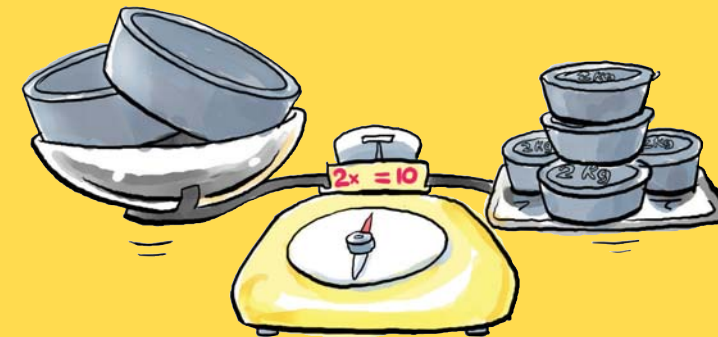


## A3 Solving equations

### This unit will show you how to:

- ▶▶ Begin to distinguish the different roles played by letter symbols.
- ▶▶ Know the meaning of *formula* and *function*.
- ▶▶ Use index notation for small positive integers.
- ▶▶ Simplify or transform linear expressions by collecting like terms.
- ▶▶ Know that algebraic operations follow the same conventions and order as arithmetic operations.
- ▶▶ Construct and solve linear equations with integer coefficients.
- ▶▶ Substitute integers into simple formulae.
- ▶▶ Begin to use graphs and set up equations to solve problems involving direct proportion.
- ▶▶ Substitute positive integers into expressions involving small powers.
- ▶▶ Solve more demanding problems and investigate in a range of contexts.
- ▶▶ Represent problems and interpret solutions in algebraic or graphical form.
- ▶▶ Solve more complex problems by breaking them down into smaller steps, choosing and using efficient techniques.



In algebra, you need to balance both sides.

### Before you start

#### You should know how to ...

- 1 Simplify expressions by collecting like terms.  
For example:
  - ▶  $a + a + a = 3a$
  - ▶  $a \times a = a^2$
- 2 Add and subtract using negative numbers.
- 3 Substitute numbers into simple expressions.
- 4 Use direct proportion in simple contexts.

#### Check in

- 1 Simplify these expressions:
  - a  $n + n + n + n + n$
  - b  $2n + 2n$
  - c  $3 \times n$
  - d  $n \times n$
  - e  $n + p + n + p$
  - f  $2n + 3p + n + p$
- 2 Work out:
  - a  $-3 + -4$
  - b  $4 - 6$
  - c  $-2 - -3$
  - d  $-3 + 3$
- 3 Substitute  $x = 4$  into each expression.
  - a  $2x$
  - b  $2x + 3$
  - c  $x - 5$
  - d  $\frac{20}{x}$
- 4 Three pencils cost 90p. How much will:
  - a six cost? b twelve cost? c ten cost?

# A3.1 Using letter symbols

This spread will show you how to:

- ▶ Distinguish the different roles played by letter symbols.
- ▶ Know the meaning of the words *expression*, *equation*, *formula* and *function*.

## KEYWORDS

Equation	Variable
Formula	Formulae
Function	Expression

You can use letter symbols in different ways.

Naomi and Sian are sorting these algebra cards into equations, formulae and functions.



Sian says:



In the **equation**  $2x + 4 = 14$  the letter  $x$  has an **unknown value**, which you can work out.

In the **formula**  $V = IR$  the letters  $V$ ,  $I$ ,  $R$  are **variables**, they can take any value. You use two of them to work out the third.

In the **function**  $y = x + 3$  the  $y$ -value can be worked out for any value of  $x$ .

$$\begin{aligned} 2x + 4 &= 14 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

$$\begin{aligned} \text{If } I &= 5 \text{ and } R = 10 \\ \text{find the value of } V. \\ V &= IR \\ V &= 5 \times 10 = 50 \end{aligned}$$

$$\begin{aligned} \text{If } y &= x + 3 \\ \text{when } x &= 2 \quad y = 5 \\ x &= 6 \quad y = 9 \end{aligned}$$

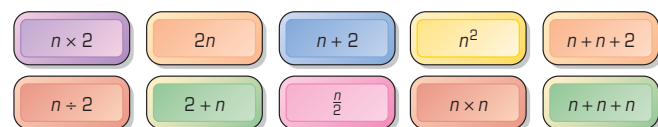
Naomi sorts the algebra cards into three piles:

Equations	Formulae	Functions
$a + 9 = 12$	$C = kd$	$y = 10 - x$
$\frac{x}{2} = 10$	$A = \frac{1}{2}bh$	$y = 2x + 4$

- ▶ Letters represent numbers.
- ▶ An expression is a collection of letter and number terms.

## example

Match equivalent expressions. Write an equivalent expression for each of the two left over.



Multiplication and addition are associative:  
 $(n \times 2) \times 3 = n \times (2 \times 3)$   
 $(n + n) + 2 = n + (n + 2)$



Simplify each of the expressions by collecting like terms.  
 $n + n + 2 = 2n + 2$     $n + n + n = 3n$

## Exercise A3.1

- Simplify these expressions by collecting like terms.
  - $3 \times a$
  - $a + a + a$
  - $d + d + d - d - d$
  - $b + b + b + a + a$
  - $b \times 5$
  - $10d \div 2$

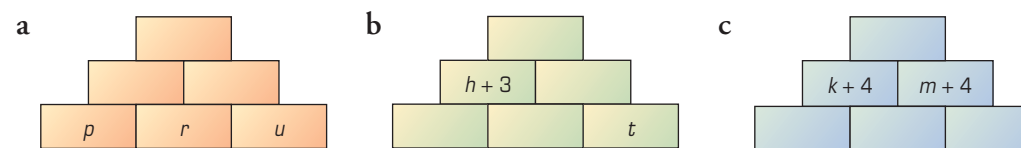
**Hint:** 'Like terms' contain exactly the same letter.

- Sort these algebra cards into three piles: equations, formulae and functions.



- The number in each brick is the result of adding the numbers in the two bricks beneath it.

Copy the walls and write expressions for the numbers in each brick. Write each expression as simply as possible.



- Here are some algebra cards.

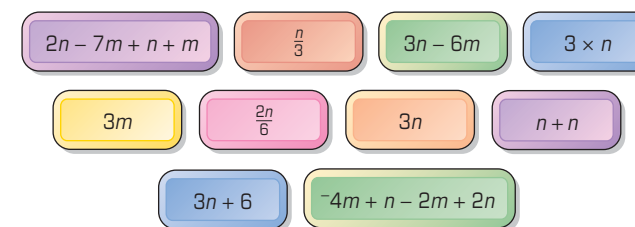
- Which card will always give the same answer as  $3 \times n$ ?

- Which cards added together will give  $6n + 6$ ?

- Which cards will always give the same answer as  $3n - 6m$ ?

- Which card simplifies to  $\frac{n}{3}$ ?

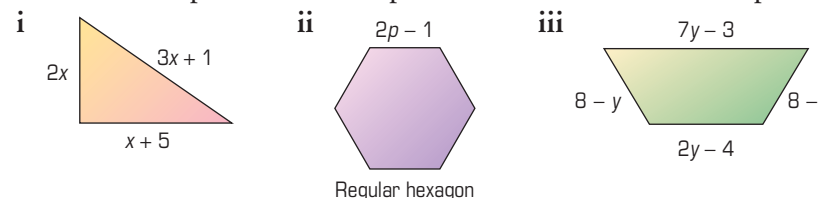
- Write a new card that will always give the same answer as  $n + n$ .



- Simplify these expressions.

- $2p + 3p$
- $5m - 6 + 2m - 3$
- $4a + 3b + 3a - 2b$
- $3s + 2p - 3s - p$
- $5x + 8 + 2x - 5 - x$
- $9w - 4x + w - 3$
- $3n + 4 - 5m + n + 1$
- $2a - 2b - 2a + 2b + 1$
- $-y + 4x - 2y + x$

- Write an expression for the perimeter of each of these shapes.



- If every shape in part **a** has a perimeter of 30 cm, find the unknown value in each.

# A3.2 Arithmetic in algebra

This spread will show you how to:

- ▶ Know that algebraic operations follow the same conventions and order as arithmetic operations.
- ▶ Substitute integers into simple expressions.

## KEYWORDS

Algebraic expression  
Substitute  
Order of operations

Algebraic operations follow the same order of operations as arithmetic:

### Order of operations

Brackets ————— **Work out:**  
 $9 - 4(3 - 1)$   
 Indices or Powers  
 Division and Multiplication —————  $= 9 - 4 \times 2$   
 Addition and Subtraction —————  $= 9 - 8$   
 $= 1$

Use the word **BIDMAS** to help you remember the order.

Remember this special rule for algebra:

- ▶ You do not use multiplication or division signs.
  - ▶  $5 \times a = 5a$
  - ▶  $2b \div 3 = \frac{2b}{3}$

You use a fraction instead of the division sign.

Brackets often contain a hidden multiplication.

- ▶ When you multiply by a bracket, you do not use the multiplication sign.
  - $2 \times (5 + 2) = 2(5 + 2)$
  - $3 \times (b + 4) = 3(b + 4)$

You can use these rules to help substitute values into algebraic expressions.

example

Find the value of these algebraic expressions when  $q = 2$ ,  $r = 4$  and  $s = 6$ .

a	$5q + 4$	b	$rs$	c	$10 - qrs$	d	$3(2s - 5)$	e	$q(r - s)$
a	$5q + 4$	b	$rs$	c	$10 - qrs$	d	$3(2s - 5)$	e	$q(r - s)$
	$= 5 \times 2 + 4$		$= 4 \times 6$		$= 10 - 2 \times 4 \times 6$		$= 3 \times (2 \times 6 - 5)$		$= 2 \times (4 - 6)$
	$= 10 + 4$		$= 24$		$= 10 - 48$		$= 3 \times (12 - 5)$		$= 2 \times -2$
	$= 14$		$= -38$		$= 3 \times 7$		$= -4$		
					$= 21$				

## Exercise A3.2

- This is a game for two players.
  - ▶ Player 1 picks a question from the table and its answer from the second table.
  - ▶ Player 2 uses a calculator to check the answer.
  - ▶ If the answer is correct Player 1 claims the question square, if incorrect Player 2 claims the square.
  - ▶ Players take turns to choose questions and answers.
  - ▶ The winner is the first player to get 4 in a row.

$4 + 2 \times 3$	$3(4 + 2)$	$10 - 2 \times 4$	$44 \div 4 + 30 \div 2$
$6(4 + 2)$	$22 - 10 \div 2$	$10 + 3(2 + 6)$	$16 - 2 \times 4$
$100 \times 5 + 40$	$2 \times 10 + 4$	$5 \times 4 + 20$	$6 \times 12 - 2 \times 9$
$40 \div 2 + 12$	$2 \times 3 + 4 \times 4$	$2(10 - 2)$	$25 \times 4 + 4$

2	8	10	16
17	18	22	24
26	32	34	36
40	54	104	540

- Find the value of each expression when  $x = 8$ .
  - i  $2x$       ii  $2x - 4$       iii  $3(2x - 4)$
  - i  $x + 1$       ii  $5(x + 1)$       iii  $2(x + 1)$
  - i  $5x$       ii  $5x - 10$       iii  $\frac{5x}{2}$
  - i  $x - 2$       ii  $3(x - 2)$       iii  $\frac{3(x - 2)}{2}$
  - i  $\frac{1}{2}x$       ii  $\frac{1}{2}x + 5$       iii  $\frac{1}{2}x - 5$
  - i  $x^2$       ii  $x^2 - 3$       iii  $2(x^2 - 3)$
- Work out the expressions in question 2 when  $x = 1$ .
- Find the value on each algebra card when  $a = 4$ ,  $b = 3$  and  $c = 6$ . Sort them into order of size, starting with the smallest.

- Challenge**
  - Find the value of each expression when  $p = 2$ ,  $q = -3$  and  $r = -4$ .
    - $3r + p$
    - $pqr$
    - $p^2 + r$
    - $\frac{3r}{p}$
    - $r^2$
    - $\frac{p+q}{r+3}$
  - Using  $p$ ,  $q$  and  $r$ , make up five expressions of your own with a value of  $-24$ . (You can use all or some of the letters.)

# A3.3 Solving linear equations

This spread will show you how to:

- ▶ Construct and solve linear equations with integer coefficients.
- ▶ Simplify or transform linear expressions by collecting like terms.
- ▶ Represent problems in algebraic form.

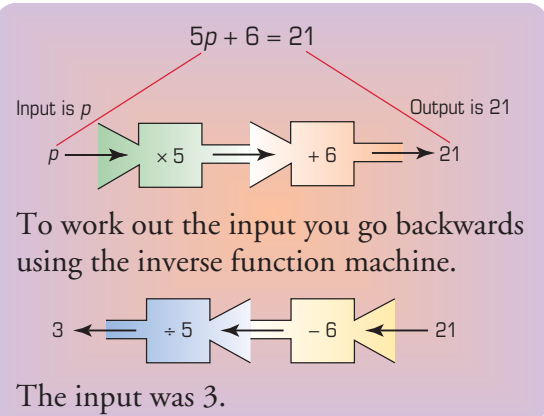
## KEYWORDS

- Function machine
- Inverse
- Equation

There are different ways of solving the equation  $5p + 6 = 21$ .

Stephen uses inverse function machines.

Paul uses inverse operations.



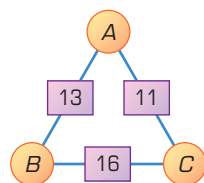
The equation is  $5p + 6 = 21$   
 Subtraction is the inverse of addition:  
 $5p = 21 - 6$   
 $5p = 15$   
 Division is the inverse of multiplication:  
 $p = 15 \div 5$   
 $p = 3$

Both methods give the same answer.

## example

In an arithmagon, the numbers in a square are the sum of the numbers in the two circles either side of it.

What are the numbers  $A$ ,  $B$  and  $C$  in this arithmagon?



Let  $A$  stand for the number in the top circle.

Form expressions for the numbers in the other circles.

$$\begin{aligned} A + B &= 13 & A + C &= 11 & B + C &= 16 \\ B &= 13 - A & C &= 11 - A \end{aligned}$$

Then form an equation and solve it.

$$\begin{aligned} (13 - A) + (11 - A) &= 16 \\ 24 - 2A &= 16 \\ 2A &= 8 \\ A &= 4 \end{aligned}$$

Think  $24 - ? = 16$   
 $24 - 8 = 16$

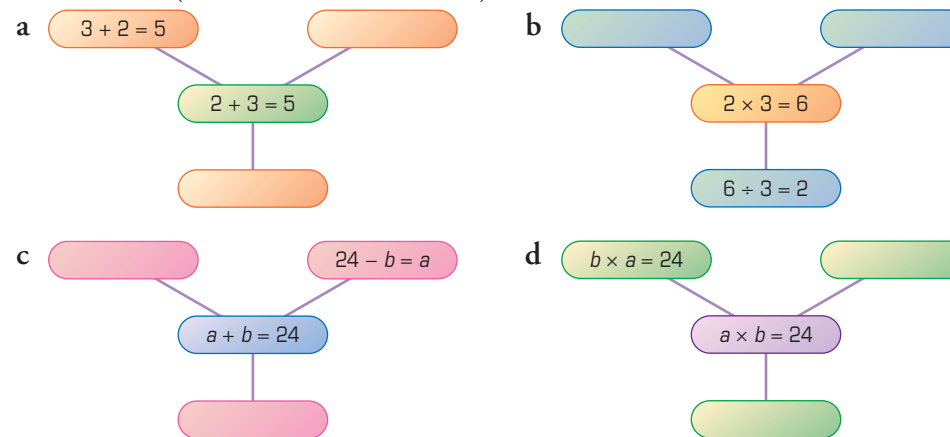
so  $A = 4$ ,  $B = 9$ ,  $C = 7$

Check:

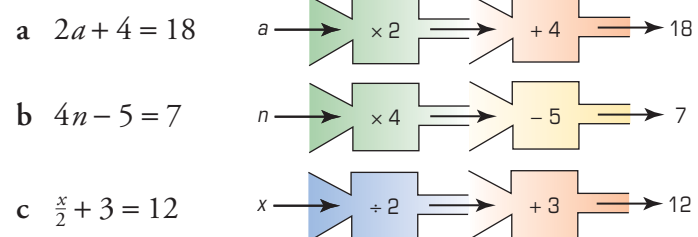
$A + B = 13$	$A + C = 11$	$B + C = 16$
$4 + 9 = 13 \checkmark$	$4 + 7 = 11 \checkmark$	$9 + 7 = 16 \checkmark$

## Exercise A3.3

1 Copy and complete these spider diagrams using related equations in the boxes (some have been filled in).

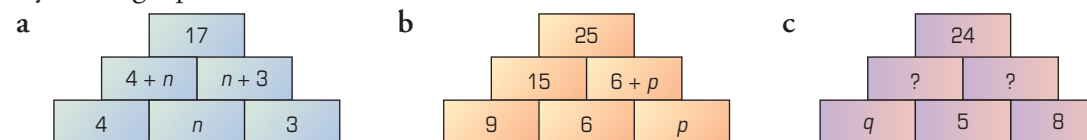


2 Use inverse function machines to solve these equations.



3 The number in each brick is the result of adding the numbers in the two bricks beneath it.

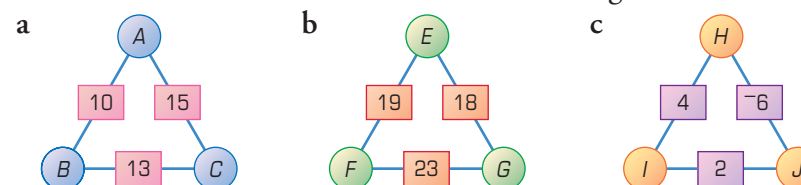
By writing equations, find the unknown letter in each wall.



4 Solve these equations.

a  $4a + 5 = 21$     b  $\frac{e}{3} + 4 = 9$     c  $4(x - 3) = 20$     d  $\frac{r+4}{5} = 2$

5 In an arithmagon, the number in a square is the sum of the numbers in the two circles on either side of it. Find the numbers in the circles in these arithmagons.



# A3.4 Balancing linear equations

This spread will show you how to:

- Construct and solve linear equations with integer coefficient unknowns on both sides, using appropriate methods.

## KEYWORDS

Equation      Substitute  
Solve

You can solve equations by using the balancing method.

You must do the same thing to both sides of the equation.

Solve this equation:

$$6a + 45 = 9a + 15$$

You can take  $6a$  off both sides:

You can take 15 off both sides:

Each  $a$  must be equal to 10.

$$3a = 30$$

$$a = 10$$

Check the solution by substituting  $a = 10$ :

Left-hand side

$$6a + 45$$

$$6 \times 10 + 45$$

$$60 + 45$$

$$105$$

Right-hand side

$$9a + 15$$

$$9 \times 10 + 15$$

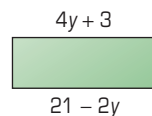
$$90 + 15$$

$$105$$

The left-hand side and right-hand side are equal so the solution is correct.

example

Work out the value of  $y$  in this rectangle.



You know that the opposite sides of a rectangle are equal.

So  $4y + 3 = 21 - 2y$

Take off 3 from both sides:  $-3 \quad -3$

$$4y = 18 - 2y$$

Add  $2y$  to both sides:  $+2y \quad +2y$

$$6y = 18$$

Divide both sides by 6:  $\div 6 \quad \div 6$

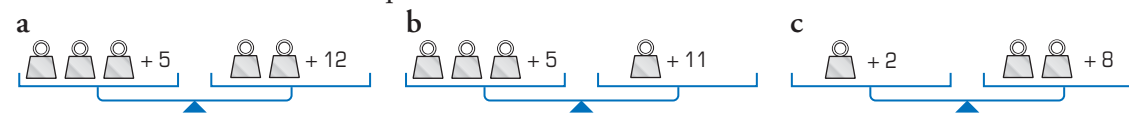
$$y = 3$$

Check: Left-hand side  $= 4y + 3 = 4 \times 3 + 3 = 12 + 3 = 15$

Right-hand side  $= 21 - 2y = 21 - 2 \times 3 = 21 - 6 = 15$  So the solution is correct.

## Exercise A3.4

1 Find the value of  $\square$  in each question.



2 Solve each of these equations.

- a  $3b + 14 = 2b + 16$       b  $5c + 4 = 3c + 22$       c  $2d + 3 = d + 7$   
 d  $5e - 5 = 2e + 16$       e  $3f + 11 = 5f + 3$       f  $5g - 1 = 2g + 14$   
 g  $9h + 4 = 5h + 20$       h  $5i - 10 = i + 2$       i  $2j + 11 = j + 20$

Transform the equations by doing the same to both sides.

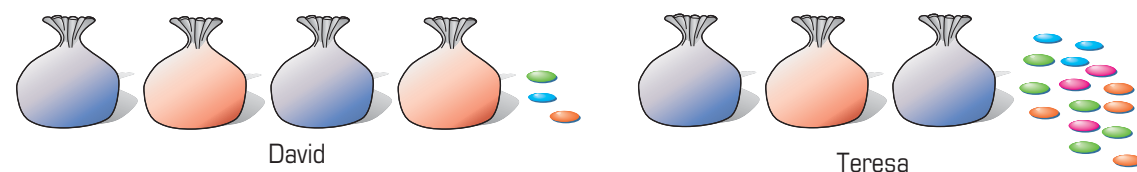
3 Work out the value of  $x$  in each of these shapes.



4 David and Teresa each have the same number of sweets.

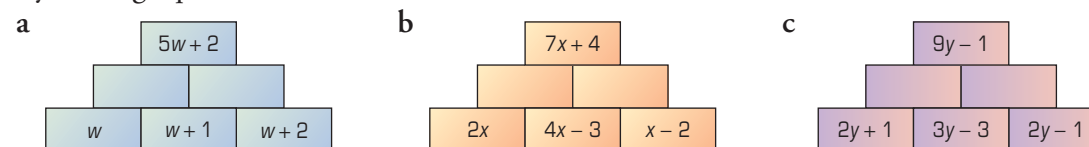
David has 4 full bags of sweets and 3 loose sweets.

Teresa has 3 full bags of sweets and 15 loose sweets.



- a Form an equation from the above information.  
 b Solve the equation to find out how many sweets there are in one bag.

5 The number in each cell is the result of adding the numbers in the two cells beneath it. By writing equations, find the unknown letter in each wall.



6 Sonia plays a number game. She says: 'Multiplying my number by 4 then subtracting 3 gives the same answer as multiplying my number by 3 then adding 2.' Work out the value of Sonia's number.

7 Which equation has a different solution to the other two?

- $4x + 3 = 5x - 4$        $4x + 1 = 6x - 9$        $3x - 4 = 2x + 3$

Write three equations that all have the same solution as the odd one out.

# A3.5 Solving proportional problems

This spread will show you how to:

- ▶ Begin to use graphs and set up equations to solve simple problems involving direct proportion.

### KEYWORDS

Direct proportion  
Origin  
Relationship

Rhiannon takes her holiday photos to be developed.

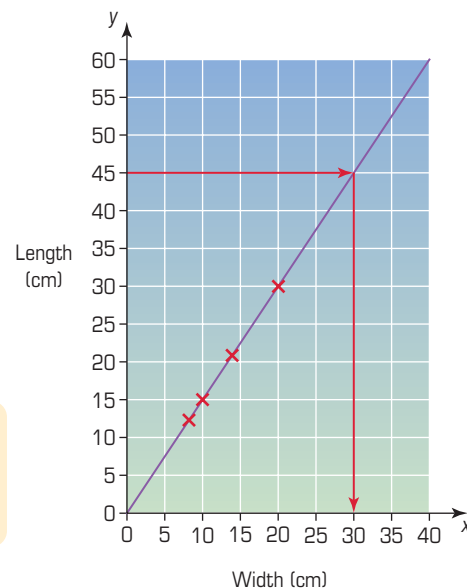
She can have them developed into different sizes.

All the photos are in the same proportion.



Rhiannon decides to choose her own size. She wants the length to be 45 cm.

She plots a graph and extends the line to work out the width of the photo.



- ▶ When two quantities are in direct proportion, the coordinate pairs will lie on a straight line which will go through the origin.

A photo of length 45 cm, in the same proportion, will have a width of 30 cm.  
The relationship between the length and the width can be written as a formula:

$$\begin{aligned} \text{length} &= 1.5 \times \text{width} \\ y &= 1.5x \end{aligned}$$

**Remember:**  
 $1.5 \times x$  is written  $1.5x$

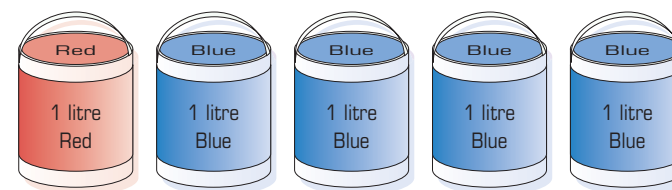
If the width is 40 cm you can work out the length using the formula:

$$y = 1.5 \times 40 = 60 \text{ cm}$$

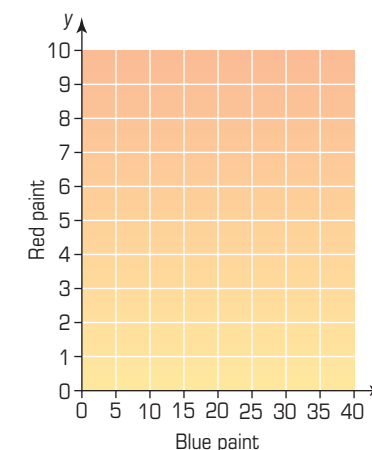
## Exercise A3.5

- James mixes 1 litre of red paint with 2 litres of white paint to make 3 litres of pink paint.  
James wants 9 litres of pink paint.  
How many litres of red paint will he need?

- You make 5 litres of purple paint by mixing 1 litre of red paint with 4 litres of blue paint.



- Copy and complete:  
To make 10 litres of purple paint, mix \_\_\_ litres of red paint with \_\_\_ litres of blue paint.  
To make 15 litres of purple paint, mix \_\_\_ litres of red paint with \_\_\_ litres of blue paint.  
To make 25 litres of purple paint, mix \_\_\_ litres of red paint with \_\_\_ litres of blue paint.
- Draw a graph to show how purple paint can be mixed. You can copy these axes.
- Use the graph to work out:
  - How many litres of blue paint you would mix with 8 litres of red paint.
  - How many litres of red paint you would mix with 14 litres of blue paint.



- Which of these pictures is not in proportion to the others?



**Hint:** Plot a graph to help.

- Draw four rectangles, with their dimensions labelled, that are all in proportion to each other.

# A3.6 Substitution

This spread will show you how to:

- ▶ Use index notation for small positive integers.
- ▶ Substitute positive integers into expressions involving small powers (e.g.  $3x^2 + 4$  or  $2x^3$ ).

### KEYWORDS

- Index
- Algebra
- To the power of  $n$
- Substitute
- Indices

When you add repeatedly, you can write the expression using a multiplication:

$$4 + 4 + 4 + 4 + 4 = 5 \times 4$$

When you multiply repeatedly, you can write the expression using an index:

$$4 \times 4 \times 4 \times 4 \times 4 = 4^5$$

This is called the **index** or the **power**.

This is the same in algebra:  $p \times p \times p \times p \times p \times p \times p \times p = p^8$

- ▶  $5 \times 5 = 5^2$  you say 'five squared'.
- ▶  $6 \times 6 \times 6 = 6^3$  you say 'six cubed'.
- ▶  $4 \times 4 \times 4 \times 4 \times 4 = 4^5$  you say '4 to the power of 5'.

You can simplify expressions with numbers and different letters combined:

$$3 \times p \times p = 3p^2 \quad 4 \times p \times p \times r \times r \times r = 4p^2 r^3$$

You can substitute values to find the value of an expression.

example

Find the value of each expression when  $p = 2$  and  $q = 4$ .

a  $q^2$    b  $pq^2$    c  $q(p^2 + 4)$

a  $q^2 = 4 \times 4 = 16$

b  $pq^2 = 2 \times 4 \times 4 = 32$

c  $q(p^2 + 4) = 4 \times (2 \times 2 + 4) = 4 \times (4 + 4) = 4 \times 8 = 32$

Remember the order of operations:  
**B**rackets  
**I**ndices  
**D**ivision  
**M**ultiplication  
**A**ddition  
**S**ubtraction

You can solve some harder equations by estimating the value and substituting.

example

Winston substitutes a number into the expression on this card. The value he gets is 85. What was the number?

$$4x^2 - 3x$$

You need to substitute different values into the expression until you find the correct number.

First guess  $x = 8$     $4 \times 8^2 - 3 \times 8 = 256 - 24 = 232$    Too big – try a smaller value

Second guess  $x = 4$     $4 \times 4^2 - 3 \times 4 = 64 - 12 = 52$    Too small – try a bigger value

Third guess  $x = 5$     $4 \times 5^2 - 3 \times 5 = 100 - 15 = 85$    Correct value!

Winston must have substituted  $x = 5$  into the expression.

## Exercise A3.6

- 1 Copy these repeated multiplications and match them to their index form. The first one has been done for you.

$2 \times 2$	$4^2$
$2 \times 2 \times 2 \times 2$	$3^1$
$1 \times 1 \times 1$	$6^4$
$4 \times 4$	$2^2$
$4 \times 4 \times 4 \times 4 \times 4 \times 4$	$2^4$
$6 \times 6 \times 6 \times 6$	$4^6$
$3$	$1^3$

- 2 Write down these expressions in their simplified form.
- a  $p \times p$    b  $r \times r \times r$    c  $p \times p \times p \times p$    d  $3 \times p \times p$   
 e  $r^2 \times r^3$    f  $2p^3 \times p^2 \times p$    g  $2 \times p \times p \times r \times r \times r$    h  $r \times r \times r + p \times p$

- 3 Match five pairs of expressions that have the same meaning. For the remaining expressions, write a simplified expression which is equivalent.

$y \times y \times y$	$2y$	$y^3 \times y^4$	$3y$	$y^2 + y^2$	$y \times y$
$y^3$	$y^2$	$y + y + y$	$2y \times 2y$	$2y^2$	$y + y$

- 4 Find the value of each expression when  $a = 3$ ,  $b = 2$  and  $c = \frac{1}{2}$ .
- i  $a^2$    ii  $10 - 3a$    iii  $b^2 + a^2$    iv  $2c + ab$    v  $(b + 1)^2 + 2a$   
 vi  $b^3 + 7$    vii  $cb^3$    viii  $(b - 1)^2$    ix  $a^3 - b^3 + 1$    x  $b^4 + a$   
 xi  $a(3b + 2c)$    xii  $(a - b)^5 + 2c$    xiii  $2b^3 + a$    xiv  $5b^2$    xv  $\frac{a^3}{3}$   
 xvi  $2a^2 + b$    xvii  $a^2 b + a$    xviii  $b^4 + b^2$    xix  $ac(b^2 + 2)$    xx  $cb^5 - 1$   
 xxi  $c(1 + a^3)$

Use this code to change your answers to letters.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Read your letters as words – what does it say?

- 5 Catherine substitutes a number into the expression on this card:

$$2x^2 + 3x$$

The value she gets for the expression is 44.

Work out what number she substituted into the expression.

- 6 Find the exact value of  $x$  for each of the following.

a  $3x^2 + 12 = 120$

b  $5x^2 - 10x = 75$

c  $10x - x^2 = 21$

d  $4x^2 + 3x = 32.5$

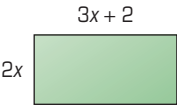
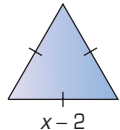
Hint:  $x$  is a decimal.

## A3 Summary

### You should know how to ...

- 1 Simplify or transform linear expressions by collecting like terms.
- 2 Construct and solve linear equations with integer coefficients.
- 3 Substitute integers into simple formulae.
- 4 Represent problems in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

### Check out

- 1 Write an expression for the perimeter of each shape.
  - a 
  - b 
- 2
  - a The perimeter of each shape in question 1 is 24 cm. Construct an equation and solve it to find the value of  $x$  in both shapes.
  - b Solve these equations:
    - i  $3x + 6 = 12$
    - ii  $4x - 10 = 2$
    - iii  $\frac{x}{2} + 3 = 9$
    - iv  $2(x - 1) = 18$
    - v  $\frac{x + 3}{3} = 21$
    - vi  $3x + 4 = 5x - 2$
    - vii  $x + 6 = 3x + 8$
- 3 Find the value of each expression when  $a = 2$  and  $b = \frac{1}{2}$ .
  - a  $a^2$
  - b  $b^2$
  - c  $a + 2b$
  - d  $a - b$
  - e  $a^3$
  - f  $2(a + b)$
  - g  $a^2b$
  - h  $\frac{2b + a}{2}$
- 4 Use algebra to write this statement correctly: 'Jez thinks of a number, adds 3, then multiplies by 2. The answer is 14.'  
Now solve the equation to find the number Jez thought of.