

5 Algebra 2

In this unit you will:

- learn how to change the subject of a formula
- learn how to solve inequalities and represent them as regions on a graph
- solve problems in direct and indirect proportion
- identify and plot curved graphs
- use graphs to solve equations.

Functional skills coverage and range:

- Understand, use, and calculate ratio and proportion, including problems involving scale
- Understand and use simple equations and simple formulae involving one- or two-step operations.

Links

If you have an idea of the shape of a graph then you can model a situation to find out the exact equation, for example, how fast a radioactive isotope is decaying. This could tell you exactly what the isotope is.

5.1 Changing the subject of a formula

The operations that you use in solving ordinary linear equations are exactly the same as the operations you need to change the subject of a formula.

5.1.1 Simple formulae

EXAMPLE

Make x the subject in these formulae.

a $ax - p = t$

b $y(x + y) = v^2$

a $ax - p = t$

$$ax = t + p$$

$$x = \frac{t + p}{a}$$

b $y(x + y) = v^2$

$$yx + y^2 = v^2$$

$$yx = v^2 - y^2$$

$$x = \frac{v^2 - y^2}{y}$$

Exercise 1

Make x the subject.

1 $x + b = e$

2 $x - t = m$

3 $x - f = a + b$

4 $x + h = A + B$

5 $x + t = y + t$

6 $a + x = b$

7 $k + x = m$

8 $v + x = w + y$

9 $ax = b$

10 $hx = m$

13 $vx = e + n$

16 $xm = h - m$

19 $ux - h = m$

11 $mx = a + b$

14 $3x = y + z$

17 $ax + t = a$

20 $ex + q = t$

12 $kx = c - d$

15 $xp = r$

18 $mx - e = k$

21 $kx - u^2 = v^2$

Now do these.

22 $gx + t^2 = s^2$

25 $a + bx = c$

28 $a + hx = 2a$

31 $a(x - b) = c$

34 $k(x - a) = t$

37 $a(x - a) = a^2$

23 $xa + k = m^2$

26 $t + sx = y$

29 $mx - b = b$

32 $c(x - d) = e$

35 $h(x - h) = k$

38 $c(a + x) = d$

24 $xm - v = m$

27 $y + cx = z$

30 $kx + ab = cd$

33 $m(x + m) = n^2$

36 $m(x + b) = n$

39 $m(b + x) = e$

In questions 31–39 multiply out the brackets first.

5.1.2 Formulae involving fractions

EXAMPLE

Make x the subject in these formulae.

a $\frac{x}{a} = p$

b $\frac{m}{x} = t$

c $\frac{a^2}{m} = \frac{d}{x}$

a $\frac{x}{a} = p$

$x = ap$

b $\frac{m}{x} = t$

$m = xt$

$\frac{m}{t} = x$

c $\frac{a^2}{m} = \frac{d}{x}$

$xa^2 = dm$

$x = \frac{dm}{a^2}$

Notice in parts **b** and **c** that when x is on the bottom you start by multiplying both sides by x .

Exercise 2

Make x the subject.

1 $\frac{x}{t} = m$

4 $am = \frac{x}{t}$

7 $\frac{x}{a} = (b + c)$

10 $\frac{x}{k} = h + i$

13 $\frac{x}{h} = \frac{c}{d}$

16 $\frac{x}{(a + b)} = c$

19 $\frac{x}{(h - k)} = t$

22 $a = \frac{e}{x}$

2 $\frac{x}{e} = n$

5 $bc = \frac{x}{a}$

8 $\frac{x}{t} = (c - d)$

11 $\frac{x}{b} = \frac{a}{c}$

14 $\frac{m}{n} = \frac{x}{e}$

17 $\frac{x}{(h + k)} = m$

20 $\frac{x}{(a + b)} = (z + t)$

23 $m = \frac{h}{x}$

3 $\frac{x}{p} = a$

6 $e = \frac{x}{y^2}$

9 $\frac{x}{m} = s + t$

12 $\frac{x}{m} = \frac{z}{y}$

15 $\frac{b}{e} = \frac{x}{h}$

18 $\frac{x}{u} = \frac{m}{y}$

21 $t = \frac{e}{x}$

24 $\frac{a}{b} = \frac{c}{x}$

When x is on the bottom, multiply both sides by x .

25 $\frac{u}{x} = \frac{c}{d}$

26 $\frac{m}{x} = t^2$

27 $\frac{h}{x} = \sin 20^\circ$

28 $\frac{e}{x} = \cos 40^\circ$

29 $\frac{m}{x} = \tan 46^\circ$

30 $\frac{a^2}{b^2} = \frac{c^2}{x}$

You will meet
sine, cos and tan
in chapter 6.

5.1.3 Formulae with negative x-terms

EXAMPLE

Make x the subject of these formulae.

a $t - x = a^2$

b $h - bx = m$

c $a(m - x) = h$

a $t - x = a^2$

b $h - bx = m$

c $a(m - x) = h$

$t = a^2 + x$

$h = m + bx$

$am - ax = h$

$t - a^2 = x$

$h - m = bx$

$am = h + ax$

or $x = t - a^2$

$\frac{h - m}{b} = x$

$am - h = ax$

$\frac{am - h}{a} = x$

Notice that in
each question the
first step is to
make the x-term
positive by taking
it to the other
side.

Exercise 3 M

Make x the subject.

1 $a - x = y$

2 $h - x = m$

3 $z - x = q$

4 $v = b - x$

5 $m = k - x$

6 $h - cx = d$

7 $y - mx = c$

8 $k - ex = h$

9 $a^2 - bx = d$

10 $m^2 - tx = n^2$

11 $v^2 - ax = w$

12 $y - x = y^2$

13 $k - t^2x = m$

14 $e = b - cx$

15 $z = h - gx$

16 $a + b = c - dx$

17 $y^2 = v^2 - kx$

18 $h = d - fx$

19 $a(b - x) = c$

20 $h(m - x) = n$

Make a the subject.

21 $m(c - a) = t$

22 $v(p - a) = w$

23 $e = d(q - a)$

24 $b^2 - a = r^2$

25 $\frac{x - a}{f} = 2f$

26 $\frac{B - Aa}{D} = E$

27 $\frac{D - Ea}{N} = B$

28 $\frac{h - fa}{b} = x$

29 $\frac{v^2 - ha}{C} = d$

30 $\frac{M(a + B)}{N} = T$

31 $\frac{f(Na - e)}{m} = B$

32 $\frac{T(M - a)}{E} = F$

Make x the subject (more difficult).

33 $\frac{2}{x} + 1 = 3y$

34 $\frac{5}{x} - 2 = 4z$

35 $\frac{A}{x} + B = C$

36 $\frac{V}{x} + G = H$

37 $\frac{r}{x} - t = n$

38 $q = \frac{b}{x} + d$

39 $t = \frac{m}{x} - n$

40 $h = d - \frac{b}{x}$

41 $C - \frac{d}{x} = e$

42 $r - \frac{m}{x} = e^2$

43 $t^2 = b - \frac{n}{x}$

44 $\frac{d}{x} + b = mn$

45 $3M = M + \frac{N}{P + x}$

46 $A = \frac{B}{c + x} - 5A$

47 $\frac{m^2}{x} - n = -p$

48 $t = w - \frac{q}{x}$

5.1.4 Formulae with squares and square roots

EXAMPLE

Make x the subject in these formulae.

a $mx^2 = b$

b $x^2 + h = k$

c $ax^2 + b = c$

d $\sqrt{x} = u$

e $\sqrt{x-e} = t$

f $\sqrt{x^2 + A} = B$

a $mx^2 = b$

$$x^2 = \frac{b}{m}$$

$$x = \pm \sqrt{\frac{b}{m}}$$

b $x^2 + h = k$

$$x^2 = k - h$$

$$x = \pm \sqrt{(k - h)}$$

c $ax^2 + b = c$

$$ax^2 = c - b$$

$$x^2 = \frac{c - b}{a}$$

$$x = \pm \sqrt{\frac{c - b}{a}}$$

d $\sqrt{x} = u$

$$x = u^2 \quad (\text{square both sides})$$

e $\sqrt{x-e} = t$

$$x - e = t^2$$

$$x = t^2 + e$$

f $\sqrt{x^2 + A} = B$

$$x^2 + A = B^2$$

$$x^2 = B^2 - A$$

$$x = \pm \sqrt{(B^2 - A)}$$

Exercise 4 (M)/H

Make x the subject.

1 $cx^2 = h$

2 $bx^2 = f$

3 $x^2t = m$

4 $x^2y = (a + b)$

5 $mx^2 = (t + a)$

6 $x^2 - a = b$

7 $x^2 + c = t$

8 $x^2 + y = z$

9 $x^2 - a^2 = b^2$

10 $x^2 + t^2 = m^2$

11 $x^2 + n^2 = a^2$

12 $ax^2 = c$

13 $hx^2 = n$

14 $cx^2 = z + k$

15 $ax^2 + b = c$

16 $dx^2 - e = h$

17 $gx^2 - n = m$

18 $x^2m + y = z$

19 $a + mx^2 = f$

20 $a^2 + x^2 = b^2$

Make x the subject.

21 $\sqrt{x} = 2z$

22 $\sqrt{(x-2)} = 3y$

23 $\sqrt{(x+C)} = D$

24 $\sqrt{(ax+b)} = c$

25 $b = \sqrt{(gx-t)}$

26 $\sqrt{(d-x)} = t$

27 $c = \sqrt{(n-x)}$

28 $g = \sqrt{(c-x)}$

29 $\sqrt{(Ax+B)} = \sqrt{D}$

30 $x^2 = g$

31 $x^2 = B$

32 $x^2 - A = M$

Make k the subject.

33 $C - k^2 = m$

34 $mk^2 = n$

35 $\frac{kz}{a} = t$

36 $n = a - k^2$

37 $\sqrt{(k^2 - A)} = B$

38 $t = \sqrt{(m + k^2)}$

39 $A\sqrt{(k+B)} = M$

40 $\sqrt{\left(\frac{N}{k}\right)} = B$

41 $\sqrt{(a^2 - k^2)} = t$

42 $2\pi\sqrt{(k+t)} = 4$

43 $\sqrt{(ak^2 - b)} = C$

44 $k^2 + b = x^2$

5.1.5 Formulae with x on both sides

EXAMPLE

Make x the subject of each formula.

a $Ax - B = Cx + D$

b $x + a = \frac{x + b}{c}$

a $Ax - B = Cx + D$

$Ax - Cx = D + B$ (x -terms on one side)

$x(A - C) = D + B$ (factorise)

$$x = \frac{D + B}{A - C}$$

b $x + a = \frac{x + b}{c}$

$c(x + a) = x + b$

$cx + ca = x + b$

$cx - x = b - ca$ (x -terms on one side)

$x(c - 1) = b - ca$ (factorise)

$$x = \frac{b - ca}{c - 1}$$

Exercise 5 (M)/(H)

Make y the subject.

1 $5(y - p) = 2(y + x)$

2 $x(y - 3) = p(3 - y)$

3 $Ny + B = D - Ny$

4 $My - D = E - 2My$

5 $ay + b = 3b + by$

6 $my - c = e - ny$

7 $xy + 4 = 7 - ky$

8 $Ry + D = Ty + C$

9 $ay - x = z + by$

10 $m(y + a) = n(y + b)$

11 $x(y - b) = y + d$

12 $\frac{a - y}{a + y} = b$

13 $\frac{1 - y}{1 + y} = \frac{c}{d}$

14 $\frac{M - y}{M + y} = \frac{a}{b}$

15 $m(y + n) = n(n - y)$

16 $y + m = \frac{2y - 5}{m}$

17 $y - n = \frac{y + 2}{n}$

18 $y + b = \frac{ay + e}{b}$

19 $\frac{ay + x}{x} = 4 - y$

20 $c - dy = e - ay$

21 $y(a - c) = by + d$

22 $y(m + n) = a(y + b)$

23 $t - ay = s - by$

24 $\frac{y + x}{y - x} = 3$

25 $\frac{v - y}{v + y} = \frac{1}{2}$

26 $y(b - a) = a(y + b + c)$

27 $\sqrt{\left(\frac{y + x}{y - x}\right)} = 2$

28 $\sqrt{\left(\frac{z + y}{z - y}\right)} = \frac{1}{3}$

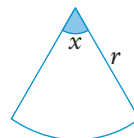
29 $\sqrt{\left[\frac{m(y + n)}{y}\right]} = p$

30 $n - y = \frac{4y - n}{m}$

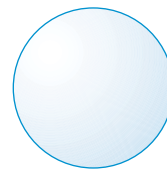
Exercise 6 (M)/(H)

1 A formula for calculating velocity is $v = u + at$.a Rearrange the formula to express a in terms of v , u and t .b Calculate a when $v = 20$, $u = 4$, $t = 8$.

2 The area of a sector of a circle is given

by the formula $A = \frac{x\pi r^2}{360}$.Express x in terms of A , π and r .

- 3 a** Express k in terms of P , m and y , when $P = \frac{mk}{y}$.
b Express y in terms of P , m and k .
- 4** A formula for calculating repair bills, R , is $R = \frac{n-d}{p}$.
a Express n in terms of R , p and d .
b Calculate n when $R = 400$, $p = 3$ and $d = 55$.
- 5** The formula for the area of a circle is $A = \pi r^2$.
 Express r in terms of A and π .
- 6** The volume of a cylinder is given by $V = \pi r^2 h$.
 Express h in terms of V , π and r .
- 7** The surface area, A , and volume, V , of a sphere are given by the formulae $A = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$. Make r the subject of each formula.



Exercise 7 (M)/(H)

Make the letter in brackets the subject.

- | | | |
|---------------------------------------|---|---------------------------------------|
| 1 $ax - d = h$ [x] | 2 $zy + k = m$ [y] | 3 $d(y + e) = f$ [y] |
| 4 $m(a + k) = d$ [k] | 5 $a + bm = c$ [m] | 6 $ae^2 = b$ [e] |
| 7 $yt^2 = z$ [t] | 8 $x^2 - c = e$ [x] | 9 $my - n = b$ [y] |
| 10 $a(z + a) = b$ [z] | 11 $\frac{a}{x} = d$ [x] | 12 $\frac{k}{m} = t$ [k] |
| 13 $\frac{u}{m} = n$ [u] | 14 $\frac{y}{x} = d$ [x] | 15 $\frac{a}{m} = t$ [m] |
| 16 $\frac{d}{g} = n$ [g] | 17 $\frac{t}{k} = (a + b)$ [t] | 18 $y = \frac{v}{e}$ [e] |
| 19 $c = \frac{m}{y}$ [y] | 20 $\frac{a^2}{m} = b$ [a] | 21 $g(m + a) = b$ [m] |
| 22 $h(h + g) = x^2$ [g] | 23 $y - t = z$ [t] | 24 $me^2 = c$ [e] |
| 25 $a(y + x) = t$ [x] | 26 $uv - t^2 = y^2$ [v] | 27 $k^2 + t = c$ [k] |
| 28 $k - w = m$ [w] | 29 $b - an = c$ [n] | 30 $m(a + y) = c$ [y] |
| 31 $pq - x = ab$ [x] | 32 $a^2 - bk = t$ [k] | 33 $v^2 z = w$ [z] |
| 34 $c = t - u$ [u] | 35 $xc + t = 2t$ [c] | 36 $m(n + w) = k$ [w] |
| 37 $v - mx = t$ [m] | 38 $c = a(y + b)$ [y] | 39 $m(a - c) = e$ [c] |
| 40 $ba^2 = c$ [a] | 41 $\frac{a}{p} = q$ [p] | 42 $\frac{a}{n^2} = e$ [n] |
| 43 $\frac{h}{f^2} = m$ [f] | 44 $\frac{v}{x^2} = n$ [x] | 45 $v - ac = t^3$ [c] |
| 46 $a(a^2 + y) = b^3$ [y] | 47 $ah^2 - d = b$ [h] | 48 $h(h + k) = bc$ [k] |
| 49 $u^2 - n^2 = v^2$ [n] | 50 $m(b - z) = b^3$ [z] | |

5.2 Inequalities and regions

5.2.1 Inequality symbols

There are four inequality symbols.

- $x < 4$ means 'x is **less than** 4'
- $y > 7$ means 'y is **greater than** 7'
- $z \leq 10$ means 'z is **less than or equal to** 10'
- $t \geq -3$ means 't is **greater than or equal to** -3'

When there are two symbols in one statement look at each part separately.

For example, if n is an **integer** and $3 < n \leq 7$, n has to be greater than 3 but at the same time it has to be less than or equal to 7.

So n could be 4, 5, 6 or 7 only.

EXAMPLE

Illustrate on a number line the range of values of x stated.

a $x > 1$



The circle at the left-hand end of the range is open. This means that 1 is not included.

b $x \leq -2$



The circle at -2 is filled in to indicate that -2 is included.

c $1 \leq x < 4$



Exercise 8

1 Write each statement with either $>$ or $<$ in the box.

a $3 \square 7$

b $0 \square -2$

c $3 \cdot 1 \square 3 \cdot 01$

d $-3 \square -5$

e $100 \text{ mm} \square 1 \text{ m}$

f $1 \text{ kg} \square 1 \text{ lb}$

2 Write the inequality displayed. Use x for the variable.



3 Draw a number line to display these inequalities.

a $x \geq 7$

b $x < 2 \cdot 5$

c $1 < x < 7$

d $0 \leq x \leq 4$

e $-1 < x \leq 5$

4 Write an inequality for each statement.

a You must be at least 16 to get married. [Use A for age.]

b Vitamin J1 is not recommended for people over 70 or for children 3 years or under. [Use A for age recommended.]

c To cook beef the oven temperature should be between 150°C and 175°C . [Use T for temperature.]

d Applicants for training as paratroopers must be at least 1.75 m tall. [Use h for height.]

5 Answer 'true' or 'false':

a n is an integer and $1 < n \leq 4$, so n can be 2, 3 or 4.

b x is an integer and $2 \leq x < 5$, so x can be 2, 3 or 4.

c p is an integer and $p \geq 10$, so p can be 10, 11, 12, 13 ...

6 Which of the numbers x , below, satisfy $x^2 < 90$?

7, -6 , 10, 8.5 , $\sqrt{95}$

7 Write one inequality to show the values of x which satisfy both of these inequalities.

$x \leq 7$

$x > 2$

8 Write one inequality to show the values of x which satisfy all three of these inequalities.

$x < 5$

$0 < x < 6$

$3 \leq x < 10$

5.2.2 Solving inequalities

Follow the same procedure that you use for solving equations except that when you multiply or divide by a **negative** number you **reverse** the inequality.

For example, $4 > -2$ but if you multiply by -2 then $-8 < 4$

It is best to avoid dividing by a negative number as in the following example, part **b**.

EXAMPLE

Solve these inequalities.

a $x + 11 < 4$

c $2x - 1 > 5$

b $8 > 13 - x$

d $x + 1 < 2x < x + 3$

a $x + 11 < 4$

$x < -7$ (subtract 11)

c $2x - 1 > 5$

$2x > 5 + 1$ (add 1)

$x > \frac{6}{2}$ (divide by 2)

$x > 3$

b $8 > 13 - x$

$8 + x > 13$ (add x)

$x > 5$ (subtract 8)

d $x + 1 < 2x < x + 3$

Solve the two inequalities separately.

$x + 1 < 2x$ $2x < x + 3$

$1 < x$ $x < 3$

The solution is $1 < x < 3$.

Exercise 9 

Solve these inequalities.

1 $x - 3 > 10$

2 $x + 1 < 0$

3 $5 > x - 7$

4 $2x + 1 \leq 6$

5 $3x - 4 > 5$

6 $10 \leq 2x - 6$

7 $5x < x + 1$

8 $2x \geq x - 3$

9 $4 + x < -4$

10 $3x + 1 < 2x + 5$

11 $2(x + 1) > x - 7$

12 $7 < 15 - x$

13 $9 > 12 - x$

14 $4 - 2x \leq 2$

15 $3(x - 1) < 2(1 - x)$

16 $7 - 3x < 0$

17 $\frac{x}{3} < -1$

18 $\frac{2x}{5} > 3$

19 $2x > 0$

20 $\frac{x}{4} < 0$

- 21** The height of this picture has to be greater than the width.
Find the range of possible values of x .



height
 $2(x + 1)$

width
 $(x + 7)$

22 $10 \leq 2x \leq x + 9$

23 $x < 3x + 2 < 2x + 6$

24 $10 \leq 2x - 1 \leq x + 5$

25 $3 < 3x - 1 < 2x + 7$

26 $x - 10 < 2(x - 1) < x$

27 $4x + 1 < 8x < 3(x + 2)$

In questions **22** to **27**, solve the two inequalities separately.

- 28** Sumitra said ‘I think of an integer.
I subtract 14.
I multiply the result by 5.
I divide by 2.
The answer is greater than the number I
thought of.’

Write an inequality and solve it to find the smallest number Sumitra could have thought of.

Take care when there are squares and square roots in inequalities.

The equation $x^2 = 4$ has solutions $x = \pm 2$, which is correct.

For the inequality $x^2 < 4$, you might wrongly write $x < \pm 2$.

Consider $x = -3$, say.

-3 is less than -2 and is also less than $+2$.

But $(-3)^2$ is not less than 4 and so

$x = -3$ does not satisfy the inequality $x^2 < 4$.

The correct solution for $x^2 < 4$ is $-2 < x < 2$.

$$(-3)^2 = 9$$

EXAMPLE

- a** Solve the inequality

$$2x^2 - 1 > 17.$$

- b** List the solutions which satisfy

$$2 \leq n < 14; n \text{ is a prime number}$$

- a** $2x^2 - 1 > 17$

$$2x^2 > 18$$

$$x^2 > 9$$

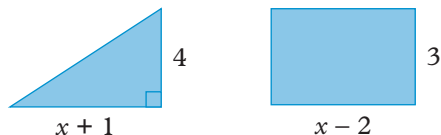
$$x > 3 \text{ or } x < -3 \quad (\text{Avoid the temptation to write } x > \pm 3!)$$

- b** The prime numbers in the range specified are 2, 3, 5, 7, 11, 13.

Exercise 10 

- 1** The area of the rectangle must be greater than the area of the triangle.

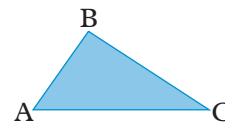
Find the range of possible values of x .



For questions **2** to **8**, list the solutions which satisfy the given condition.

- 2** $3a + 1 < 20$; a is a positive integer.

- 3 $b - 1 \geq 6$; b is a prime number less than 20.
- 4 $1 < z < 50$; z is a square number.
- 5 $2x > -10$; x is a negative integer.
- 6 $x + 1 < 2x < x + 13$; x is an integer.
- 7 $0 \leq 2z - 3 \leq z + 8$; z is a prime number.
- 8 $\frac{a}{2} + 10 > a$; a is a positive even number.
- 9 Given that $4x > 1$ and $\frac{x}{3} \leq 1\frac{1}{3}$, list the possible integer values of x .
- 10 State the smallest integer n for which $4n > 19$.
- 11 Given that $-4 \leq a \leq 3$ and $-5 \leq b \leq 4$, find
- the largest possible value of a^2
 - the smallest possible value of ab
 - the largest possible value of ab
 - the value of b if $b^2 = 25$.
- 12 For any shape of triangle ABC, complete the statement
 $AB + BC \square AC$, by writing $<$, $>$ or $=$ inside the box.



- 13 Find a simple fraction r such that $\frac{1}{3} < r < \frac{2}{3}$.
- 14 Find the largest prime number p such that $p^2 < 400$.
- 15 Find the integer n such that $n < \sqrt{300} < n + 1$.
- 16 If $f(x) = 2x - 1$ and $g(x) = 10 - x$ for what values of x is $f(x) > g(x)$?
- 17 a The solution of $x^2 < 9$ is $-3 < x < 3$.
 [The square roots of 9 are -3 and 3 .]
- b Copy and complete.
- If $x^2 < 100$, then $\square < x < \square$
 - If $x^2 < 81$, then $\square < x < \square$
 - If $x^2 > 36$, then $x > \square$ or $x < \square$

Solve the inequalities.

- 18 $x^2 < 25$ 19 $x^2 \leq 16$ 20 $x^2 > 1$
- 21 $2x^2 \geq 72$ 22 $3x^2 + 5 > 5$ 23 $5x^2 - 2 < 18$
- 24 Given $2 \leq p \leq 10$ and $1 \leq q \leq 4$, find the range of values of
- pq
 - $\frac{p}{q}$
 - $p - q$
 - $p + q$

- 25 If $2^r > 100$, what is the smallest integer value of r ?

- 26 Given $\left(\frac{1}{3}\right)^x < \frac{1}{200}$, what is the smallest integer value of x ?
- 27 Find the smallest integer value of x which satisfies $x^x > 10\,000$.
- 28 What integer values of x satisfy $100 < 5^x < 10\,000$?
- *29 If x is an acute angle and $\sin x > \frac{1}{2}$, write the range of values that x can take.
- *30 If x is an acute angle and $\cos x > \frac{1}{4}$, write the range of values that x can take.

5.2.3 Shading regions

You can represent inequalities on a graph, particularly where two variables (x and y) are involved.

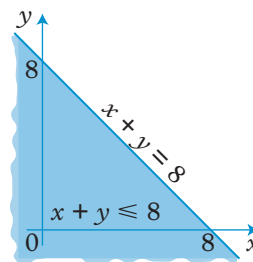
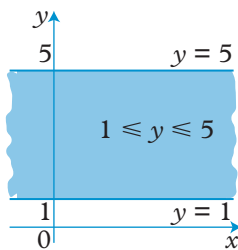
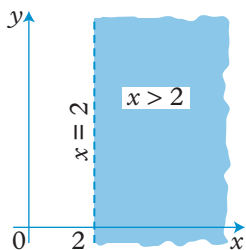
EXAMPLE

Draw a sketch graph and shade the area which represents the set of points that satisfy each of these inequalities.

a $x > 2$

b $1 \leq y \leq 5$

c $x + y \leq 8$



In each graph, the required region is shaded.

In **a**, the line $x = 2$ is shown as a broken line to indicate that the points on the line are not included.

In **b** and **c** points on the line **are** included 'in the region' and the lines are drawn unbroken.

To decide which side to shade when the line is sloping, take a **trial point**. This can be any point which is not actually on the line.

In **c** above, the trial point could be $(1, 1)$.

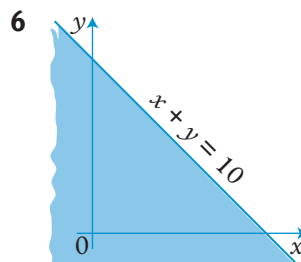
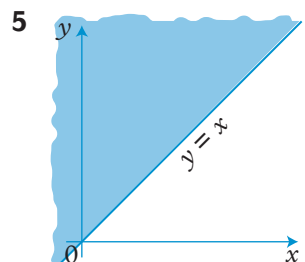
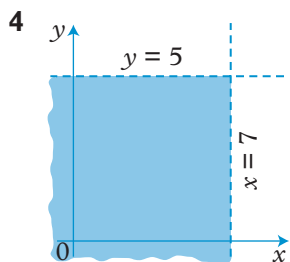
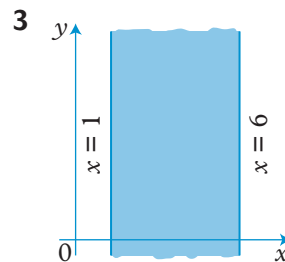
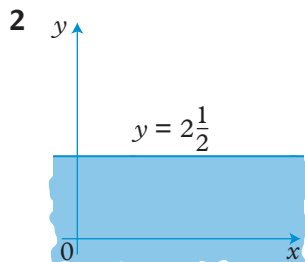
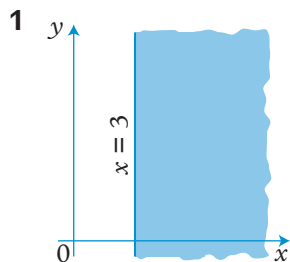
Is $(1, 1)$ in the region $x + y \leq 8$?

It satisfies $x + y < 8$ because $1 + 1 = 2$, which is less than 8.

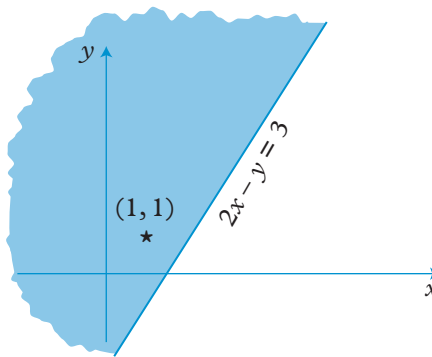
So below the line is $x + y < 8$ which is in the shaded region.

Exercise 11 (M)/(H)

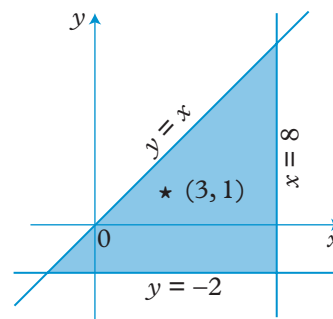
In questions 1 to 6, describe the region which is shaded.



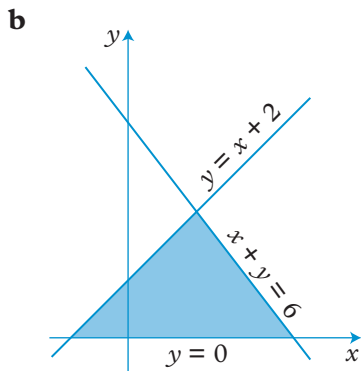
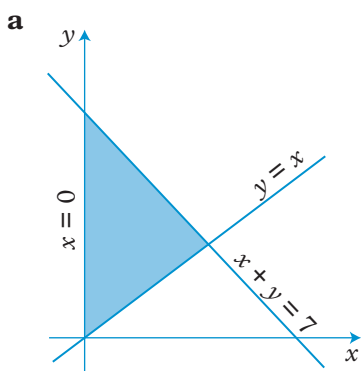
- 7 The point (1, 1), marked *, lies in the shaded region.
 Use this as a trial point to describe the shaded region as follows:
 Is the shaded region $2x - y > 3$?
 Try $x = 1, y = 1$. Is $2 - 1 > 3$? No.
 Copy and complete 'So the shaded region is $2x - y \square 3$.'



- 8 The point (3, 1), marked *, lies in the shaded triangle.
 Use this as a trial point to write the three inequalities which describe the shaded region.



9 A trial point (1, 2) lies inside the shaded triangles. Write the three inequalities which describe each shaded region.



For questions 10 to 27, draw a sketch graph similar to those in question 9 and indicate the set of points which satisfy the inequalities by shading the required region.

10 $2 < x < 7$

11 $0 < y < 3\frac{1}{2}$

12 $-2 < x < 2$

13 $x < 6$ and $y < 4$

14 $0 < x < 5$ and $y < 3$

15 $1 < x < 6$ and $2 < y < 8$

16 $-3 < x < 0$ and $-4 < y < 2$

17 $y < x$

18 $x + y < 5$

19 $y > x + 2$ and $y < 7$

20 $x > 0$ and $y > 0$ and $x + y < 7$

21 $x > 0$ and $x + y < 10$ and $y > x$

22 $8 > y > 0$ and $x + y > 3$

23 $x + 2y < 10$ and $x > 0$ and $y > 0$

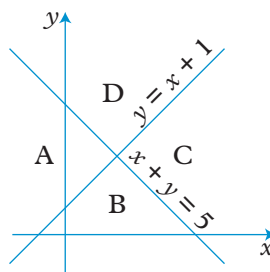
24 $3x + 2y < 18$ and $x > 0$ and $y > 0$

25 $x > 0, y > x - 2, x + y < 10$

26 $3x + 5y < 30$ and $y > \frac{x}{2}$

27 $y > \frac{x}{2}, y < 2x$ and $x + y < 8$

28 The two lines $y = x + 1$ and $x + y = 5$ divide the graph into four regions A, B, C, D. Write the two inequalities which describe each of the regions A, B, C, D.



*29 Using the same axes, draw the graphs of $xy = 10$ and $x + y = 9$ for values of x from 1 to 10. Hence find all pairs of positive integers with products greater than 10 and sums less than 9.

5.3 Direct and inverse proportion

5.3.1 Direct proportion

- a** When you buy petrol, the more you buy the more money you have to pay. So if 2.2 litres costs 198p, then 4.4 litres will cost 396p. The cost of petrol is **directly proportional** to the quantity bought. To show that quantities are proportional, you use the symbol ' \propto '. So in the example if the cost of petrol is c pence and the number of litres of petrol is l , you write

$$c \propto l$$

The ' \propto ' sign can always be replaced by ' $= k$ ' where k is a constant.

$$\text{So } c = kl$$

From above, if $c = 198$ when $l = 2.2$
then $198 = k \times 2.2$

$$k = \frac{198}{2.2} = 90$$

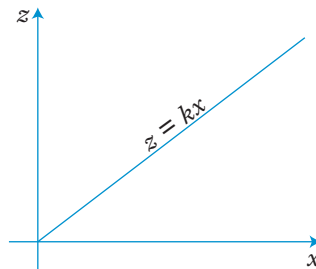
You can then write $c = 90l$, and this allows you to find the value of c for any value of l , and **vice versa**.

- b** If a quantity z is proportional to a quantity x , you can write

$$z \propto x \quad \text{or} \quad z = kx$$

Two other expressions are sometimes used when quantities are directly proportional. You can say

' z varies as x '
or ' z varies directly as x '.



- When z and x are directly proportional the graph connecting z and x is a straight line which passes through the origin.

EXAMPLE

If y varies as z , and $y = 2$ when $z = 5$, find

- a** the value of y when $z = 6$
b the value of z when $y = 5$.

Because $y \propto z$, then $y = kz$ where k is a constant.

$$y = 2 \text{ when } z = 5$$

So $2 = k \times 5$

$$k = \frac{2}{5}$$

So $y = \frac{2}{5}z$

a When $z = 6$, $y = \frac{2}{5} \times 6 = 2\frac{2}{5}$.

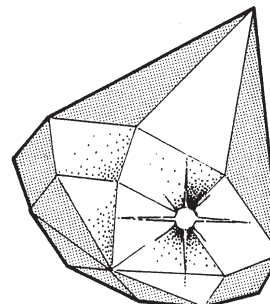
b When $y = 5$, $5 = \frac{2}{5}z$; $z = \frac{25}{2} = 12\frac{1}{2}$.

EXAMPLE

The value V of a diamond is proportional to the square of its weight W .

If a diamond weighing 10 grams is worth £200, find

- a** the value of a diamond weighing 30 grams
b the weight of a diamond worth £5000.



$$V \propto W^2$$

or $V = kW^2$ where k is a constant.

$$V = 200 \text{ when } W = 10$$

So $200 = k \times 10^2$

$$k = 2$$

So $V = 2W^2$

a When $W = 30$,

$$V = 2 \times 30^2 = 2 \times 900$$

$$V = \text{£}1800$$

So a diamond of weight 30 grams is worth £1800.

b When $V = 5000$,

$$5000 = 2 \times W^2$$

$$W^2 = \frac{5000}{2} = 2500$$

$$W = \sqrt{2500} = 50$$

So a diamond of value £5000 weighs 50 grams.

Exercise 12  

- 1 Rewrite the statement connecting each pair of variables using a constant k instead of ' \propto '.

a $S \propto e$

b $v \propto t$

c $x \propto z^2$

d $y \propto \sqrt{x}$

e $T \propto \sqrt{L}$

Reminder: If $x \propto d$,
then $x = kd$.

- 2 y is proportional to t so that $y = kt$. If $y = 6$ when $t = 4$, calculate the value of k and hence find
- a** the value of y when $t = 6$
b the value of t when $y = 4$.
- 3 z is proportional to m . If $z = 20$ when $m = 4$, calculate
- a** the value of z when $m = 7$
b the value of m when $z = 55$.
- 4 A varies directly as r^2 . If $A = 12$, when $r = 2$, calculate
- a** the value of A when $r = 5$
b the value of r when $A = 48$.

- 5 Given that $z \propto x$, copy and complete the table.

x	1	3		$5\frac{1}{2}$
z	4		16	

- 6 Given that $V \propto r^3$, copy and complete the table.

r	1	2		$1\frac{1}{2}$
V	4		256	

- 7 The pressure of the water, P , at any point below the surface of the sea varies as the depth of the point below the surface, d . If the pressure is 200 newtons/cm² at a depth of 3 m, calculate the pressure at a depth of 5 m.
- 8 The distance d through which a stone falls from rest is proportional to the square of the time taken, t . If a stone falls 45 m in 3 seconds, how far will it fall in 6 seconds? How long will it take to fall 20 m?
- 9 The energy, E , stored in an elastic band is proportional to the square of the extension, x . When the elastic is extended by 3 cm, the energy stored is 243 joules.
- a** What is the energy stored when the extension is 5 cm?
b What is the extension when the stored energy is 36 joules?

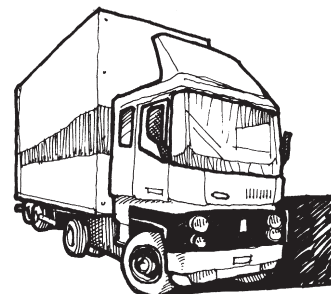
- 10 The resistance to motion of a car is proportional to the square of the speed of the car.
- If the resistance is 4000 newtons at a speed of 20 m/s, what is the resistance at a speed of 30 m/s?
 - At what speed is the resistance 6250 newtons?
- 11 In an experiment, Julie made measurements of w and p .

w	2	5	7
p	1.6	25	68.6

Which of these laws fits the results?

$$p \propto w, \quad p \propto w^2, \quad p \propto w^3.$$

- 12 A road research organisation recently claimed that the damage to road surfaces was proportional to the fourth power of the axle load. The axle load of a 44-ton HGV is about 15 times that of a car. Calculate the ratio of the damage to road surfaces made by a 44-ton HGV and a car.



5.3.2 Inverse proportion

If you travel a distance of 200 m at 10 m/s, the time taken is 20 s.

If you travel the same distance at 20 m/s, the time taken is 10 s.

As you **double** the speed, you **halve** the time taken.

For a fixed journey, the time taken is **inversely proportional** to the speed at which you travel.

If s is inversely proportional to t , you write

$$s \propto \frac{1}{t}$$

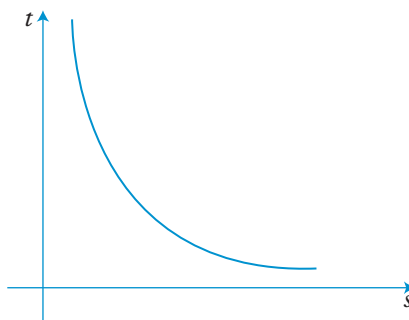
or $s = k \times \frac{1}{t}$

Notice that the product $s \times t$ is constant.

The graph connecting s and t is a curve.

The shape of the curve is similar

to that of $y = \frac{1}{x}$.



Sometimes you will see 'x varies inversely as y'. It means the same as 'x is inversely proportional to y'.

EXAMPLE

z is inversely proportional to t^2 and $z = 4$ when $t = 1$.
Calculate z when $t = 2$.

$$z \propto \frac{1}{t^2} \quad \text{or} \quad z = k \times \frac{1}{t^2} \quad (k \text{ is a constant})$$

$$z = 4 \text{ when } t = 1$$

$$\text{So } 4 = k \left(\frac{1}{1^2} \right)$$

$$\text{so } k = 4$$

$$\text{So } z = 4 \times \frac{1}{t^2}$$

$$\text{When } t = 2, \quad z = 4 \times \frac{1}{2^2} = 1$$

Exercise 13



- Rewrite the statements connecting the variables using a constant of variation, k .
 - $x \propto \frac{1}{y}$
 - $s \propto \frac{1}{t^2}$
 - $t \propto \frac{1}{\sqrt{q}}$
 - m varies inversely as w
 - z is inversely proportional to t^2 .
- T is inversely proportional to m . If $T = 12$ when $m = 1$, find
 - T when $m = 2$
 - T when $m = 24$.
- L is inversely proportional to x . If $L = 24$ when $x = 2$, find
 - L when $x = 8$
 - L when $x = 32$.
- b varies inversely as e . If $b = 6$ when $e = 2$, calculate
 - the value of b when $e = 12$
 - the value of e when $b = 3$.
- x is inversely proportional to y^2 . If $x = 4$ when $y = 3$, calculate
 - the value of x when $y = 1$
 - the value of y when $x = 2\frac{1}{4}$.
- p is inversely proportional to \sqrt{y} . If $p = 1.2$ when $y = 100$, calculate
 - the value of p when $y = 4$
 - the value of y when $p = 3$.

Start by writing
 $T = \frac{k}{m}$ and then
find k .

- 7 Given that $z \propto \frac{1}{y}$, copy and complete the table.

y	2	4		$\frac{1}{4}$
z	8		16	

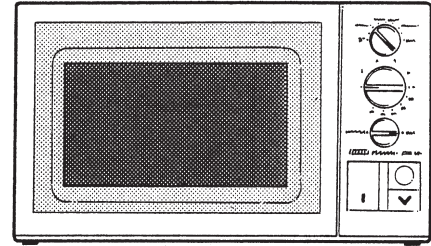
- 8 Given that $v \propto \frac{1}{t^2}$, copy and complete the table.

t	2	5		10
v	25		$\frac{1}{4}$	

- 9 e varies inversely as $(y - 2)$. If $e = 12$ when $y = 4$, find
a e when $y = 6$ **b** y when $e = \frac{1}{2}$.
- 10 The volume, V , of a given mass of gas varies inversely as the pressure, P . When $V = 2 \text{ m}^3$, $P = 500 \text{ N/m}^2$.
a Find the volume when the pressure is 400 N/m^2 .
b Find the pressure when the volume is 5 m^3 .
- 11 The number of hours, N , required to dig a certain hole is inversely proportional to the number of people available, x .
 When 6 people are digging, the hole takes 4 hours.
a Find the time taken when 8 people are available.
b If it takes $\frac{1}{2}$ hour to dig the hole, how many people are there?
- 12 The force of attraction, F , between two magnets varies inversely as the square of the distance, d , between them. When the magnets are 2 cm apart, the force of attraction is 18 newtons. How far apart are they if the attractive force is 2 newtons?
- 13 The number of tiles, n , that can be pasted using one tin of tile paste is inversely proportional to the square of the side, d , of the tile. One tin is enough for 180 tiles of side 10 cm. How many tiles of side 15 cm can be pasted using one tin?
- 14 The life expectancy, L , of a rat varies inversely as the square of the density, d , of poison distributed around its home. When the density of poison is 1 g/m^2 the life expectancy is 50 days. How long will the rat survive if the density of poison is
a 5 g/m^2 ? **b** $\frac{1}{2} \text{ g/m}^2$?

- 15 When cooking snacks in a microwave oven, a cook assumes that the cooking time is inversely proportional to the power used. The five levels on his microwave have the powers shown in the table.

Level	Power used
Full	600 W
Roast	400 W
Simmer	200 W
Defrost	100 W
Warm	50 W



- a Escargots de Bourgogne take 5 minutes on 'Simmer'. How long will they take on 'Warm'?
- b Escargots a la Provençale are normally cooked on 'Roast' for 3 minutes. How long will they take on 'Full'?
- 16 Given $z = \frac{k}{x^n}$, find k and n , then copy and complete the table.

x	1	2	4	
z	100	$12\frac{1}{2}$		$\frac{1}{10}$

- *17 Given $y = \frac{k}{\sqrt[n]{v}}$, find k and n , then copy and complete the table.

v	1	4	36	
y	12	6		$\frac{3}{25}$

$\sqrt[n]{v}$ means the n th root of v .

5.4 Curved graphs


5.4.1 Common curves


It is helpful to know the general shape of some of the more common curves.

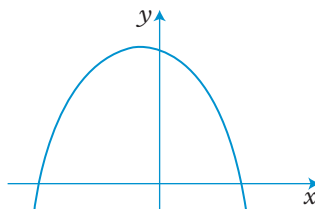
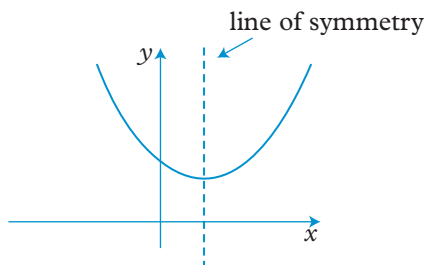
- **Quadratic curves** have an x^2 -term as the highest power of x .

The curve also has a line of symmetry.

For example, $y = 2x^2 - 3x + 7$ and $y = 5 + 2x - x^2$

a When the x^2 -term is positive, the curve is -shaped.

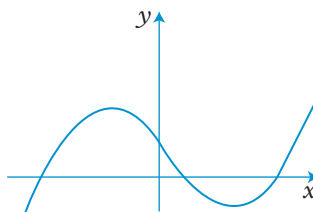
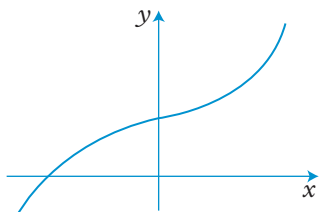
b When the x^2 -term is negative, the curve is an inverted .



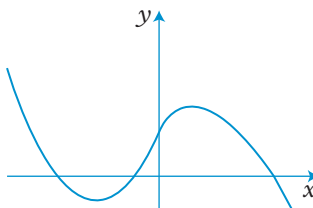
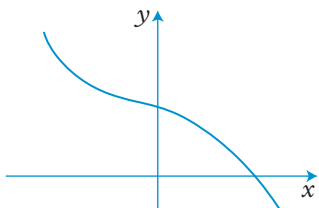
- **Cubic curves** have an x^3 -term as the highest power of x .

For example, $y = x^3 + 7x - 4$ and $y = 8x - 4x^3$

a When the x^3 -term is positive, the curve can be like one of the two shown below. Notice that as x gets larger, so does y .



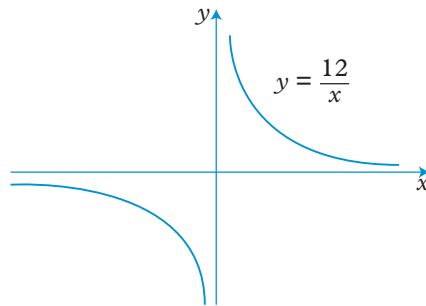
b When the x^3 -term is negative, the curve can be like one of the two shown below. Notice that when x is large, y is large but negative.



- **Reciprocal** curves have a $\frac{1}{x}$ term.

For example, $y = \frac{12}{x}$ and

$$y = \frac{6}{x} + 5$$



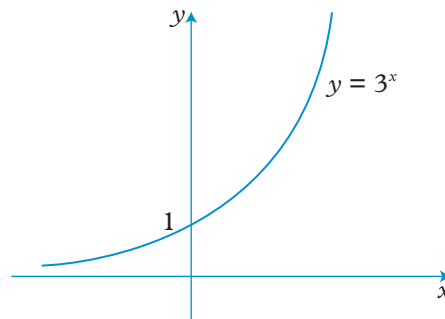
The curve has a break at $x = 0$. The x -axis and the y -axis are called **asymptotes** to the curve. The curve gets very near but never actually touches the asymptotes.

- **Exponential curves** have a term involving a^x , where a is a constant.

For example, $y = 3^x$

$$y = \left(\frac{1}{2}\right)^x$$

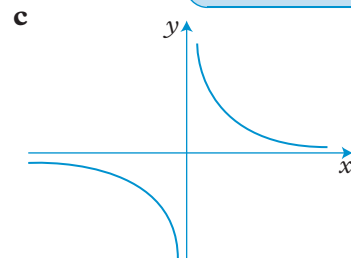
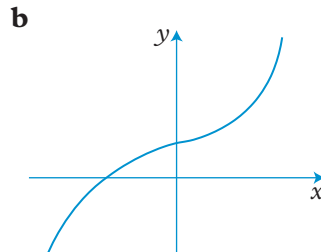
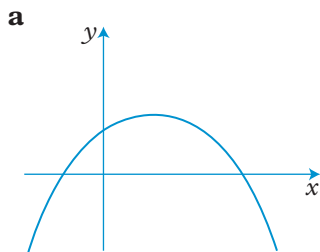
The x -axis is an asymptote to the curve.

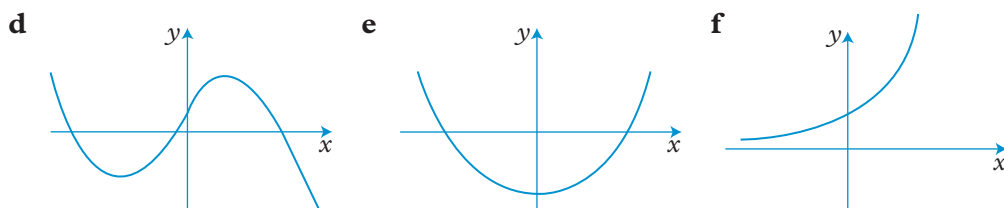


Exercise 14 (M)/(H)

- 1 What sort of curves are these? Give as much information as you can.

For example:
'quadratic,
positive x^2 '





2 Draw the general shape of these curves. (Do not draw accurate graphs.)

a $y = 3x^2 - 7x + 11$

b $y = 2^x$

c $y = \frac{100}{x}$

d $y = 8x - x^2$

e $y = 10x^3 + 7x - 2$

***f** $y = \frac{1}{x^2}$

3 Here are the equations of the six curves in question 1, but not in the correct order.

i $y = \frac{8}{x}$

ii $y = 2x^3 + x + 2$

iii $y = 5 + 3x - x^2$

iv $y = x^2 - 6$

v $y = 5^x$

vi $y = 12 + 11x - 2x^2 - x^3$

Write which equations fit the curves **a** to **f**.

4 Sketch the two curves given and state the number of times the curves intersect.

a $y = x^3,$

b $y = x^2,$

c $y = x^3,$

***d** $y = 3^x$

$y = 10 - x$

$y = 10 - x^2$

$y = x$

$y = x^3$

5.4.2 Plotting curved graphs

EXAMPLE

Draw the graph of the function
 $y = 2x^2 + x - 6$, for $-3 \leq x \leq 3$.

a Make a table of x - and y -values.

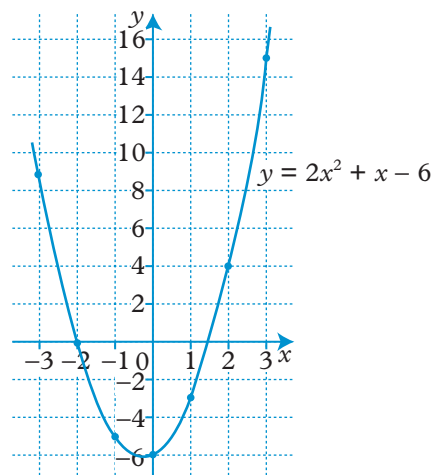
x	-3	-2	-1	0	1	2	3
$2x^2$	18	8	2	0	2	8	18
x	-3	-2	-1	0	1	2	3
-6	-6	-6	-6	-6	-6	-6	-6
y	9	0	-5	-6	-3	4	15

b Draw and label axes using suitable scales.

c Plot the points and draw a smooth curve through them with a pencil.

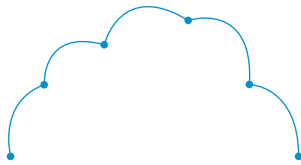
d Check any points which interrupt the smoothness of the curve.

e Label the curve with its equation.

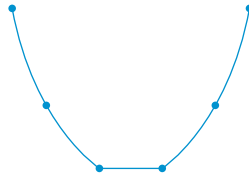


Common errors with graphs

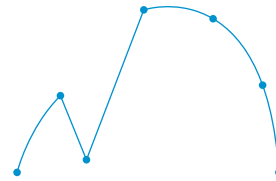
Avoid these mistakes. Your curve should be smooth.



A series of 'mini curves'



Flat bottom



Wrong point

Exercise 15

- 1 a** Copy and complete the table for
 $y = x^2 + 2x$.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x$	-6	-4		0		4	
y	3	0		0			

- b** Draw the graph of $y = x^2 + 2x$ using a scale of 2 cm for 1 unit on the x -axis and 1 cm for 1 unit on the y -axis.

- 2 a** Copy and complete the table for
 $y = x^2 - 3x$.

x	-3	-2	-1	0	1	2	3
x^2	9			0		4	
$-3x$	9			0		-6	
y	18						

- b** Draw the graph of $y = x^2 - 3x$ using the same scales as in question 1.

Draw the graphs of these functions using a scale of 2 cm for 1 unit on the x -axis and 1 cm for 1 unit on the y -axis.

- 3** $y = x^2 + 4x$, for $-3 \leq x \leq 3$
- 4** $y = x^2 + 2$, for $-3 \leq x \leq 3$
- 5** $y = x^2 - 7$, for $-3 \leq x \leq 3$
- 6** $y = x^2 + x - 2$, for $-3 \leq x \leq 3$
- 7** $y = x^2 + 3x - 9$, for $-4 \leq x \leq 3$
- 8** $y = x^2 - 3x - 4$, for $-2 \leq x \leq 4$
- 9** $y = x^2 - 5x + 7$, for $0 \leq x \leq 6$
- 10** $y = 2x^2 - 6x$,
for $-1 \leq x \leq 5$
- 11** $y = 2x^2 + 3x - 6$, for $-4 \leq x \leq 2$
- 12** $y = 3x^2 - 6x + 5$,
for $-1 \leq x \leq 3$
- 13** $y = 2 + x - x^2$, for $-3 \leq x \leq 3$
- 14** $f(x) = 1 - 3x - x^2$, for $-5 \leq x \leq 2$
- 15** $f(x) = 3 + 3x - x^2$, for $-2 \leq x \leq 5$
- 16** $f(x) = 7 - 3x - 2x^2$, for $-3 \leq x \leq 3$
- 17** $f(x) = 6 + x - 2x^2$, for $-3 \leq x \leq 3$
- 18** $y = 8 + 2x - 3x^2$, for $-2 \leq x \leq 3$
- 19** $y = x(x-4)$, for $-1 \leq x \leq 6$
- 20** $y = (x+1)(2x-5)$, for $-3 \leq x \leq 3$.

In question 10,
remember:
 $2x^2 = 2(x^2)$.

Draw the graph of $y = \frac{12}{x} + x - 6$, for $1 \leq x \leq 8$.

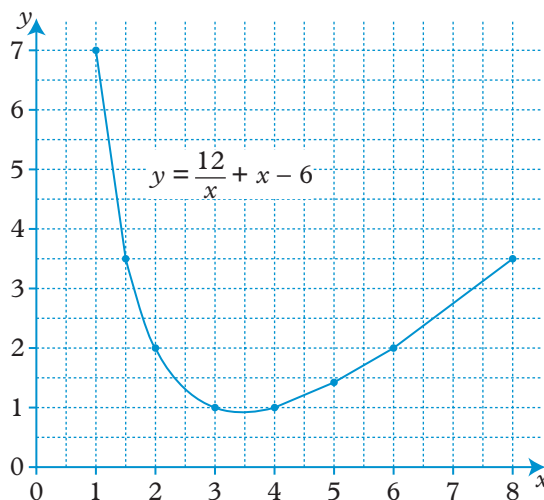
Use the graph to find approximate values for

- a** the minimum value of $\frac{12}{x} + x - 6$
b the value of $\frac{12}{x} + x - 6$, when $x = 2.25$.

Here is the table of values.

x	1	2	3	4	5	6	8
$\frac{12}{x}$	12	6	4	3	2.4	2	1.5
x	1	2	3	4	5	6	8
-6	-6	-6	-6	-6	-6	-6	-6
y	7	2	1	1	1.4	2	3.5

- a** From the graph, the minimum value of $\frac{12}{x} + x - 6$, (that is y) is approximately 0.9.
b At $x = 2.25$, y is approximately 1.6.



Exercise 16 (M)/H

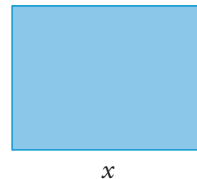
Draw these curves. The scales given are for one unit of x and y .

- $y = \frac{12}{x}$, for $1 \leq x \leq 10$. (Scales: 1 cm for x and y)
- $y = \frac{9}{x}$, for $1 \leq x \leq 10$. (Scales: 1 cm for x and y)
- $y = \frac{12}{x+1}$, for $0 \leq x \leq 8$. (Scales: 2 cm for x , 1 cm for y)
- $y = \frac{8}{x-4}$, for $-4 \leq x \leq 3.5$. (Scales: 2 cm for x , 1 cm for y)
- $y = \frac{x}{x+4}$, for $-3.5 \leq x \leq 4$. (Scales: 2 cm for x and y)
- $y = \frac{x+8}{x+1}$, for $0 \leq x \leq 8$. (Scales: 2 cm for x and y)

- 7 $y = \frac{10}{x} + x$, for $1 \leq x \leq 7$. (Scales: 2 cm for x , 1 cm for y)
- 8 $y = 3^x$, for $-3 \leq x \leq 3$. (Scales: 2 cm for x , $\frac{1}{2}$ cm for y)
- 9 $y = \left(\frac{1}{2}\right)^x$, for $-4 \leq x \leq 4$. (Scales: 2 cm for x , 1 cm for y)
- 10 $y = 5 + 3x - x^2$, for $-2 \leq x \leq 5$. (Scales: 2 cm for x , 1 cm for y)
Find
 a the maximum value of the function $5 + 3x - x^2$
 b the two values of x for which $y = 2$.
- 11 $y = \frac{15}{x} + x - 7$, for $1 \leq x \leq 7$. (Scales: 2 cm for x and y)
From your graph find
 a the minimum value of y
 b the y -value when $x = 5.5$.
- 12 $y = x^3 - 2x^2$, for $0 \leq x \leq 4$. (Scales: 2 cm for x , $\frac{1}{2}$ cm for y)
From your graph find
 a the y -value at $x = 2.5$
 b the x -value at $y = 15$.
- 13 $y = \frac{1}{10}(x^3 + 2x + 20)$, for $-3 \leq x \leq 3$. (Scales: 2 cm for x and y)
From your graph find
 a the x -value where $x^3 + 2x + 20 = 0$
 b the x -value where $y = 3$.
- *14 Draw the graph of
 $y = \frac{x}{x^2 + 1}$, for $-6 \leq x \leq 6$. (Scales: 1 cm for x , 10 cm for y)
- *15 Draw the graph of
 $E = \frac{5000}{x} + 3x$ for $10 \leq x \leq 80$.
 (Scales: 1 cm to 5 units for x and 1 cm to 25 units for E)
 From the graph find
 a the minimum value of E ,
 b the value of x corresponding to this minimum value,
 c the range of values of x for which E is less than 275.

Exercise 17 (M)/(H) (Mixed questions)

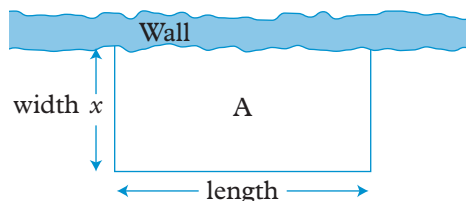
- 1** A rectangle has a perimeter of 14 cm and length x cm. Show that the width of the rectangle is $(7 - x)$ cm and hence that the area, A , of the rectangle is given by the formula, $A = x(7 - x)$.



Draw the graph, plotting x on the horizontal axis with a scale of 2 cm to 1 unit, and A on the vertical axis with a scale of 1 cm to 1 unit. Take x from 0 to 7. From the graph find

- the area of the rectangle when $x = 2.25$ cm
- the dimensions of the rectangle when its area is 9 cm^2
- the maximum area of the rectangle
- the length and width of the rectangle corresponding to the maximum area
- what shape of rectangle has the largest area.

- 2** A farmer has 60 m of wire fencing which he uses to make a rectangular pen for his sheep. He uses a stone wall as one side of the pen so the wire is used for only three sides of the pen.



- If the width of the pen is x m, what is the length (in terms of x)?
 - What is the area, A , of the pen?
 - Draw a graph with area, A , on the vertical axis and the width, x , on the horizontal axis. Take values of x from 0 to 30.
 - What dimensions should the pen have if the farmer wants to enclose the largest possible area?
- 3** A ball is thrown in the air so that t seconds after it is thrown, its height h metres above its starting point is given by the function $h = 25t - 5t^2$. Draw the graph of the function of $0 \leq t \leq 6$, plotting t on the horizontal axis with a scale of 2 cm to 1 second, and h on the vertical axis with a scale of 2 cm for 10 metres. Use the graph to find
- the time when the ball is at its greatest height
 - the greatest height reached by the ball
 - the interval of time during which the ball is at a height of more than 30 m.

4 Consider the equation $y = \frac{1}{x}$.

When $x = \frac{1}{2}$, $y = \frac{1}{\frac{1}{2}} = 2$. When $x = \frac{1}{100}$, $y = \frac{1}{\frac{1}{100}} = 100$.

As the denominator of $\frac{1}{x}$ gets smaller, the answer gets larger.

An 'infinitely small' denominator gives an 'infinitely large' answer.

You write $\frac{1}{x} \rightarrow \infty$ as $x \rightarrow 0$.

Draw the graph of $y = \frac{1}{x}$ for

$x = -4, -3, -2, -1, -0.5, -0.25, 0.25, 0.5, 1, 2, 3, 4$.

(Scales: 2 cm for x and y)

The symbol for infinity is ∞ .

5 Draw the graph of $y = x + \frac{1}{x}$ for

$x = -4, -3, -2, -1, -0.5, -0.25, 0.25, 0.5, 1, 2, 3, 4$.

(Scales: 2 cm for x and y)

6 Draw the graph of $y = \frac{2^x}{x}$, for $-4 \leq x \leq 7$, including

$x = -0.5, x = 0.5$.

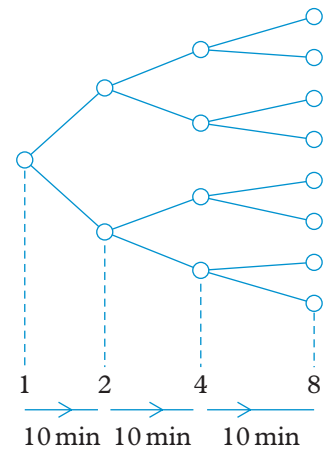
(Scales: 1 cm to 1 unit for x and y)

7 At time $t = 0$, one bacterium is placed in a culture in a laboratory. The number of bacteria doubles every 10 minutes.

a Draw a graph to show the growth of the bacteria from $t = 0$ to $t = 120$ min.

Use a scale of 1 cm to 10 minutes across the page and 1 cm to 100 units up the page.

b Use your graph to estimate the time taken to reach 800 bacteria.



*8 Draw the graph of $y = \frac{x^4}{4^x}$,
for $x = -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$,

$1, 1.5, 2, 2.5, 3, 4, 5, 6, 7$.

(Scales: 2 cm to 1 unit for x , 5 cm to 1 unit for y)

a For what values of x is the gradient of the function zero?

b For what values of x is $y = 0.5$?

5.5 Graphical solution of equations

With an accurately drawn graph you can find an approximate solution for a wide range of equations, many of which are impossible to solve exactly by other methods.

EXAMPLE

Draw the graph of the function $y = 2x^2 - x - 3$ for $-2 \leq x \leq 3$.

Use the graph to find approximate solutions to these equations.

a $2x^2 - x - 3 = 6$ **b** $2x^2 - x = x + 5$

- a** To solve the equation $2x^2 - x - 3 = 6$, draw the line $y = 6$. At the points of intersection (A and B), y simultaneously equals both 6 and $(2x^2 - x - 3)$.

So you can write $2x^2 - x - 3 = 6$

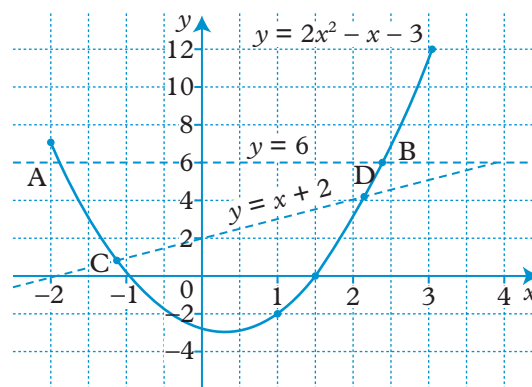
The solutions are the x -values of the points A and B, that is $x = -1.9$ and $x = 2.4$ approx.

- b** To solve the equation $2x^2 - x = x + 5$, rearrange the equation to obtain the function $(2x^2 - x - 3)$ on the left-hand side. In this case, subtract 3 from both sides.

$$2x^2 - x - 3 = x + 5 - 3$$

$$2x^2 - x - 3 = x + 2$$

If you now draw the line $y = x + 2$, the solutions of the equation are given by the x -values of C and D, the points of intersection, that is, $x = -1.2$ and $x = 2.2$ approx.



EXAMPLE

Assuming that the graph of $y = x^2 - 3x + 1$ has been drawn, find the equation of the line which you should draw to solve the equation $x^2 - 4x + 3 = 0$.

Rearrange $x^2 - 4x + 3 = 0$ in order to obtain $(x^2 - 3x + 1)$ on the left-hand side.

$$x^2 - 4x + 3 = 0$$

add x $x^2 - 3x + 3 = x$

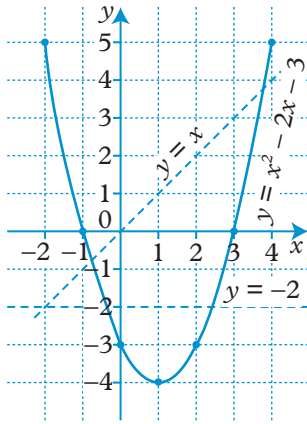
subtract 2 $x^2 - 3x + 1 = x - 2$

Therefore draw the line $y = x - 2$ to solve the equation.

Remember:
'Rearrange the
equation to be
solved.'

Exercise 18 H

- 1 In the diagram, the graphs of $y = x^2 - 2x - 3$, $y = -2$ and $y = x$ have been drawn.



Use the graphs to find approximate solutions to these equations.

a $x^2 - 2x - 3 = -2$

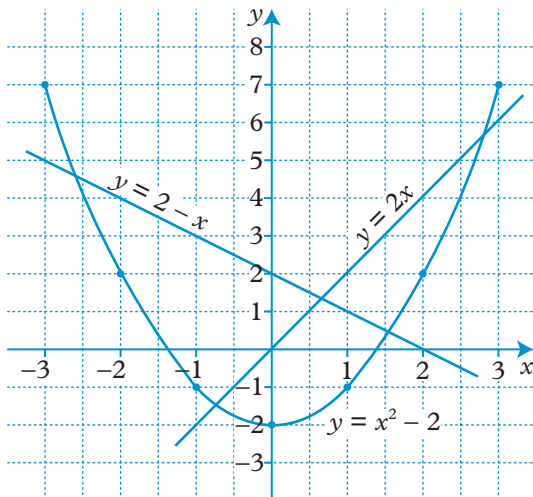
b $x^2 - 2x - 3 = x$

c $x^2 - 2x - 3 = 0$

d $x^2 - 2x + 1 = 0$

Reminder: Only the x -values are required.

- 2 The graphs of $y = x^2 - 2$, $y = 2x$ and $y = 2 - x$ are shown.



Use the graphs to solve these equations.

a $x^2 - 2 = 2 - x$

b $x^2 - 2 = 2x$

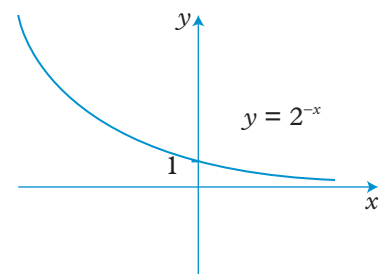
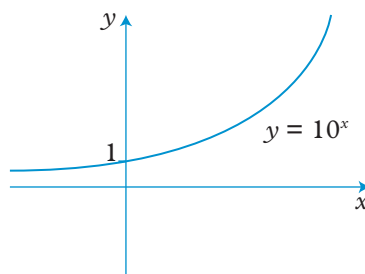
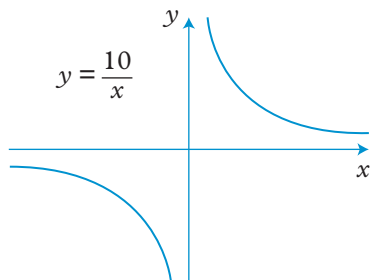
c $x^2 - 2 = 2$

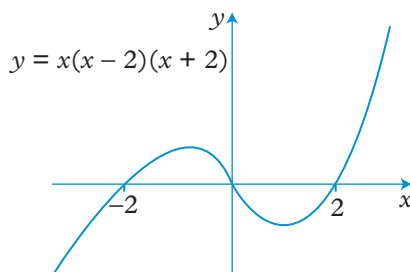
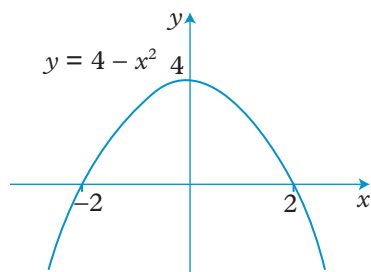
For questions **10** to **13**, use scales of 2 cm to 1 unit for x and 1 cm to 1 unit for y .

- 10 a** Complete the table and then draw the graph of $y = x^2 + 3x - 1$.

x	-5	-4	-3	-2	-1	0	1	2
y	9	3		-3	-3			9

- b** By drawing other graphs, solve these equations.
- $x^2 + 3x - 1 = 0$
 - $x^2 + 3x = 7$
 - $x^2 + 3x - 3 = x$
- 11** Draw the graph of $y = x^2 - 2x + 2$ for $-2 \leq x \leq 4$. By drawing other graphs, solve these equations.
- $x^2 - 2x + 2 = 8$
 - $x^2 - 2x + 2 = 5 - x$
 - $x^2 - 2x - 5 = 0$
- 12** Draw the graph of $y = x^2 - 7x$ for $0 \leq x \leq 7$. Draw suitable straight lines to solve these equations.
- $x^2 - 7x + 9 = 0$
 - $x^2 - 5x + 1 = 0$
- 13** Draw the graph of $y = 2x^2 + 3x - 9$ for $-3 \leq x \leq 2$. Draw suitable straight lines to find approximate solutions of these equations.
- $2x^2 + 3x - 4 = 0$
 - $2x^2 + 2x - 9 = 1$
- 14** Draw the graph of $y = \frac{18}{x}$ for $1 \leq x \leq 10$, using scales of 1 cm to one unit on both axes. Use the graph to solve these equations approximately.
- $\frac{18}{x} = x + 2$
 - $\frac{18}{x} + x = 10$
 - $x^2 = 18$
- 15** Here are five sketch graphs (see overleaf for the other two).





Use the graphs to make your own sketch graphs to find the **number of solutions** of each of these equations.

a $\frac{10}{x} = 10^x$

b $4 - x^2 = 10^x$

c $x(x-2)(x+2) = \frac{10}{x}$

d $2^{-x} = 10^x$

e $4 - x^2 = 2^{-x}$

f $x(x-2)(x+2) = 0$

- 16** Draw the graph of $y = \frac{1}{2}x^2 - 6$ for $-4 \leq x \leq 4$, taking 2 cm to 1 unit on each axis.

a Use your graph to solve approximately the equation

$$\frac{1}{2}x^2 - 6 = 1.$$

b Using tables or a calculator confirm that your solutions are approximately $\pm\sqrt{14}$ and explain why this is so.

c Use your graph to find the two square roots of 8.

- 17** Draw the graph of $y = 3x^2 - x^3$ for $-2 \leq x \leq 3$. Use your graph to find the range of values of k for which the equation $3x^2 - x^3 = k$ has three solutions.

- *18** Draw the graph of $y = 6 - 2x - \frac{1}{2}x^3$ for $x = \pm 2, \pm 1\frac{1}{2}, \pm 1, \pm \frac{1}{2}, 0$.

Take 4 cm to 1 unit for x and 1 cm to 1 unit for y .

Use your graph to find approximate solutions of these equations.

a $\frac{1}{2}x^3 + 2x - 6 = 0$

b $x - \frac{1}{2}x^3 = 0$

Confirm that two of the solutions to the equation in part **b** are $\pm\sqrt{2}$ and explain why this is so.

- *19** Draw the graph of $y = 2^x$ for $-4 \leq x \leq 4$, taking 2 cm to one unit for x and 1 cm to one unit for y . Find approximate solutions to these equations.

a $2^x = 6$

b $2^x = 3x$

c $x(2^x) = 1$

Find also the approximate value of $2^{2.5}$.

Test yourself

- 1 Make p the subject of the formula

$$5p + 2q = 6 - q$$

Simplify your answer as much as possible.

- 2 Make x the subject of this formula.

$$4(2x - y) = 2y + 5$$

(OCR, 2003)

- 3 Solve the inequality $7 + n > 13 - 2n$

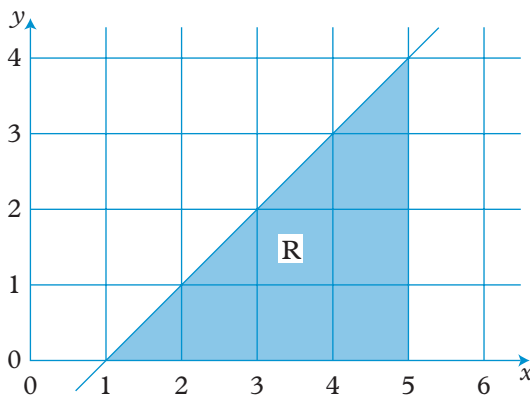
- 4 a Solve the inequality $3x - 1 \leq 8$

b Write down the inequality shown by the following diagram.



- c Write down all the integers that satisfy both inequalities shown in parts (a) and (b).

- 5 Write down three inequalities which together describe the shaded region R .



- 6 y is inversely proportional to x^2 and $y = 0.8$ when $x = 2$.

a Find an equation connecting y and x .

b Find the value of y when $x = 3$.

(OCR, 2003)

- 7 y is directly proportional to the square of x .

$$y = 72 \quad \text{when} \quad x = 2$$

a Express y in terms of x .

b Work out the value of y when $x = \frac{1}{2}$.

- 8** b is inversely proportional to the square of a .
 When $b = 6$, $a = 2$.
 Calculate the value of b when $a = 6$.

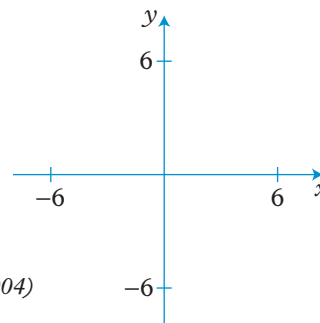
- 9 a** Complete the table for $y = x^2 - 3x + 1$

x	-2	-1	0	1	2	3	4
y	11		1	-1		1	5

- b** Draw the graph of $y = x^2 - 3x + 1$
c Use your graph to find an estimate for the minimum value of y .
d Use a graphical method to find estimates of the solutions to the equation $x^2 - 3x + 1 = 2x - 4$

(Edexcel, 2003)

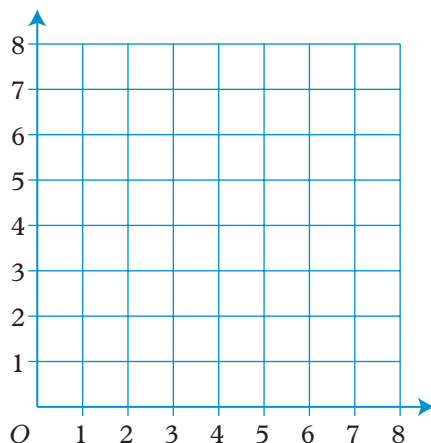
- 10 a** Draw the graph of $y = \frac{6}{x}$
 for values of x from -6 to $+6$.
b On the same axes draw the graph of $y = x$.
c Show why you can use these graphs to solve $x^2 = 6$.
d Use these graphs to solve $x^2 = 6$.



(OCR, 2004)

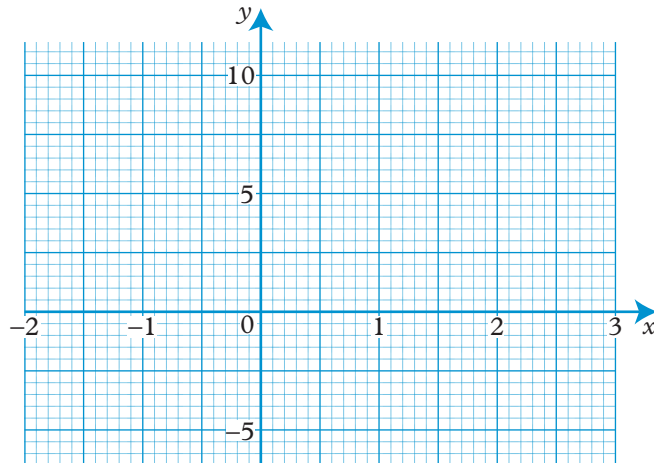
- 11** The region **R** satisfies the inequalities
 $x \geq 2, y \geq 1, x + y \leq 6$

On a copy of the grid below, draw straight lines and use shading to show the region **R**.



(Edexcel, 2008)

- 12 a Draw the graph of $y = 2^x$ for values of x from -2 to 3 .



- b Use your graph to solve $2^x = 6$.

(OCR, 2004)

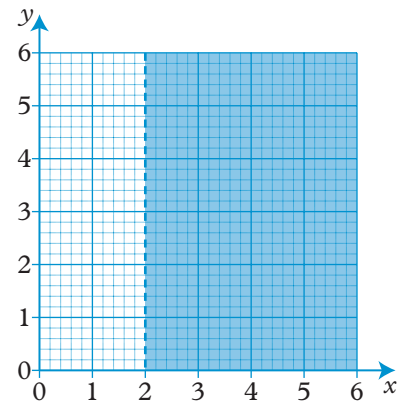
- 13 a Which inequality is shown shaded on the grid?
Choose the correct answer.

$$y > 2$$

$$x > 2$$

$$y \geq 2$$

$$x \geq 2$$



- b On a copy of the grid draw lines to find the region satisfied by the three inequalities

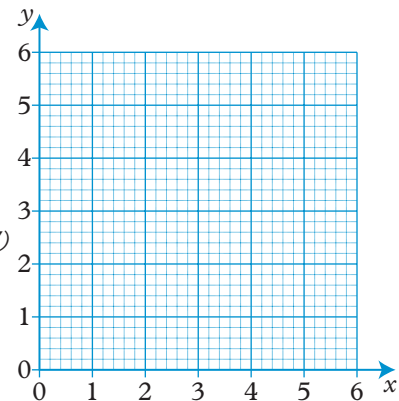
$$y > 2$$

$$y < x + 1$$

$$x + y < 5$$

Label the region with the letter R.

(AQA, 2007)



- 14 You are given that y is inversely proportional to x , and that $y = 9$ when $x = 4$.

a i Find an equation connecting y and x .

ii Find y when $x = \frac{1}{2}$

b Find y when $x = y$.

(OCR, 2008)

Functional task 4

Mr Roe's horses

Mr Roe has a farm and needs to work out the cost of keeping his horses. The main costs are feed, vet's bills and keeping the horse healthy. He has volunteers to clean the stables and look after the horses in return for being allowed to ride the horses every week.



Feed

The horses are given a mixture of hay and oats. The hay is free from the farm but a 40 kg bag of oats costs £9.50. Big horses needs more feed than small horses as shown in the **feed chart**.

Horse measurement

By tradition the height of a horse is given in hands and inches. There are four inches in one hand. The formula for converting hands and inches into centimetres is:

$$c = \frac{5(4h + i)}{2}$$

Where c = number of centimetres
 h = number of hands
 i = number of inches

Feed chart

Height of horse (cm)	Feed per day (kg)
135	4.2
140	4.5
145	4.8
150	5.2
155	5.6
160	6.0
165	6.5
170	7.1

Mr Roe's horses

Number	Height (hands, inches)	Age (years)
1	17 hands	9
2	14 hands 2 inches	7
3	15 hands	13
4	13 hands 2 inches	6
5	16 hands 2 inches	14
6	14 hands	9
7	15 hands	15
8	13 hands 2 inches	5

Vet's bills

Each year horses need injections against Influenza and Tetanus. Illness or injury can be very expensive so Mr Roe takes out insurance to cover the cost of fees. This cost is £30 per month per horse.

Keeping the horse healthy

A farrier is needed to care for the horses's feet as the grow all the time (like human fingernails). A farrier is needed eight times a year and it costs £20 per horse per visit. Each horse also needs to be wormed seven times a year at a cost of £15 per horse per visit.

Task 1

Work out the height of each horse in centimetres.

Task 2

Work out the total cost for the eight horses, including feed, vet's bills and keeping the horse healthy.

