19}{20}$     e  $\frac{11}{20}$   
 f  $\frac{9}{20}$     g  $\frac{8}{20}$     h  $\frac{15}{20}$     i  $\frac{16}{20}$

- 2 a 30  
 b i  $\frac{7}{30}$     ii  $\frac{15}{30}$     iii  $\frac{12}{30}$   
 iv  $\frac{9}{30}$     v  $\frac{9}{30}$     vi  $\frac{21}{30}$

3 a 0.45    b 0.25    c 0.3

- 4 a red, orange, blue    b yellow  
 c i 0.2    ii 0.32    iii 0.23

### C25.3 Rich task

Patsy should get each colour 20 times (there will be some fluctuation because her sample size is small). However, the fluctuation in favour of red is more than 30% so it is likely to be biased towards red.

#### Ex C25.3

- 1 140
- 2 120
- 3 a  $P(3) = \frac{44}{120}$ ,  $P(1) = \frac{22}{120}$     b  $\frac{34}{120}$     c 50
- 4 Area of circle: area of square =  $\pi r^2$ :  $(2r)^2$  where side of square is  $2r$ , so  $P(\text{dart lands in circle}) = \frac{\pi}{4}$
- 5 a i  $\frac{1}{8}$     ii  $\frac{3}{8}$     iii  $\frac{3}{8}$     iv  $\frac{1}{8}$   
 b owner;  $P(\text{owner wins}) = P(B) + P(C) = \frac{6}{8}$

#### Ex C25.4

- 1 a  $\frac{4}{10}, \frac{7}{20}, \frac{11}{30}, \frac{13}{40}, \frac{18}{50}, \frac{22}{60}, \frac{26}{70}, \frac{29}{80}, \frac{32}{90}, \frac{35}{100}$   
 b 0.35  
 c yes; if fair probability would be close to 0.5
- 2 a i 6    ii 5  
 b i third 10 spins    ii fourth 10 spins  
 c 3
- 3 a  $0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{2}{5}, \frac{2}{6}, \frac{2}{7}, \frac{2}{8}, \frac{2}{9}, \frac{2}{10}, \frac{3}{11}, \frac{3}{12}, \frac{3}{13}, \frac{4}{14}, \frac{4}{15}, \frac{4}{16}, \frac{5}{17}, \frac{6}{18}, \frac{6}{19}, \frac{6}{20}$   
 b  $P(B) = \frac{7}{20}$ ,  $P(O) = \frac{7}{20}$ ,  $P(G) = \frac{6}{20}$   
 c fair, all probabilities approximately equal (note: with 20 throws and 3 outcomes could not be exactly equal)  
 d 1050 (B), 1050 (O), 900 (G)

#### Ex C25.5

- 1 a 0.3    b 0.49    c 0.42
- 2 a i  $\frac{1}{9}$     ii  $\frac{3}{9}$   
 b two possible outcomes for 3 (1, 2 and 2,1) and two for 5 (2, 3 and 3,2)
- 3 a  $\frac{4}{9}$     b  $\frac{8}{27}$

4 a  $\frac{99}{400}$       b  $\frac{532}{1875}$

5 0.375

### Ex C25.6

1 a 0.2

b diagram

c i 0.64      ii 0.32      iii 0.96

2 a diagram

b i  $\frac{25}{64}$       ii  $\frac{30}{64}$       iii  $\frac{55}{64}$

3 a 0.99

b diagram

c 0.0001

d 0.0198

### Ex C25.7

1 a diagram

b i  $\frac{3}{55}$       ii  $\frac{24}{55}$       iii  $\frac{27}{55}$

2 a diagram

b i  $\frac{1}{20}$       ii  $\frac{2}{5}$       iii  $\frac{9}{20}$

3 a diagram      b i  $\frac{21}{25}$       ii  $\frac{2}{25}$

4 a diagram      b i 0.62      ii 0.02

### Exam practice

1  $\frac{15}{36}$

2 a diagram      b  $\frac{19}{28}$

3 16p to make a profit

