

KEY WORDS

equation	like term
expression	solve
formula	

- In an **equation** a letter represents a specific unknown number.
You **solve** an equation by finding the value of the letter.
For example in the equation $5x = 20$ the value of $x = 4$

An equation always has an = sign.

- An **expression** is a collection of terms separated by + or - signs.
 $3x + y - x$ is an expression

An expression never has an = sign.

- A **formula** tells you how two (or more) different quantities are related.
For example, the formula $A = \pi d$ tells you that the area of a circle is related to its diameter. You can substitute a values of either A or d into the formula to find the corresponding values of d or A .

A formula contains two or more letters.

- Expressions can be simplified by collecting **like terms** or using powers.

EXAMPLE

Simplify

a $r \times r \times r \times r \times r$

b $5k + 2k$

c $6p + 5r + 3p - 4r$

Write r rather than 1r.

a $r \times r \times r \times r \times r = r^5$

b $5k + 2k = 7k$

c $6p + 5r + 3p - 4r = 9p + r$

EXAMPLE

Decide whether the following are equations, expressions or formulae.

a $3m + 2n$ **b** $3m + 6 = 18$ **c** $v = u + at$ **d** $\frac{r}{2} = r + 2$

a $3m + 2n$ is an expression – it has no = sign.

b $3m + 6 = 18$ is an equation.

c $v = u + at$ is a formula.

d $\frac{r}{2} = r + 2$ is an equation.

Exercise A1

1 Simplify

a $b + b + b$

b $3v + 2v + 6v$

c $y + 6y + 2y - 5y$

d $c \times c \times c \times c$

e $2s + 5t + 7s + 8t$

f $7j + 2k - 3j + 5k$

g $4 \times z \times z \times z$

h $4g - 2h - 5g - 7h$

i $6q + r - 5q - 5r + 4r - q$

2 Decide whether the following are equations, expressions or formulae.

a $8 - 2f = 10$

b $2r + 10$

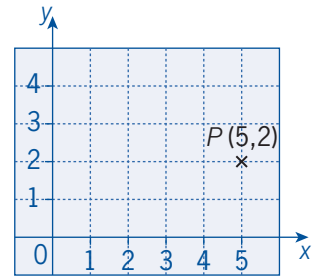
c $A = 4\pi r^2$

d $v^2 = u^2 + 2as$

e $y = 8y + 21$

f $8x + 9y - 6z$

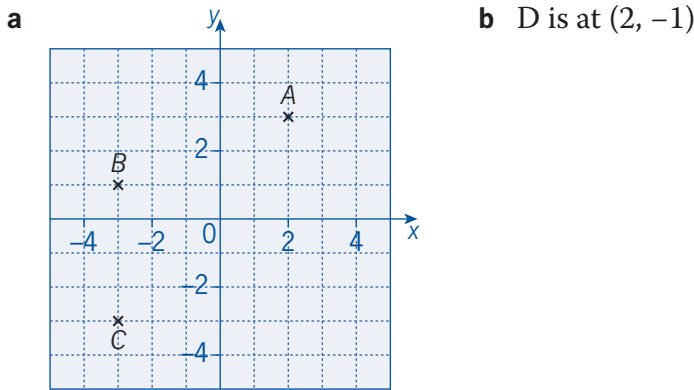
- The point (0, 0) on a graph where the axes cross is called the **origin**.
- The **coordinates** of a point P, for example (5, 2), tell you that P is 5 units along and 2 units up from the origin (0, 0).
- The first number, 5, is the **x-coordinate**.
- The second number, 2, is the **y-coordinate**.



EXAMPLE

Simplify.

- Mark an label these points on a grid.
A (2, 3), B (-3, 1) and C (-3, -3)
- Write down the coordinates of point D such that ABCD is a parallelogram.



EXAMINER'S TIP:
When marking points use a small neat cross and write labels nearby. If you just write the letter on the point it may not be accurate enough.

EXAMINER'S TIP:
It is OK to join up the points even if the question does not ask you to.

- The **midpoint** of a line is halfway between the ends.
- The length of a line can be calculated using Pythagoras' theorem.

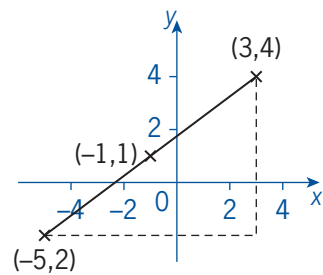
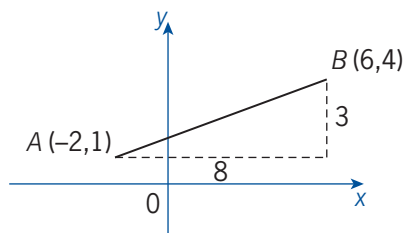
EXAMPLE

- Point A is at (-2, 1), point B is at (6, 4).
- Find the midpoint of AB.
 - Calculate the length of line AB, giving your answer in units to 3 sf.

Start by drawing a sketch.

a The midpoint is at $\left(\frac{-2+6}{2}, \frac{1+4}{2}\right) = (2, 2.5)$

b By Pythagoras:
 $c^2 = a^2 + b^2$
 $c^2 = 8^2 + 3^2$
 $= 64 + 9$
 $= 73$
 $c = \sqrt{73} = 8.54$ units



Technically AB should be called a line segment, as a line is infinitely long.

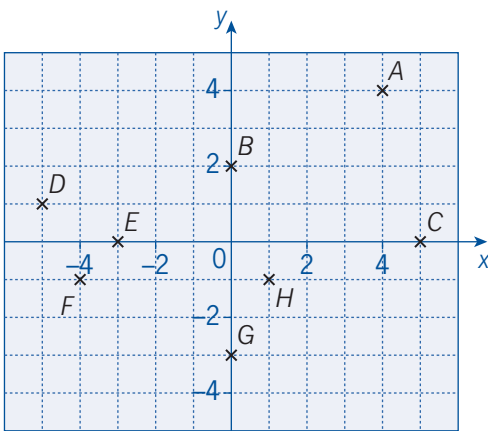
You can think of the midpoint as the 'mean average' point.

$\sqrt{73}$ is an 'exact' answer.
8.54 is correct to 3 significant figures

Overmatter exercise A2

Exercise A2

- 1 Write down the coordinates of points A to H.



- 2 **a** Mark and label these points on a grid.
 $P(3, 5)$, $Q(-2, 4)$, and $R(-2, 0)$
- b** PQRS is a trapezium.
Find two possible positions for point S. You must use four different whole numbers for the coordinates.
- 3 Pete marks the points $U(-3, 1)$ and $V(5, 1)$ on a grid.
He finds points, W, so that UVW is an isosceles triangle.
He says that W must always have 1 as its x -coordinate.
Jennie disagrees.
Find a possible point for W so that Jennie can demonstrate to Pete that he is wrong.
- 4 For each of these pairs of points, A and B
- find the midpoint of AB
 - the length of AB. Give your answer to 3 significant figures where appropriate.
- a** $A(1, 1)$ $B(4, 5)$ **b** $A(1, 4)$ $B(7, 0)$ **c** $A(-1, -4)$ $B(6, 2)$
d $A(2, -3)$ $B(-4, -1)$ **e** $A(-5, -2)$ $B(3, 13)$ **f** $A(0, -2)$ $B(7, 0)$
- 5 PQRS is a rectangle.
P, Q and R are the points $(-1, -2)$, $(1, 4)$ and $(7, 2)$ respectively.
- Find the coordinates of point S.
- M is the midpoint of PQ and N is the midpoint of QR.
- Show that the length of MN is exactly $\sqrt{20}$.

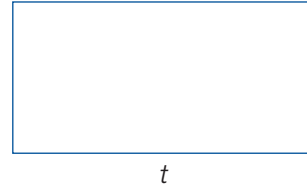
- A **formula** tells you how two (or more) quantities are related.

A formula can be in words or letters.

For example the formula $A = \pi r^2$ tells you that the area of a circle, A , is proportional to the square of its radius, r .

- You can **substitute** values into a formula.

For example if $y = 3x - 10$ find y when $x = 6$.
 $y = 3 \times 6 - 10 = 18 - 10 = 8$



- You can **derive** (write in letters) a formula using information you know.

The letter on its own is called the **subject** of the formula.

P is the subject of the formula
 $P = 4l$

For example the perimeter of a square of side l is given by the formula

$$P = 4l$$

- You can **rearrange** or **change the subject** of a formula by doing the same operation to both sides.

For example, make p the subject of the formula

$$\begin{aligned} r &= 3p + q && \text{Subtract } q \text{ from both sides} \\ r - q &= 3p && \text{Divide both sides by 3} \\ p &= \frac{r - q}{3} \end{aligned}$$

EXAMPLE

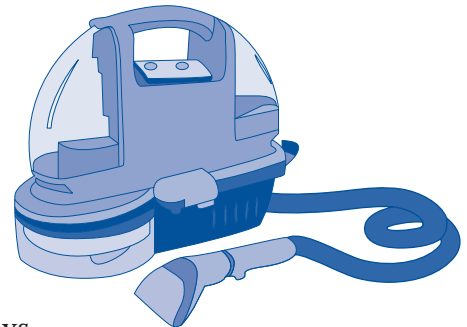
Hanif hires a carpet cleaner from *Coventry Cleaners*. The hire charge, in £, is worked out using this word formula.

Charge = 20 + (number of days hire × 8) + (number of litres of cleaning fluid used × 5)

- a** Hanif hires the carpet cleaner for 3 days and uses 12 litres of cleaning fluid.

How much is the charge?

- b** Write a formula for the total charge C , if he hires it for d days and uses c litres of cleaning fluid.



a Charge = $20 + 3 \times 8 + 12 \times 5$
 $= 20 + 24 + 60 = \text{£}104$

b $C = 20 + 8d + 5c$

EXAMPLE

If $s = 3t + 2u$, find s when $t = 4$ and $u = -2$.

$$\begin{aligned} s &= 3t + 2u \\ &= 3 \times 4 + 2 \times -2 \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

Remember the correct order of operations - \times before $+$.

Also remember the rules

$$+ \times + = +$$

$$- \times - = + \text{ and}$$

$$+ \times - = -$$

Rearrange these formulae to make p the subject.

a $s = \frac{p}{r}$ **b** $t = \frac{p-s}{r}$

a $s = \frac{p}{r}$ multiply both sides by r
 $rs = p$

b $t = \frac{p-s}{r}$ multiply both sides by r
 $tr = p - s$ add s to both sides $tr + s = p$

Compare the operations used here to those in solving equations. The processes are very similar.

Exercise A3

- 1** A water company charges its customers for water using the following formula.

$$\text{cost} = \text{standing charge} + (\text{cost per cubic metre} \times \text{number of cubic metres used})$$

Karl uses 157 cubic metres of water in one year.

The standing charge is £27 and the cost per cubic metre is £1.41.

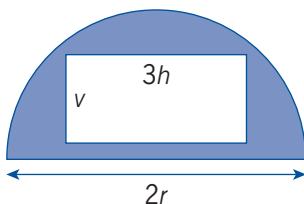
How much does he pay for his water?

- 2** If $p = 6$ and $q = 3$, find the value of w if

a $w = 2p + q$ **b** $w = 8p + 3q$ **c** $w = pq$
d $w = 2p - q$ **e** $w = 2p - 7q$ **f** $w = \frac{p+q}{9}$

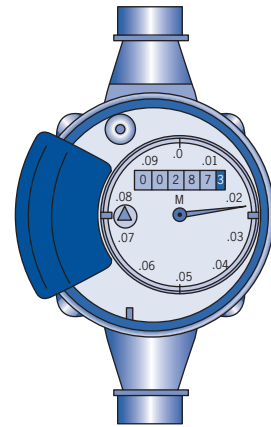
- 3 a** A window cleaner charges £ p for each window she cleans plus an extra £4. Write down a formula for the amount C she charges for cleaning w windows.

- b** Write down a formula for the shaded area.



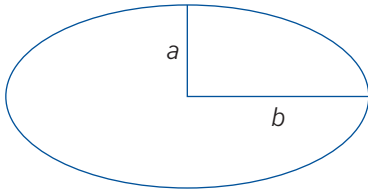
- 4** Rearrange these formulas to make x the subject.

a $y = ax$ **b** $m = x - t$ **c** $p = \frac{x}{r}$ **d** $k = x + z$
e $a = 5x - 7$ **f** $y = mx + c$ **g** $t = wx - g$ **h** $s = ab + rx$
i $q = r(x - z)$ **j** $f = 3m - x$ **k** $p = \frac{y}{x}$ **l** $c = \frac{x}{t} - d$



Overmatter exercise A3

- 5 a The area of an ellipse is given by the formula $A = \pi ab$ where a and b are the lengths shown in the diagram.



Rearrange the formula $A = \pi ab$ to make a the subject.

- b A machine cuts ellipses out of a rectangular sheet of metal, 40 cm by 50 cm, to make cases for thermometers. The ellipses have $a = 3$ cm and $b = 5$ cm. The axes of symmetry of the ellipses have to be parallel to the edges of the sheet of metal.

Calculate how much more area of metal is wasted if the ellipses are cut with b parallel to the 50 cm side rather than the 40 cm side.

Explain why this calculation may not be accurate in real life.

You should be able to continue sequences and explain the pattern.

EXAMPLE

Find the next two terms in these sequences and explain how you worked them out.

a 5 9 13 17

b 160 80 40 20

a 21 25 Add 4 each time.

b 10 $5\frac{1}{2}$ Divide by 2 each time.

- Sequences are described by a **term-to-term rule** or a **position-to-term rule**.

You can generate a sequence with an n th term by putting $n = 1, 2, 3, \dots$ etc

The first three terms of the sequence with n th term $n^2 + 3$ are 4, 7, 12.

- A **linear sequence** is one where the term-to-term rule is to add the same number on each time.
7, 11, 15, 19,

Here 4 is added each time.

You can find the **n th term** of a linear sequence by looking at the differences.

The n th term of 7, 11, 15, 19, is $4n + 3$

You should be familiar with these sequences:

Odds and Evens	Squares 1, 4, 9, 16, ...	Cubes 1, 8, 27, 64, ...
Triangle numbers 1, 3, 6, 10, 15, ...	Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, ...	Primes 2, 3, 5, 7, 11, ...

1 is not a prime number


EXAMPLE

Calculate the 1st term and the 10th term of the sequences with these n th term is $3n - 2$.

$$\begin{aligned} \text{1st term} &= 3 \times 1 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{10th term} &= 3 \times 10 - 2 \\ &= 30 - 2 \\ &= 28 \end{aligned}$$

Find the n th term of the sequence with first five terms
11, 17, 23, 29, 35, ...

Look at the differences between the terms 11 17 23


It goes up in 6s so the n th term starts $6n$.
 Look at the first few terms of the sequence with n th term $6n$ that is 6, 12, 18, ...
 The terms we want are all 5 more than these.
 So the n th term is $6n + 5$

EXAMINER'S TIP:

Don't be like the many candidates who just write '+6' on the answer line and score nothing.

Exercise A4

1 Write down the next 2 terms of each of these sequences.

Explain how you worked out your answer.

a 11 14 17 20

b 3 6 12 24

c 33 27 21 15

d 243 81 27 9

e 38 30 22 14

f 72 36 18 9

2 Write down the first two terms of the sequences with these n th terms.

a $5n + 11$

b $7n + 1$

c $10n$

h $0.2n + 1.4$

e $12 - 2n$

f $40 - 2n$

g $n^2 + n$

h $n^3 - 2^n$

3 Find the 8th term of the sequences with these n th terms.

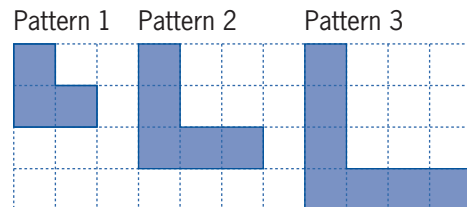
a $12n - 9$

b $8 - n$

4 a Draw the next L shape in this pattern.

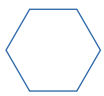
b Copy and complete this table to show the number of shaded squares.

Pattern	1	2	3	4
Number of squares	3	5		

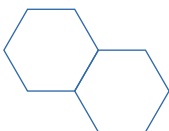


c How many squares will be shaded in Pattern 100?

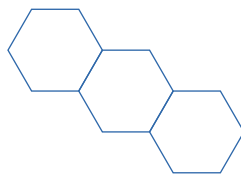
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1 Hexagon



2 Hexagon



3 Hexagon

Find a formula for the number of lines in a similar pattern of n Hexagons.

6 Marcus is adding 10g weights onto a spring. He measures the length of the spring after he has added each weight. Here are his results.

Number of 10g weights added	1	2	3	4
Length of spring (cm)	38	41.5	45	48.5

a Explain how Marcus can tell that there is a linear relationship between the number of weights and the length of the spring.

b Find a formula for the length of the spring when n weights have been added

c How many weights must Marcus add to make the spring 1 metre long?

You can **solve** an **equation** to find the value of an unknown quantity.

$$3x + 1 = 10 \quad \text{Find the value of } x$$

You should apply the same operation to each side of the equation so that it still balances.

$$3x + 1 - 1 = 10 - 1 \quad \text{Subtract 1 from each side}$$

Full working:

$$3x + 1 = 10$$

$$3x = 9 \quad \text{Subtract 1 from each side}$$

$$x = 3 \quad \text{divide both sides by 3}$$

The **solution** is $x = 3$

EXAMPLE

Solve these equations.

a $5x = 40$ **b** $x + 12 = 5$ **c** $4x - 7 = 3$

EXAMINER'S TIP:

Equations in the exam will often have solutions that are negative or fractions.

a $5x = 40$

$$x = \frac{40}{5} \quad \text{Divide both sides by 5}$$

$$x = 8$$

$$4x - 7 = 3$$

$$4x = 10 \quad \text{Add 7 to both sides}$$

c $x = \frac{10}{4}$ Divide both sides by 4

$$x = 2\frac{1}{2}$$

b $x + 12 = 5$

$$x = 5 - 12 \quad \text{Subtract 12 from each side}$$

$$x = -7$$

You can check your answer by substituting your solution into each side of the equation and seeing if you get the same number.

Try it with this example and you should get 3 on each side.

EXAMPLE

Solve these equations.

a $5x + 6 = 2x - 9$

b $\frac{6m+7}{2} = 5$

a $5x + 6 = 2x - 9$

$$3x + 6 = -9 \quad \text{Subtract } 2x \text{ from each side}$$

$$3x = -15 \quad \text{Subtract 6 from each side}$$

$$x = -5 \quad \text{Divide each side by 3}$$

b $\frac{6m+7}{2} = 5$

$$6m + 7 = 10 \quad \text{Multiply each side by 2}$$

$$6m = 3 \quad \text{Subtract 7 from each side}$$

$$m = \frac{1}{2} \quad \text{Divide each side by 6}$$

EXAMPLE

Form and solve an equation to find g .

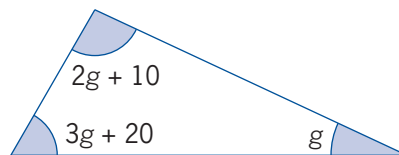
Because angles in a triangle add to 180° ,

$$g + 2g + 10 + 3g + 20 = 180$$

$$6g + 30 = 180$$

$$6g = 150$$

$$g = 25$$



EXAMINER'S TIP:

When questions say 'Form and solve', the answer by itself will not score full marks

Exercise A5

1 Solve these equations.

a $8x = 32$

b $x - 12 = 7$

c $\frac{x}{5} = 12$

d $x + 15 = 22$

e $7 = 2x$

f $x + 9 = 5$

g $2w - 3 = 11$

h $\frac{x}{4} - 11 = 9$

i $3n + 6 = 3$

j $7x + 5 = 40$

k $11 = 4k - 3$

l $12 - x = 8$

m $\frac{x}{5} = 15$

n $\frac{24}{x} = 3$

o $20 - 2d = 30$

p $5x + 2 = 2x + 17$

q $8x - 9 = 2x + 15$

r $5x + 4 = 3x + 11$

s $20 - 3x = 2x + 5$

t $9x + 15 = 5x - 1$

u $8x + 9 = 5x$

v $\frac{7y+6}{4} = 12$

w $1 = \frac{3m+8}{2}$

x $\frac{6-x}{5} = 5$

2 a Jo hires a carpet cleaner from this company for 3 hours.

How much does it cost her?

b Alex is charged £38 to hire a carpet cleaner for h hours.

Form and solve an equation in h to find how long he hired the carpet cleaner for.

Karl's Carpet Cleaner hire
 £2 per hour
 +
 £10

3 The angles of a quadrilateral are $5x$, $10x$, $4x + 12$ and $8x + 24$.

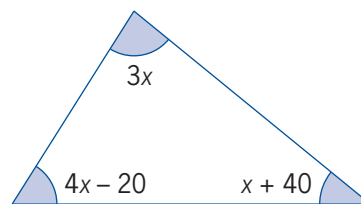
a Form and solve an equation in x and write down the size of all four angles.

b What sort of quadrilateral is this?

4 Form and solve an equation to find x .

a Find the size of each angle of the triangle.

b What type of triangle is this?



5 Catherine is solving an equation but has made an error.

$$4x + 3 = 7x + 15$$

$$3x = 12$$

$$x = 4$$

a Show by substitution that her solution is wrong.

b Describe her error.

c Solve the equation correctly.

You can **expand** or '**multiply out**' brackets by multiplying every term inside the bracket by the term outside. $5(x + 2) = 5x + 10$

EXAMPLE

Expand these expressions.

a $4(x - 6)$ **b** $r(r + 2)$ **c** $a(a + b - 3c)$

a $4x - 24$ **b** $r^2 + 2r$ **c** $a^2 + ab - 3ac$

Remember to multiply every term inside the bracket by the term outside.

r^2 means $r \times r$

- Putting expressions into brackets (the opposite of multiplying out) is called **factorising**. $9y - 6 = 3(3y - 2)$

Find the HCF of all the terms (in this case 3).

EXAMPLE

Factorise these expressions.

a $3c - 12$ **b** $36 - 24k$ **c** $2m^2 + 8m$

a $3c - 12$ Look for a number (or letter) that is a factor of each term in this case 3.

$3(\dots - \dots)$ Write the 3, then a set of brackets

$3(c - 4)$ In the spaces, write in the numbers or letters that would expand to give the original expression

b $12(3 - 2k)$

c $2m(m + 4)$ Here, both 2 and m are factors

You can check your answer by expanding:
 $3(c - 4) = 3c - 12$

You should choose the largest factor, in this case 12

EXAMINER'S TIP:

If there are 2 factors the question will normally be worth 2 marks.

When an equation has brackets, your first step could be to expand the brackets.

EXAMPLE

Solve $8(x + 5) = 3(6 - x)$

$8(x + 5) = 3(6 - x)$

$8x + 40 = 18 - 3x$ Multiply out the brackets

$11x + 40 = 18$ Add $3x$ to both sides

$11x = -22$ Subtract 40 from both sides

$x = -2$ Divide both sides by 11

Exercise A6

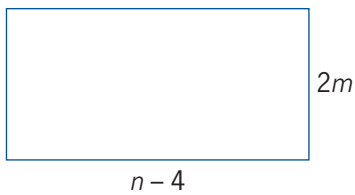
1 Expand these expressions.

- a** $4(c + 5)$ **b** $7(y + 7)$ **c** $p(p - 7)$
d $y(y + z)$ **e** $2e(e + 5f)$ **f** $3f(3f - h + 5h)$

2 Factorise these expressions.

- a** $5h - 20$ **b** $28 + 7k$ **c** $f^2 + 9f$
d $4j + 12$ **e** $18d - 24$ **f** $40 - 60m$
g $pq - q^2$ **h** $3g^2 + 6g$ **i** $8mn + 12n^2$

3 Find an expression for the area of this rectangle.



4 Solve

- a** $2(x + 3) = 12$ **b** $5(w - 1) = 20$ **c** $15 = 6(q + 5)$
d $4(2 - x) = -4$ **e** $x = 3(x - 4)$ **f** $3(y - 2) = 2(y + 11)$
g $6(z + 1) = 2(1 - z)$

5 Sam, Kabir and Patrick collect game cards.

Sam has x cards, Kabir has 10 cards more than Sam and Patrick has twice as many cards as Kabir. Altogether they have 182 cards.

- a** Write an expression in terms of x for the number of cards that
i Kabir has,
ii Patrick has.
b Form and solve an equation in x to find out how many cards Sam has.

6 Form and solve an equation to find x .

- a** Find the size of each angle of the triangle.
b What type of triangle is this?

