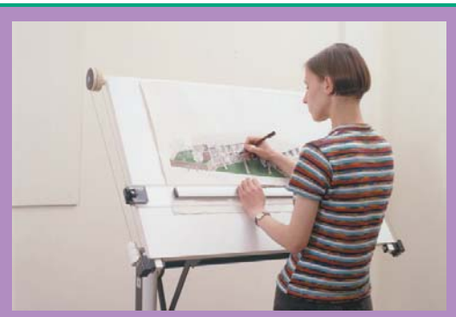
S3 Triangles and quadrilaterals

This unit will show you how to:

- Solve geometrical problems using angle properties of triangles and special quadrilaterals.
- Use straight edge and compasses to construct:
  - a triangle
  - the mid-point and perpendicular bisector of a line segment
  - the bisector of an angle.
- Find simple loci to produce paths.
- Know and use geometric properties of cuboids and shapes made from cuboids.
- Begin to use plans and elevations.
- Solve problems and investigate in the context of shape and space.
- Represent problems and interpret solutions in geometric form.



Architects use plans and views to help them construct buildings.

Before you start

You should know how to ...

1 Calculate angles on:

a a straight line

b in a triangle

c around a point.

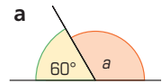
2 Know the names of polygons.

3 Recognise and sketch solids.

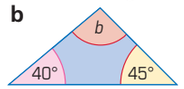
Check in

1 Calculate the unknown angles.

a



b



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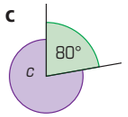
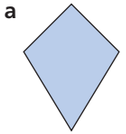


2 Name these shapes.

a



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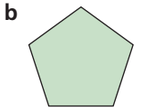

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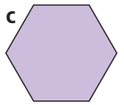
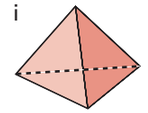
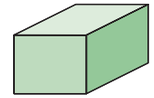


3 a Name these solids.

i



ii



b Sketch **i** a cylinder **ii** a cone **iii** a triangular prism

S3.1 Calculating angles

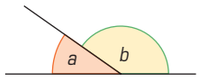
This spread will show you how to:

- Calculate angles in triangles and special quadrilaterals.

KEYWORDS

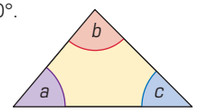
Angle	Triangle
Equilateral	Quadrilateral
Axis of symmetry	
Vertically opposite	

Angles on a straight line add up to 180°.



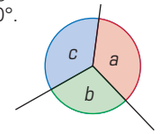
$a + b = 180°$

Angles in a triangle add up to 180°.



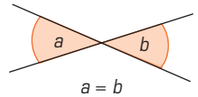
$a + b + c = 180°$

Angles in a full turn add up to 360°.



$a + b + c = 360°$

Vertically opposite angles are equal.

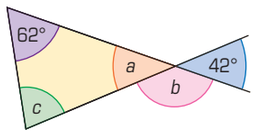


$a = b$

You can use these rules to find unknown angles.

example

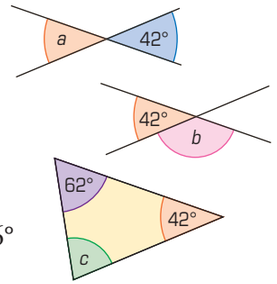
Find angles a, b and c.



$a = 42°$
(vertically opposite angles)

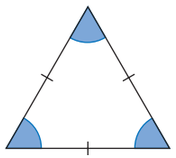
$b = 180° - 42° = 138°$
(angles in a straight line)

$c = 180° - 42° - 62° = 76°$
(angles in a triangle)



Equal lengths can often give you information about missing angles.

example



a What is the name of this triangle? **b** What is the size of each angle?

a The marks show that the lengths are equal.
It is an equilateral triangle.

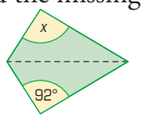
b The angles of an equilateral triangle are equal.
Angles in a triangle add up to 180°.
So each angle = 180° ÷ 3 = 60°

▶ The angles of an equilateral triangle are all 60°.

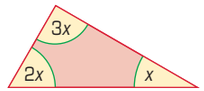
example

Find the missing angles in these shapes.

a



b



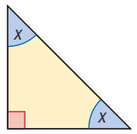
a The shape has an axis of symmetry.
This means that the lengths and angles above the line are the same as those below the line.
So $x = 92°$.

b $x + 2x + 3x = 180°$
$6x = 180°$
$x = 30°$
So the angles are 30°, 60° and 90°.

Exercise S3.1

Find the missing angles in these diagrams.

1



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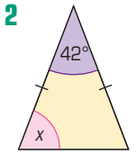

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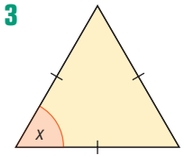

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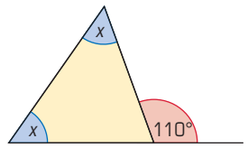

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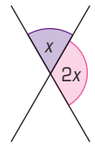

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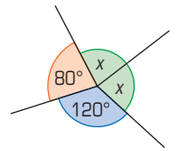

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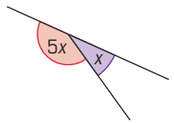

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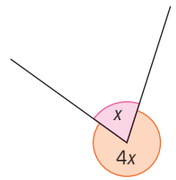

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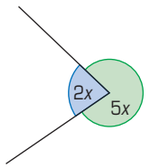

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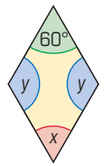

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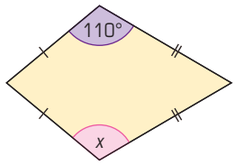

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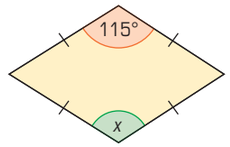

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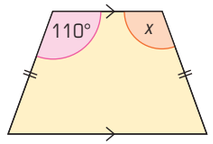

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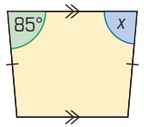

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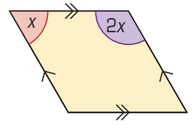


S3.2 Angles in shapes

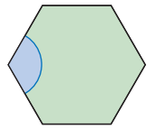
This spread will show you how to:

- Understand a proof that the sum of the angles in a quadrilateral are 360°.

KEYWORDS

Exterior	Protractor
Hexagon	Quadrilateral
Interior	Triangle

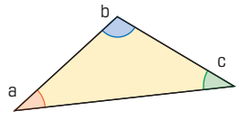
You find angles both inside and outside shapes.
Here is a hexagon:



A hexagon is a shape with six sides.

The angle marked is an **interior** angle of the hexagon. It is inside the shape.

- The angles inside a shape are called **interior** angles.
- The sum of the interior angles of a triangle is 180°.

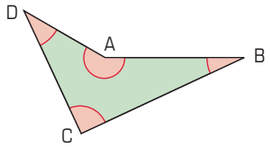


a + b + c = 180°

example

a Estimate and then measure accurately the interior angles of this quadrilateral.
b What do the interior angles add up to?

a **Estimate**: A is between 180° and 270° but closer to 180°, say 200°.
B is between 0° and 90° but closer to 0°, say 20°.
C looks like a right angle, so about 90°.
D is about halfway between 0° and 90°, say 45°.

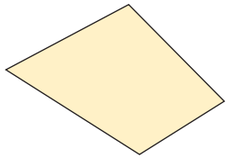


Measure: Using a protractor the angles are 210°, 25°, 90° and 35°.

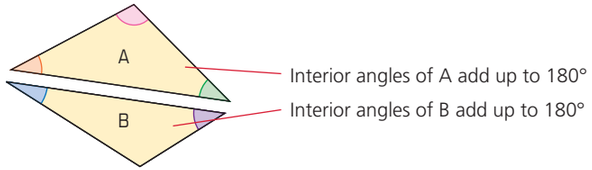
b 210° + 25° + 90° + 35° = 360°

You can identify properties of quadrilaterals from your knowledge of triangles.

Draw a quadrilateral …



… split it into triangles

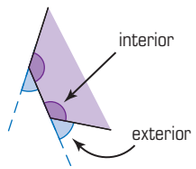


Any quadrilateral has 2 × the angle sum of a triangle.

- The interior angles of a quadrilateral add up to 360°.

If you extend the lines of a shape you can make angles outside the shape.

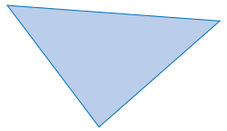
- The angles outside a shape are called **exterior** angles.
- Exterior angle + interior angle = 180°.



Exercise S3.2

1 Draw polygons with these numbers of sides:
a 3 **b** 4 **c** 5 **d** 6 **e** 7 **f** 8
Estimate, and then measure accurately each of their interior angles .
You can use the shapes supplied if you prefer.

a



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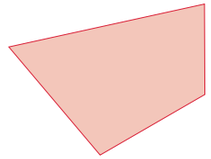

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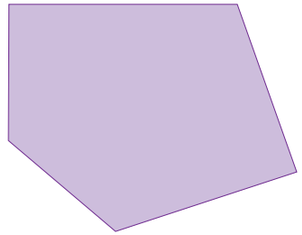
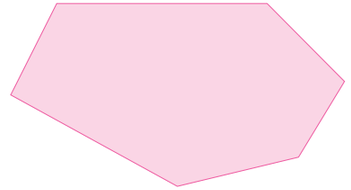
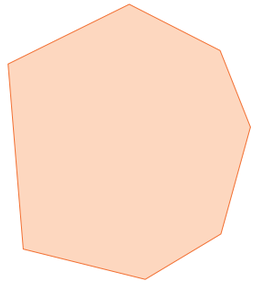


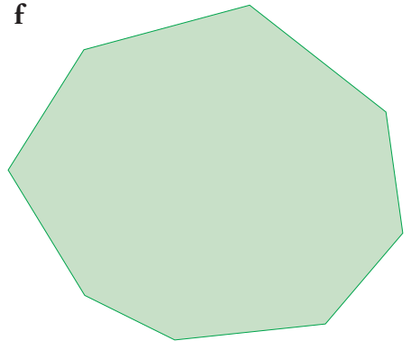
d



e



f



2 Copy and complete this table to show your results.

Number of Sides	Estimates of angles	Accurate angle sizes	Interior angle total	Exterior angle total
3				
4				
5				
6				
7				
8				

3 Extend the sides of your polygons and measure the exterior angles. Complete the last column of the table.

4 What do you notice about your results?

S3.3 Constructing triangles

This spread will show you how to:
- Construct triangles using a ruler, protractor and compasses.

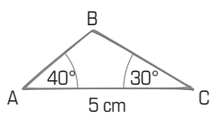
KEYWORDS
Arc, Compasses, Construct, Intersection

As long as you have certain information you can construct any triangle.

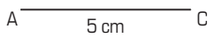
1. Two angles and the included side (ASA)
You need a ruler and protractor.

Construct the triangle ABC where AC = 5 cm, BAC = 40° and BCA = 30°.

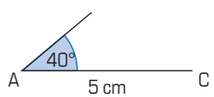
First sketch the triangle



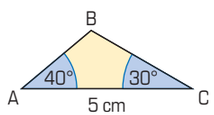
Draw the base AC with a ruler



Draw angle BAC at A



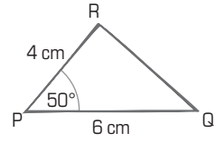
Draw angle BCA at C



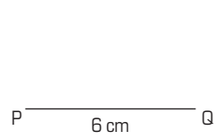
2. Two sides and the included angle (SAS)
You need a ruler and protractor.

Construct the triangle PQR, where PQ = 6 cm, ∠P = 50° and PR = 4 cm.

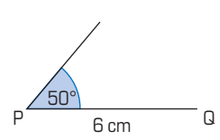
First sketch the triangle



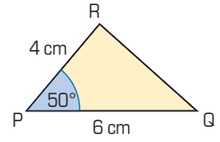
Make the longest side PQ the base



Draw an angle of 50° at P



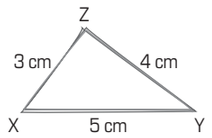
Mark R, 4 cm from P



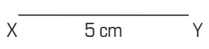
3. Three sides (SSS)
You need a ruler and compasses.

Construct the triangle XYZ with lengths XZ = 3 cm, YZ = 4 cm and XY = 5 cm.

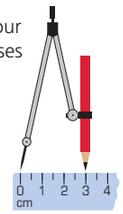
First sketch the triangle – label the vertices



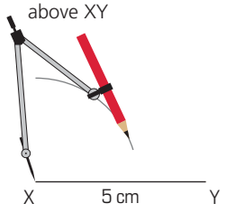
Draw a base line XY 5 cm long



Open your compasses to 3 cm

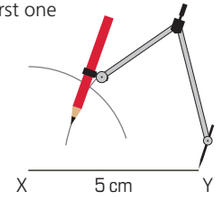


Place the point at X and draw an arc above XY

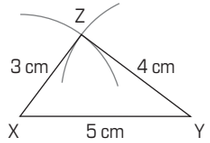


Open your compasses to 4 cm

Place the point at Y and draw an arc that crosses the first one



Label the intersection Z and join up the lines

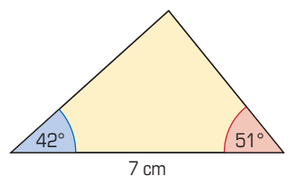


Exercise S3.3

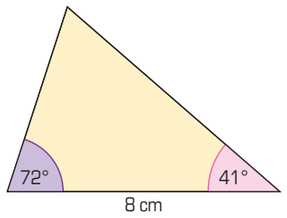
1 Without drawing these triangles, describe:
- What type they are (ASA, SAS or SSS)
- What special equipment you need (protractor or compasses)

a △ABC where AB = 4 cm, AC = 3.5 cm and ∠CAB = 80°.
b △PQR where ∠P = 30°, ∠Q = 40° and PQ = 2 cm.
c △XYZ where XY = 5 cm, YZ = 3 cm and XZ = 6 cm.

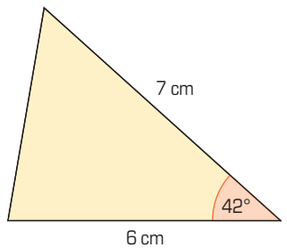
2 Construct accurately the following triangle.



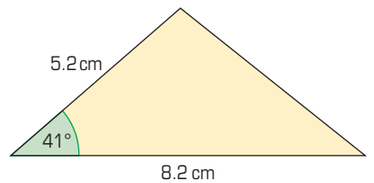
3 Construct accurately the following triangle.



4 Construct accurately the following triangle.



5 Construct accurately the following triangle.



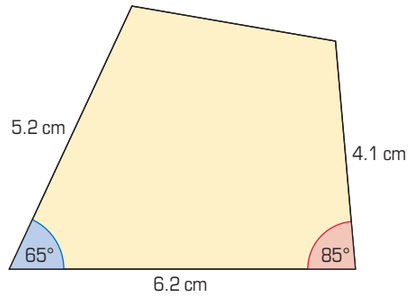
6 Construct △ABC where AB = 4 cm, ∠A = 35°, ∠B = 42°

7 Construct △PQR where PQ = 5 cm, PR = 4.5 cm and ∠P = 35°.

8 Construct an equilateral triangle ABC with sides 5 cm.

9 Construct an isosceles triangle EFG with EF = 3.2 cm and FG = EG = 4.7 cm.

10 Construct the quadrilateral as shown in the diagram:



11 Construct WXYZ where WX = 4.5 cm, WZ = 3.5 cm, XY = 5.5 cm, ∠W = 75° and ∠X = 65°. Sketch it first!

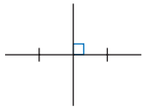
S3.4 Constructing bisectors

This spread will show you how to:
- Construct the midpoint and perpendicular bisector of a line segment.
- Construct the bisector of an angle.
- Find simple loci to produce paths.

KEYWORDS

Bisector	Midpoint
Construct	Perpendicular
Equidistant	Line segment
Locus	

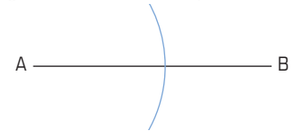
► A perpendicular bisector divides a straight line into two equal parts at right angles.



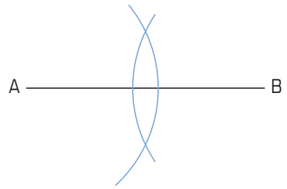
Method

1 Draw a line AB.

2 Open your compasses so that they are at least half the length of AB. Place the point at A and draw an arc.

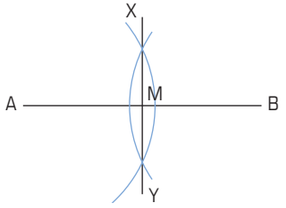


3 Keeping your compasses exactly the same, place the point at B and draw another arc.



All the points on XY are **equidistant** from A and B.
M is the **midpoint** of AB.

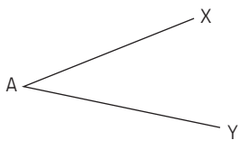
4 Join X to Y to make the perpendicular bisector of AB.



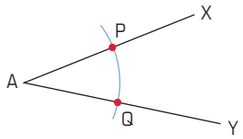
► An angle bisector divides an angle into two equal parts.

Method

1 Draw an angle XAY.

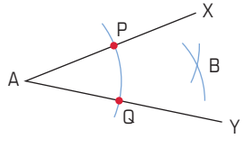


2 Open your compasses, put the point at A and draw an arc that cuts both lines.



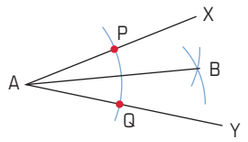
Do not remove any construction lines. These show evidence of your working.

3 Draw arcs from P and Q using the same compass settings for each one.



All the points on AB are equidistant from AX and AY.

4 Join AB. This bisects the angle XAY.

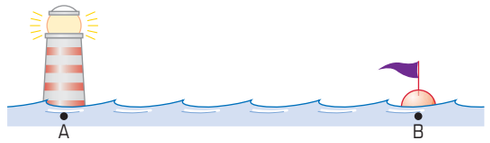


Exercise S3.4

1 **a** Draw a line AB = 10 cm. Construct the perpendicular bisector of AB.
b Label the midpoint of the line X
c Construct an angle of 45° at X.

2 Using a protractor draw an angle of 60°.
a Bisect this angle to make 30°.
b Bisect one of these angles to make 15°.

3 The diagram below shows a lighthouse at A and a buoy at B.

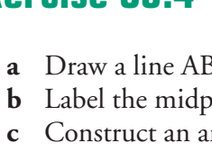


The path traced by a point, satisfying certain conditions, is called a **locus**.

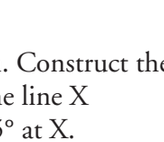
A boat sails so that it is always equidistant from A and B.
Copy the diagram and construct the path of the boat.

4 **a** Follow these instructions to construct a rhombus:
Construct a rhombus WXYZ where ∠W = 60° and the sides are 5 cm.

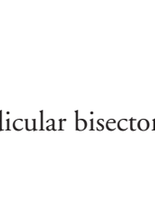
1. First sketch the rhombus.



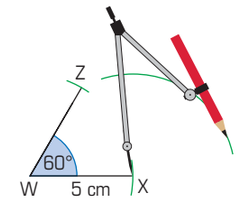
2. Draw an angle of 60° at W and extend the arms.



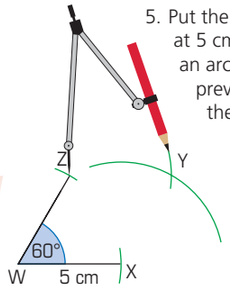
3. Open your compasses to 5 cm and draw arcs on the arms – label X and Z.



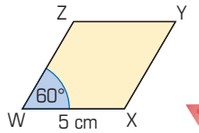
4. Put the compasses, set at 5 cm, on X and draw an arc as shown.



5. Put the compasses, set at 5 cm, on Z and draw an arc that cuts the previous arc – label the intersection Y.



6. Join X to Y and Y to Z.



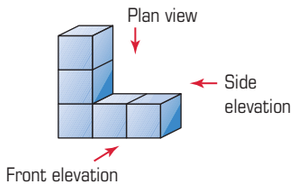
b Draw the diagonals ZX and YW. Label the intersection M.
c Measure WM and MY. What do you notice?
d Copy and complete this sentence: ZX is the _____ bisector of WY.
e Measure angle ZWY. What do you notice?
f Copy and complete this sentence: WY is the _____ bisector of ∠ZWX.

S3.5 2-D representations of 3-D shapes

This spread will show you how to:
- Visualise and describe 3-D shapes.
- Draw plans and elevations of 3-D shapes.
- Construct shapes from their plans and elevations.

KEYWORDS
Elevation, Isometric, Parallel, Perpendicular, Plan

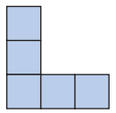
- You can describe a 3-D shape by its:
  - **Plan view**, as seen from the top
  - **Front elevation**, as seen from the front
  - **Side elevation**, as seen from the side



The plan view is a rectangle

The side elevation is a rectangle

The front elevation is an L-shape

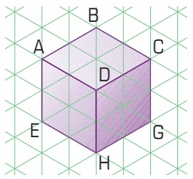


You can draw 3-D shapes using isometric paper.

example

Look at the cube ABCDEFGH. Identify:

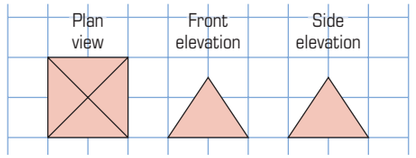
a Two edges that meet at a vertex
b Two faces that meet at an edge
c Two lines that are neither parallel nor perpendicular



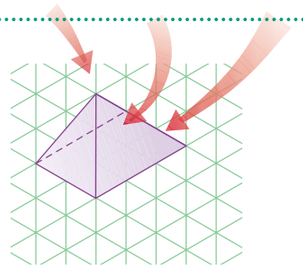
a Edges AB and BC meet at the vertex B.
b Faces ABCD and CDHG meet at the edge CD.
c Line AB is not parallel nor perpendicular to the diagonal line CH.

example

Here are the plan, front elevation and side elevation of a solid shape.
Draw the solid on isometric paper, and describe it.



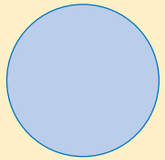
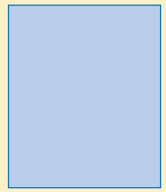
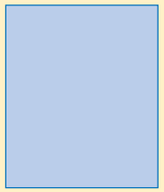
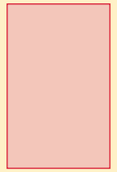
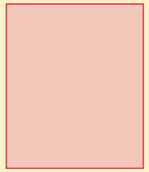
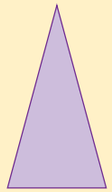
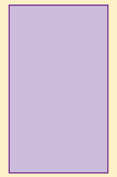
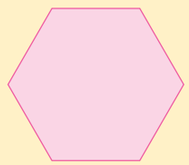
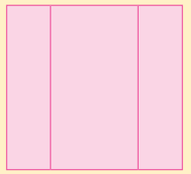
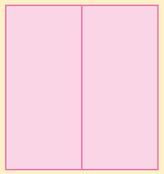
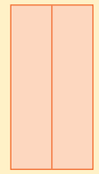
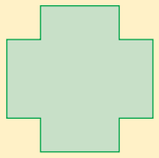
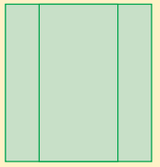
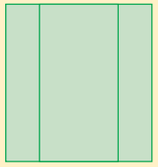
Look at the shapes and dimensions of the different views and piece the final shape together.



The shape is a square-based pyramid.

Exercise S3.5

For each of questions 1 to 6:
- name the 3-D shape
- sketch the 3-D shape on isometric paper.

	Plan View	Front Elevation	Side Elevation
1			
2			
3			
4			
5			
6			

S3 Summary

You should know how to …

1 Solve geometrical problems using angle properties of triangles and quadrilaterals.

2 Use straight edge and compasses to construct:
- the midpoint and perpendicular bisector of a line segment.
- the bisector of an angle.

3 Construct a triangle given three sides.

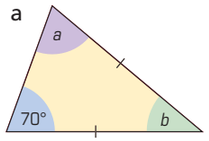
4 Interpret solutions in geometric form.

5 Begin to use plans and elevations.

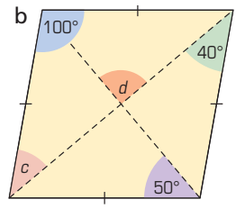
Check out

1 Find the missing angles:

a



b



2 a Draw a line exactly 5.7 cm long
Use compasses to bisect your line.
Check your construction by measuring the mid point of the line.

b Use a ruler and a protractor to draw two lines that are 4 cm long and that meet at 80°.
Use compasses to bisect this angle.

c Bisect an angle of 52°.

3 Construct these triangles:

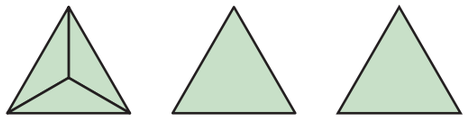
a ABC where AB = 10 cm, $\hat{B}$ = 50° and $\hat{C}$ = 60°.

b PQR where PQ = 6 cm, QR = 7 cm, $\hat{P}$ = 50°.

c EFG where EF = 3 cm, FG = 4 cm and EG = 5 cm.

4 In question 3 which of your triangles are right-angled?

5 Sketch this solid.



plan view | front elevation | side elevation