S3 Triangles and quadrilaterals

This unit will show you how to:

- Use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes.
- Begin to identify and use angle, side and symmetry properties of triangles and quadrilaterals and use them to solve problems.
- Use 2-D representations, including plans and elevations, to visualise 3-D shapes and deduce some of their properties.
- Use a ruler and protractor to:
  - measure and draw lines and angles
  - construct a triangle.
- Solve word problems and investigate in the context of shape and space.
- Explain and justify methods and conclusions.
- Begin to generalise and to understand the significance of a counter-example.

Buildings use properties of shapes.

Before you start

You should know how to …

1 Recognise these special quadrilaterals:

- kite
- parallelogram
- rectangle
- rhombus
- square
- trapezium

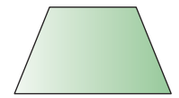
2 Estimate angles in degrees.

3 Collect like terms:
$a + a + a = 3a$

Check in

1 What are each of these quadrilaterals?

a



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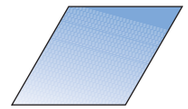
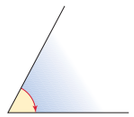


2 **Estimate** the size of these angles.

a



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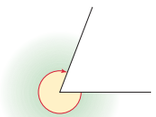


3 Simplify these expressions:

a $a + a + 2a$ **b** $b + 2b + 3b$

c $3 + c - c$ **d** $d + 2d + 3e - e$

S3.1 Finding angles

This spread will show you how to:

- Know the sum of angles at a point and on a straight line.
- Use a protractor to measure and draw angles, including reflex angles, to the nearest degree.

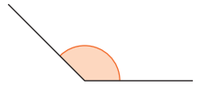
KEYWORDS

Angle, Vertex, Protractor, Measure, Angles at a point, Angles on a straight line

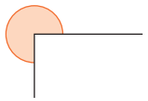
Remember:

An angle smaller than 90° is an acute angle.

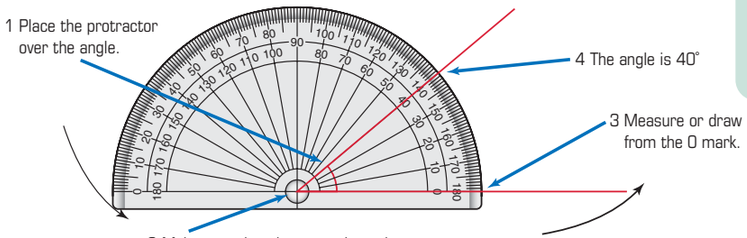
An angle between 90° and 180° is an obtuse angle.



An angle between 180° and 360° is a reflex angle.



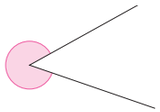
You measure and draw angles in degrees using a protractor:



Note: There are two scales – to make sure you use the right one you should estimate the size of the angle first.

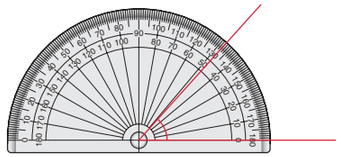
example

Measure this angle in degrees:



Hint: To draw a reflex angle, first subtract the angle from 360° then draw the associated smaller angle.

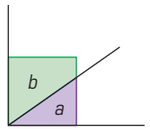
- Estimate the size of the angle: it is more than $\frac{3}{4}$ of a turn which is 270° – it is roughly 300°.
- Then measure the acute angle:
- Then subtract this from 360°:



360° – 48° = 312°

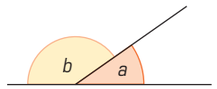
You should know these angle facts:

There are 90° on a corner.



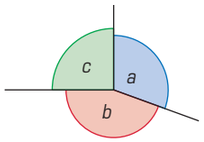
$a + b = 90°$

There are 180° on a straight line.



$a + b = 180°$

There are 360° on a full turn.

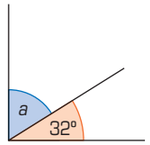


$a + b + c = 360°$

Exercise S3.1

1 Find the missing angles in these diagrams by calculating.

a



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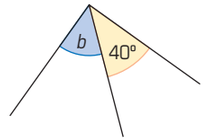

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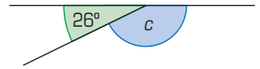

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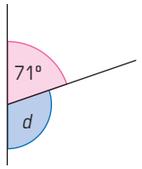

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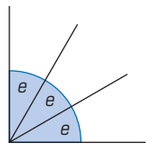

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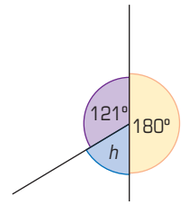

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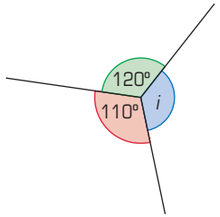

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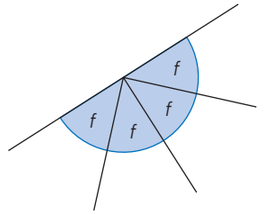

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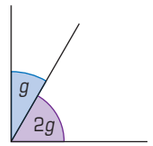
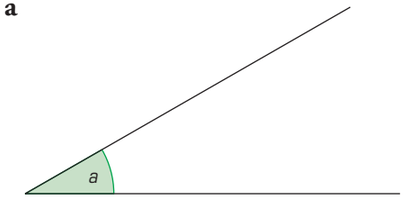


2 Measure these angles:

a



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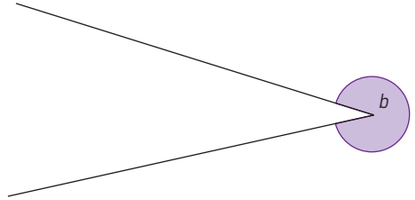

3 Draw an angle ABC = 55°.
What is the associated reflex angle?

4 Draw an angle of 300°.

5 Draw a line AB = 5 cm.
Draw an angle ABC = 50° on the same diagram.
Draw an angle BAC = 70° on the same diagram.
Mark the point C where the lines meet.
Measure the angle BCA.

6

- Sketch a quadrilateral. Use a ruler to make sure the sides are straight.
- Cut the quadrilateral out.
- Tear off the corners. Put them together carefully.
- What do you notice?

S3.2 Angles in shapes

This spread will show you how to:
- Know the sum of angles in a triangle.
- Recognise angles on a straight line.
- Visualise and sketch 2-D shapes.

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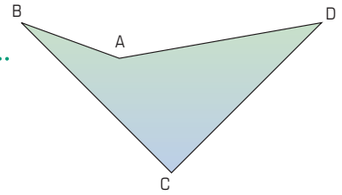
KEYWORDS

Quadrilateral	Diagonal
Concave	Shape
Convex	2-D
Triangle	

► The angles inside a shape are called **interior** angles.

example

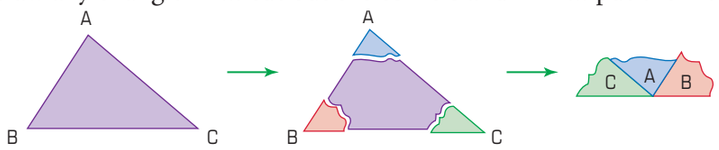
Measure the interior angles of this quadrilateral:



Interior angles are inside the shape.

Using a protractor the angles are:
A = 210°, B = 25°, C = 90° and D = 35°

If you ...
... draw any triangle ... cut out the corners and ... put them together



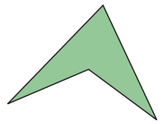
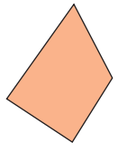
the angles should always make a straight line, so long as you cut them out carefully.

► The angles in a triangle add to 180°. They make a straight line.

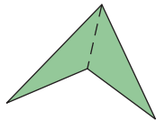
You can use this fact to find the angle sum for other shapes.

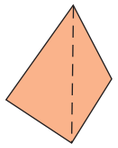
Quadrilaterals

Draw any quadrilateral:

concave  or convex 

Split it into triangles by drawing a diagonal:





Cut out the angles and fit them together to make a full turn, 360°.

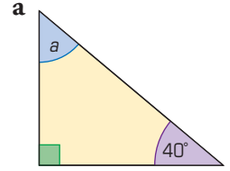
Any quadrilateral can be split into two triangles.
A quadrilateral has 2 × the angle sum of a triangle.

► The interior angles of a quadrilateral add up to 360°.

Exercise S3.2

1 Find the missing angles in these triangles

a



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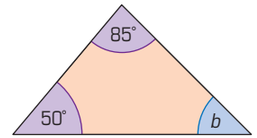

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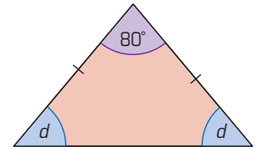

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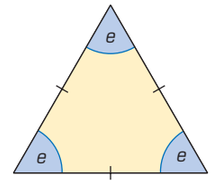

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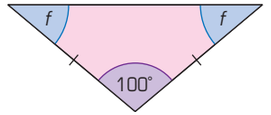

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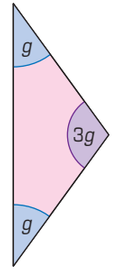

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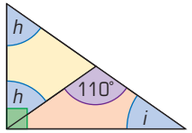

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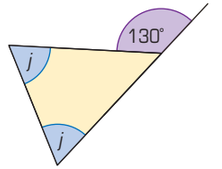
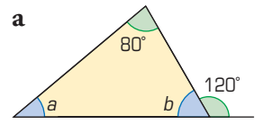


2 State whether these triangles are possible or impossible:
- **a** a triangle with an obtuse angle
- **b** a triangle with an reflex angle
- **c** a triangle with two obtuse angles

3 Is this statement sometimes true, always true or never true?
Half a reflex angle is an obtuse angle.

4 Calculate the missing angles. Explain each of your answers.

a



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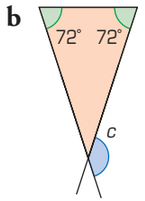

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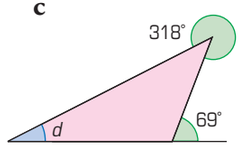

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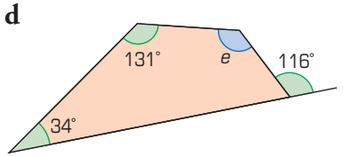

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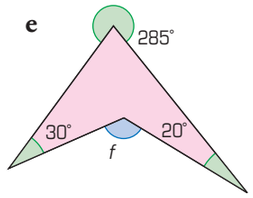

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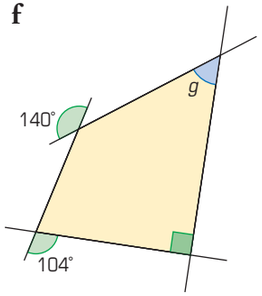

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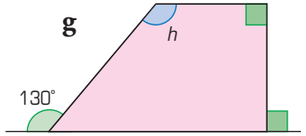

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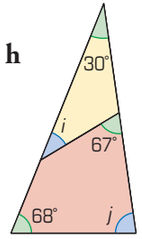

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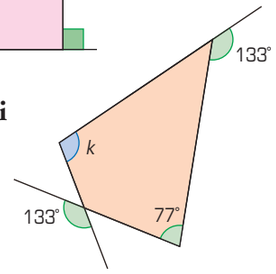


S3.3 Triangles and quadrilaterals

This spread will show you how to:
- Identify parallel and perpendicular lines.
- Visualise and sketch 2-D shapes.
- Identify and use the geometric properties of triangles and quadrilaterals to solve problems.

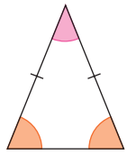
KEYWORDS

Parallel	Triangle
Perpendicular	Quadrilateral
Adjacent	Equal
Opposite	

You can describe a shape by its angles and sides.
Some common shapes have special names and properties that you need to know.

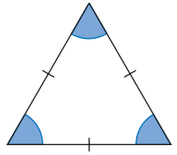
Triangles

Any triangle with exactly two equal angles is **isosceles**.



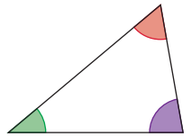
Two equal angles means two equal sides.

If a triangle has all three angles equal it is **equilateral**.



Three equal angles means three equal sides.

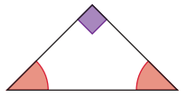
If all the angles are different the triangle is **scalene**.



No equal angles means no equal sides.

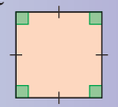
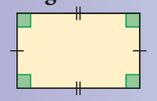
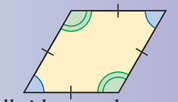
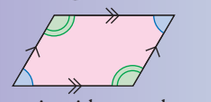
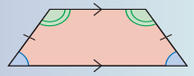
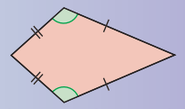
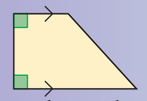
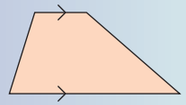
example

Describe this triangle. Be as specific as possible.



The triangle has two equal angles so it has two equal sides – it is isosceles.
It also has a right angle so you can say it is a **right-angled isosceles** triangle.

Quadrilaterals

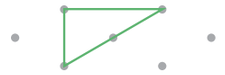
All angles equal	**Square**  all sides equal	**Rectangle**  opposite sides equal	
Two pairs of angles equal	**Rhombus**  all sides equal	**Parallelogram**  opposite sides equal	**Isosceles trapezium**  one pair of opposite sides equal
One pair of angles equal	**Kite**  adjacent sides equal	**Right trapezium**  may have sides equal	This trapezium has no sides or angles equal: 

Exercise S3.3

You will need some 7-dot isometric grids.

1 This is a 7-dot isometric grid:

On 7-dot isometric grids, draw different triangles and quadrilaterals.
Name each of the shapes you draw.
For example:



right-angled triangle

Shapes that are the same but in different orientations only count once.
For example, these are drawings of the same shape:

They are both equilateral triangles.

2 Describe the shapes you drew in question 1.
Use these terms as appropriate:
- opposite sides
- adjacent sides
- symmetrical
- acute
- reflex
- parallel
- perpendicular
- regular
- obtuse
- right angle

3 Name any triangles or quadrilaterals that you cannot draw on the isometric grid.
Try to explain why they cannot be drawn.

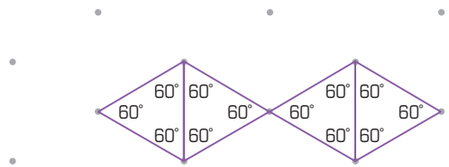
4 Sort the shapes you drew in question 1 into this two-way table:

You put a rectangle here because it has two pairs of parallel sides and two pairs of perpendicular sides.

	At least one pair of parallel sides	No parallel sides
At least one pair of perpendicular sides	Rectangle	
No perpendicular sides		

Are there any empty spaces in your table?
Give reasons.

5 On an isometric grid the lines intersect to form equilateral triangles:



Use this fact to work out the size of the interior angles for each of the shapes drawn in question 1.

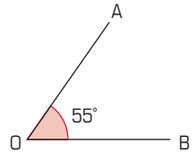
S3.4 Constructing triangles

This spread will show you how to:
- Use accurately the labelling conventions for lines, angles and shapes.
- Use a ruler and a protractor to measure and draw lines to the nearest millimetre and angles to the nearest degree and construct triangles.

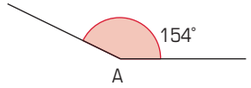
KEYWORDS

Construct	Protractor
Sketch	Measure
Ruler	

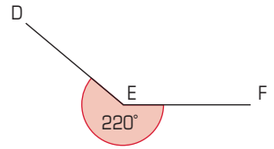
There are lots of shorthand ways of writing 'angle'. These are the most common:



AÔB is 55°



∠A is 154°



∠DEF is 220°

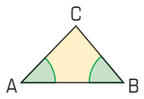
You can **construct** a triangle using a protractor and a ruler.

Construct means draw accurately. The examples are drawn to scale, 1 cm : 4 cm.

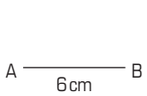
example

Draw triangle ABC where AB = 6 cm, B = 50° and A = 46°

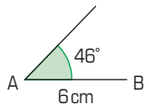
First sketch the triangle.



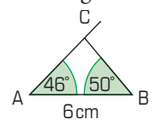
Draw the base – label it.



Draw angle A at point A.



Draw angle B at point B.



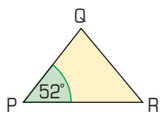
The joined up triangle is ABC.

You can check your drawing is correct – just measure angle C.
A + B = 96° so C should be 180° – 96° = 84°

example

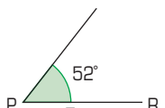
Draw triangle PQR where PQ = 5.5 cm, PR = 7 cm and P = 52°

First sketch the triangle.

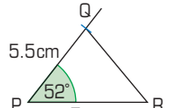


Make the longest side the base – label it.

Draw angle P at point P.



Mark Q 5.5 cm from P.



Draw in the last side by joining PR.

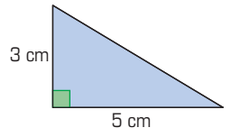
You can construct a triangle when you know:
- two angles and one side (ASA)
- two sides and the included angle (SAS).

You need to know these facts to ensure your triangle is **unique** – there are no other possible triangles that you can draw.

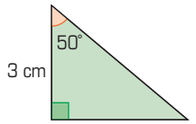
Exercise S3.4

1 Construct these triangles:

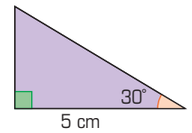
a



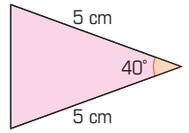
b



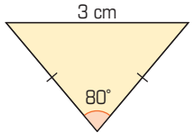
c



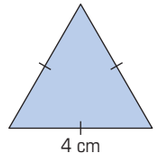
d



e



f

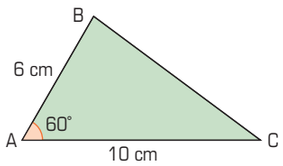


2 Construct these triangles.
Remember to make a sketch of your triangle first.
- **a** ABC, where AC = 7 cm, BC = 6 cm, C = 72°
- **b** PQR, where PR = 7.3 cm, P = 52°, R = 43°
- **c** LMN, where LM = 8.5 cm, L = 31°, M = 127°
- **d** GHI, where HI = 6.8 cm, GI = 4.8 cm, I = 134°
- **e** ABC, where A = 64°, B = 40°, BC = 6.3 cm
- **f** ABC, where AB = 5.6 cm, B = 24°, C = 53°

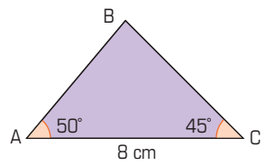
Hint: Do you have all the information you need?

3 Construct these triangles:

a



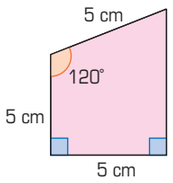
b



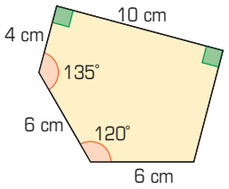
- **c** Write down all the information you were given for your shapes.
- **d** Measure and write down any other information you can.

4 Construct these shapes.

a



b



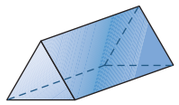
S3.5 2-D representations of 3-D shapes

This spread will show you how to:
- Use 2-D representations and oral descriptions to visualise 3-D shapes and deduce some of their properties.

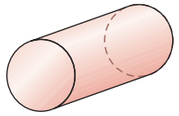
KEYWORDS
3-D, Faces, Edges, Vertices, Base

You should be able to name different 3-D shapes.

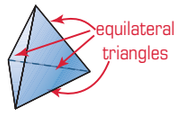
- **Prisms** have a constant cross-section.
- **Pyramids** taper to a point.



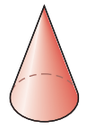
Triangular prism



Circular prism or cylinder

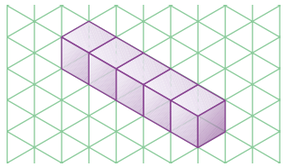


Triangular-based pyramid – when all the faces are equilateral triangles it is a tetrahedron



Circular-based pyramid or cone

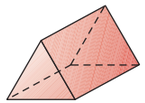
3-D shapes are all around us, but the paper you draw them on is flat!
Isometric paper makes it easier to draw 3-D shapes.



The uprights should be upright!

It is not easy to describe a 3-D shape to someone so they can draw or make it.
Here are some terms you may find useful:

Faces, edges and vertices

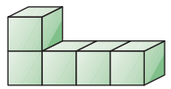


The shape is a **triangular prism** because the triangular face is constant along the whole length.

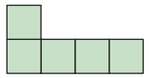
This prism has: 5 faces, 9 edges and 6 vertices.

Views

This shape is made up of five multilink cubes.
It has: 8 faces, 18 edges and 12 vertices.

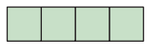


The **front** looks like an L shape.



The **side** looks like a rectangle.

From **above** you see a rectangle.

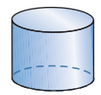


If your information is specific, it is more likely someone else can draw your shape!
It can help if you try to sketch the shape on paper first.

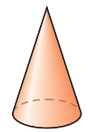
Exercise S3.5

1 Match each solid with its name:
Choose from:
- cylinder
- cone
- square-based pyramid
- cube
- cuboid
- tetrahedron
- triangular-based pyramid
- triangular prism.

a

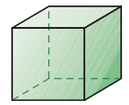


b

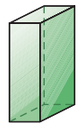


c

d

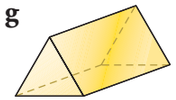


e



f

g



h

2 For each object, name the solid that closely matches its shape:
- pound coin
- cornflake packet
- can of beans
- dice.

3 Name three other objects that are in the shape of:
a a cube **b** a cone
c a cuboid **d** a cylinder

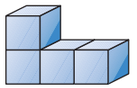
4 Platonic solids have all their faces identical and regular.
Which solids in question 1 are Platonic?

5 Copy and complete the table for solids c to h in question 1.

Name of solid	No. of faces	No. of vertices	No. of edges

Try to find a formula to link:
- the number of faces f
- the number of edges e
- the number of vertices v

6 Use four multilink cubes.
Make as many different shapes as you can.
For example:



For each different shape you make:
a Draw what it looks like from above, the front and the side.
b Draw the shape on isometric paper.
c Describe the shape in words for someone else to construct.

S3 Summary

You should know how to ...

1 Know the sum of angles at a point, on a straight line and in a triangle.

2 Identify parallel and perpendicular lines.

3 Construct lines, angles and shapes using a ruler and a protractor.

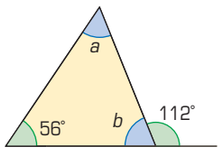
4 Explain and justify methods and conclusions.

5 Solve word problems and investigate in the context of shape and space.

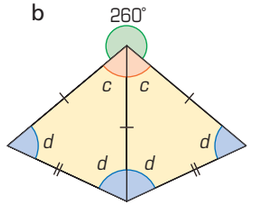
Check out

1 Find the missing angles:

a

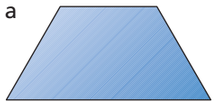


b

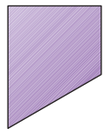


2 Copy and mark all the parallel and perpendicular sides on these shapes:

a



b

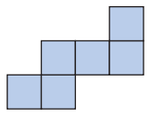


3 a Construct the triangle ABC where AB = 6.8 cm, AC = 6.2 cm, BAC= 70°

b Construct the triangle where AB = 8 cm, BAC= 50°, ABC= 45°

4 Is it possible to have a triangle with interior angles 41°, 62° and 80°?
Explain your answer.

5 This is the net of a closed cube.



Copy the net onto squared paper.
Mark the pairs of edges that fit together to make the cube.