# Symbols



- In **equations** a letter represents a specific unknown number (or numbers).
  eg $5x = 20$

An equation is true for some values of $x$, an identity is true for all values of $x$.

- An **identity** is true for all values of the letters used and is used when writing expressions more simply.
  eg $x + x = 2x$

Technically an identity should be written with the identity symbol $\equiv$ rather than the equals sign $=$.

- A **formula** is used when we need to substitute different values of a value that can change.
  eg $A = \pi r^2 h$

- **Expressions** can be simplified by **collecting like terms** or using powers.

**EXAMPLE**

Simplify.

**a** $r \times r \times r \times r \times r$ **b** $6p + 5r + 3p - 4r$

**a** $r^5$ **b** $9p + r$

It is simpler to write $r$ rather than $1r$.

**EXAMPLE**

Decide whether the following are equations, identities or formulae.

$3m + 6 + m + 11 = 4m + 17$ $3m + 6 = 18$

$v = u + at$ $\frac{r}{2} = r + 2$

$3m + 6 + m + 11 = 4m + 17$ is an identity as it is true for all values of $m$.

$v = u + at$ is a formula

$3m + 6 = 18$ and $\frac{r}{2} = r + 2$ are both equations and are only true when $m = 4$ and $r = -4$ respectively.

A question like this may or may not use the identity symbol ($\equiv$)

# Exercise A1

**1** Simplify

**a** $y + 6y + 2y - 5y$ **b** $c \times c \times c \times c$ **c** $s + 5t + 7s - 8t$

**g** $4 \times z \times z \times 3 \times z$ **h** $4g - 2h - 5g - 7h$ **i** $6q + r - 5q - 5r + 4r - q$

**2** Decide whether the following are equations, expressions or formulae.

**a** $8 - 2f = 10$ **b** $2(r - 5) = 2r - 10$ **c** $A = 4\pi r^2$

**d** $v^2 = u^2 + 2as$ **e** $y = 8y + 21$ **f** $6r \times 4 = 8 \times (2r + r)$

# Brackets

A2

You can **expand (multiply out)** brackets by multiplying every term inside the bracket by the term outside, for example

$5(x + 2) = 5x + 10$

**KEY WORDS**

common factor
expand
factorise
multiply out

EXAMPLE

Expand

**a** $4(x - 6)$ **b** $r(r + 2)$ **c** $a(a + b - 3c)$

**a** $4(x - 6) = 4x - 24$ **b** $r(r + 2) = r^2 + 2r$
**c** $a(a + b - c) = a^2 + ab - 3ac$

Find the HCF of all the terms (in this case 3).

Remember to multiply every term inside the bracket by the term outside.

$r^2$ means $r \times r$

You can factorise an expression by putting it in brackets with a **common factor** outside (the opposite of multiplying out), for example

$9y - 6 = 3(3y - 2)$

EXAMPLE

Factorise

**a** $3c - 12$ **b** $36 - 24k$ **c** $2m^2 + 8m$

**a**
- Look for a number (or letter) that is a factor of each term – in this case 3.
- Write the 3, then a set of brackets – like this $3(\ldots . - \ldots .)$.
- In the spaces, write in the numbers or letters that would expand to give the original expression – in this case $3(c - 4)$

**b** $36 - 24k = 12(3 - 2k)$

**c** Here, both 2 and $m$ are factors, so $2m^2 + 8m = 2m(m + 4)$

Check your answer by expanding. $3(c - 4) = 3c - 12$

Choose the largest factor, in this case 12.

## Exercise A2

**1** Expand these expressions.

**a** $4(c + 5)$ **b** $7(y + 7)$ **c** $p(p - 7)$
**d** $y(y + z)$ **e** $2e(e + 5f)$ **f** $3f(3f - h + 5h)$

**2** Factorise.

**a** $5h - 20$ **b** $28 + 7k$ **c** $f^2 + 9f$
**d** $4j + 12$ **e** $18d - 24$ **f** $40 - 60m$
**g** $pq - q^2$ **h** $3g^2 + 6g$ **i** $8mn + 12n^2$

**EXAMINER'S TIP:**

➜ If there are 2 factros the question will normally be worth 2 marks.

# Coordinates



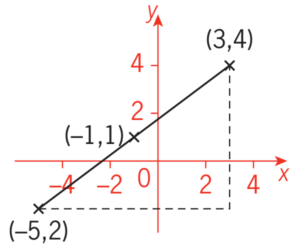
You use **coordinates** to specify the position of a point on a grid.

**KEY WORDS**

coordinates, line segment, midpoint

- The **midpoint** of a line segment is halfway between its ends.

You can calculate the length using Pythagoras' theorem.




**EXAMPLE**

Point A is at (−2, 1), point B is at (6, 4).

**a** Find the midpoint of AB.

**b** Calculate the length of line AB

Technically AB should be called a **line segment**, as a line is infinitely long

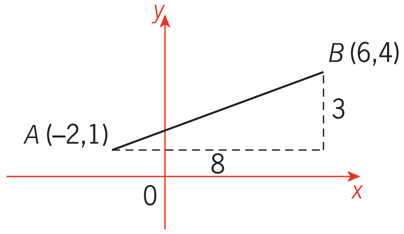
Start by drawing a sketch.

**a** The midpoint is at $\left(\frac{-2+6}{2}, \frac{1+4}{2}\right) = (2, 2.5)$

You can think of the midpoint as the 'mean average' point.

**b** By Pythagoras:

$$c^2 = a^2 + b^2$$
$$c^2 = 8^2 + 3^2$$
$$= 64 + 9$$
$$= 73$$
$$c = \sqrt{73} = 8.54 \text{ units}$$




$\sqrt{73}$ is an 'exact' answer.
8.54 is correct to 3 significant figures

# Exercise A3

**1** For each of these pairs of points, A and B

**i** find the midpoint of AB

**ii** the length of AB. Give your answer to 3 significant figures where appropriate.

**a** A(1, 1) B(4, 5) **b** A(1, 4) B(7, 0) **c** A(−1, −4) B(6, 2)

**d** A(2, −3) B(−4, −1) **e** A(−5, −2) B(3, 13) **f** A(0, −2) B(7, 0)

**2** PQRS is a rectangle.
P, Q and R are the points (−1, −2), (1, 4) and (7, 2) respectively.

**a** Find the coordinates of point S.

M is the midpoint of PQ and N is the midpoint of QR.

**b** Show that the length of MN is exactly $\sqrt{20}$.

# Formulae

A4

- A **formula** is work out unknown values from known values. This is called **substituting**.
  If $y = x^2 - 10$, find $y$ when $x = 5$

**ANSWER:**

$y = 5^2 - 10$
$= 25 - 10$
$= 15$

You can be **derive** a formula using information you know.
The area of a square of side $t$ is given by $A = t^2$.

**EXAMPLE**

If $s = ut + \frac{1}{2}at^2$, find $s$ when $u = 4$, $t = 5$ and $a = -2$.

$s = ut + \frac{1}{2}at^2$

$= 4 \times 5 + \frac{1}{2} \times -2 \times 5^2$

$= 20 - 25$

$= -5$

**EXAMPLE**

Find and simplify a formula for the mean average, $M$, of the values, $3f$, $4f$, $9f$ and $12g$.

Mean, $M = \dfrac{3f + 4f + 9f + 12g}{4}$

$M = \dfrac{16f + 12g}{4}$

$M = 4f + 3g$

You can **rearrange** or **change the subject** of a formulae by applying the same operation to each side

The letter on its own is called the **subject** of the formula

**EXAMPLE**

Rearrange these equations to make $p$ the subject.

**a** $r = 3p + q$ **b** $t = p^2 - w$ **c** $u(p + r) = 6w + z$

**a** $r = 3p + q$

$r - q = 3p$ subtract $q$ from both sides

$p = \dfrac{r-q}{3}$ divide both sides by 3

**b** $t = p^2 - w$

$t + w = p^2$ add $w$ to both sides

$p = \sqrt{t+w}$ square root both sides

Technically it should be $p = \pm\sqrt{t + w}$

**c** $u(p+r) = 6w + z$

$up + ur = 6w + z$ expand the brackets

$up = 6w + z - ur$ subtract $ur$ from each side

$p = \dfrac{6w + z - ur}{u}$ divide both sides by $u$

Compare the operations used here to those in solving equations. The processes are very similar but tend to be used on letters in rearranging and numbers in solving.

EXAMPLE

Rearrange this equation to make $d$ the subject.

**a** $3d + e = f - gd$

$3d + e = f - gd$

$3d + gd + e = f$ Collect the terms in $d$ on one side …

$3d + gd = f - e$ … and any other terms on the other side

$d(3 + g) = f - e$ Factorise

$d = \dfrac{f - e}{3 + g}$ Divide bothe sides by $3 + g$

If the letter you are trying to make the subject appears on both sides, you collect any terms it appears in on one side and then **factorise**.

# Exercise A4

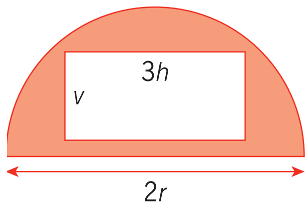
**1** If $p = 6$, $q = -2$ and $r = 3$, find the value of $w$ if

**a** $w = 8q - 5$ **b** $w = pq + qr$ **b** $w = pqr^2$

**d** $w = \sqrt{pq^2r}$ **e** $w = \dfrac{p(q = r)}{12}$ **f** $w = \sqrt[3]{(pr)^2 + 11q^2}$

**2** **a** A window cleaner charges £$p$ for each window she cleans plus an extra £4. Write down a formula for the amount she charges, $C$ for cleaning $w$ windows

**b** Write down a formula for the shaded area.




**3** Rearrange these formulas to make $x$ the subject.

**a** $y = mx + c$ **b** $t = wx - g$ **c** $s = ab + rx$ **d** $q = r(x - z)$

**e** $f = 3m - x$ **f** $p = \dfrac{y}{x}$ **g** $c = \dfrac{x}{t} - d$ **h** $k(x + w) = 5w - 7$

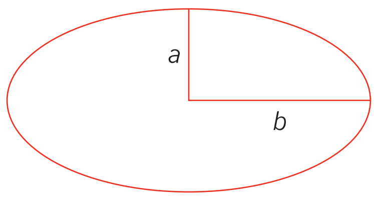
**i** $x^2y = z$ **j** $\dfrac{x^2 + a}{c} = d$ **k** $r = \sqrt{a + x}$ **l** $m = \sqrt{ax + b}$

**4** Rearrange these formulas to make t the subject.

**a** $3t = mt + d$ **b** $rt + f = 7 - 2t$ **c** $3p + 6t - m = m + dt$ **d** $t = r(t - s)$

**e** $t(m - 5) = a(12 - t)$ **f** $3(t + 1) + 2(t - 6) = g(t + 5)$ **g** $\dfrac{pt}{t + 1} = 3$ **h** $4(t - k) = \dfrac{kt}{3}$

**5** **a** The area of an ellipse is given by the formula $A = \pi ab$ where $a$ and $b$ are the lengths shown in the diagram. Rearrange $A = \pi ab$ to make $a$ the subject.




**b** A machine cuts ellipses out of a rectangular sheet of metal, 40 cm by 50 cm, to make cases for thermometers.

The ellipses have $a$ = 3 cm and $b$ = 5 cm.

The axes of symmetry of the ellipses have to be parallel to the edges of the sheet of metal.

Calculate how much more area of metal is wasted if the ellipses are cut with $b$ parallel to the 50 cm side rather than the 40 cm side.

Explain why this calculation may not be accurate in real life.

# Sequences

A5

**KEY WORDS**

| | |
|---|---|
| Linear sequence | term-to-term |
| *n*th term | position-to-term |

You describe a sequence with a **term-to-term** rule or a **position-to-term** rule.

You can generate a sequence with an $n$th term by putting $n = 1, 2, 3, \ldots$ and so on in the rule.
The first three terms of the sequence whose $n$th term is $n^2 + 3$ are $1^2 + 3 = \mathbf{4}$, $2^2 + 3 = 7$, $3^2 + 3 = \mathbf{12}$.

In a **linear sequence** you add the same number each time to get the next term, for example

7, 11, 15, 19, … and 26, 24, 22, 20, … are linear sequences.

You can add a positive or a negative number.

You can find the **$n$ th term** of a linear sequence by looking at the differences between the terms.
The $n$th term of the sequence 7, 11, 15, 19, … is $4n + 3$

You should be familiar with these sequences.

| | | |
|---|---|---|
| Odds and Even numbers | Squares 1, 4, 9, 16, … | Cubes 1, 8, 27, 64, … |
| Triangle numbers 1, 3, 6, 10, 15, … | Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, … | Primes 2, 3, 5, 7, 11, … |

1 is not a prime number.

**EXAMPLE**

Calculate the first term and the tenth term of the sequences with these $n$th terms

**a** $3n - 2$ **b** $6n - 2$ **c** $2^n - 2$

**a** first term $= 3 \times 1 - 2$
$= 1$

tenth term $= 3 \times 10 - 2$
$= 30 - 2$
$= 28$

**b** first term $= 6 - 1^2$
$= 6 - 1$
$= 5$

tenth term $= 6 - 10^2$
$= 6 - 100$
$= -94$

**c** first term $= 2^1 - 1$
$= 2 - 1$
$= 1$

tenth term $= 2^{10} - 1$
$= 1024 - 1$
$= 1023$

**EXAMPLE**

Find the $n$th term of the sequence with first five terms 11, 17, 23, 29. 35, …

Look at the differences between the terms 11 17 23

+6 +6

It goes up in 6s so the $n$th term starts $6n$.

Look at the first few terms of the sequence with $n$th term $6n$ that is 6, 12, 18, …

The terms you want are all 5 more than these.

So the nth term is $6n + 5$

**EXAMINER'S TIP:**

➜ Don't just write '+6' on the answer line – that will score nothing.

# Exercise A5

**1** Write down the first two terms of the sequences with these $n$th terms.

**a** $5n + 11$ **b** $7n + 1$ **c** $10n$ **d** $0.2n + 1.4$

**e** $12 - 2n$ **f** $40 - 20n$ **g** $n^2 + n$ **h** $n^3 - n^n$

**2** Find the eighth term of the sequences with these $n$th terms.

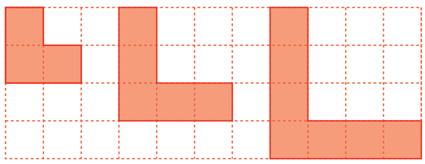
**a** $12n - 9$ **b** $8 - n$

**3 a** Draw the next L shape in this pattern.

**b** Copy and complete this table to show the number of shaded squares.

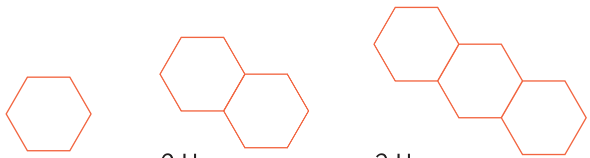
| **Pattern** | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Number of squares** | 3 | 5 | | |

Pattern 1 Pattern 2 Pattern 3



**c** How many squares will be shaded in Pattern 100?

**4** Find a formula for the number of lines in a pattern like these with $n$ hexagons.



1 Hexagon 2 Hexagon 3 Hexagon

**5** Marcus is adding 10 g weights to a spring. He measures the length of the spring after he has added each weight. Here are his results.

| Number of 10 g weights added | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Length of spring (cm) | 38 | 41.5 | 45 | 48.5 |

**a** Explain how Marcus can tell that there is a linear relationship between the number of weights and the length of the spring.

**b** Find a formula for the length of the spring when $n$ weights have been Added.

**c** How many weights must Marcus add to make the spring 1 metre long?

**6** A sequence has $n$th term $7n + k$ where $k$ is an integer between –5 and 5.
The eighth term is a 2-digit Fibonacci number with two identical digits.
The fourth term is a cube number.

Find $k$ and hence find the first term of the sequence that is a prime number.

**KEY WORDS**

equation solution solve

You **solve** an **equation** by finding the value of an unknown quantity.
For example if $3x + 1 = 10$ the value of $x = 3$

You do the same operation to each side of the equation so that it still balances.

Full working: $3x+1 = 10$
$3x = 9$ Subtract 1 from each side
$x = 3$ divide both sides by 3
The **solution** is $x = 3$

**EXAMPLE**

Solve **a** $5x + 6 = 2x - 9$ **b** $\frac{6m+7}{2} = 5$

**a**
$5x + 6 = 2x - 9$
$3x + 6 = -9$ subtract $2x$ from both sides
$3x = -15$ subtract 6 from both sides
$x = -5$ divide both sides by 3

**b**
$\frac{6m+7}{2} = 5$
$6m + 7 = 10$ multiply both sides by 2
$6m = 3$ subtract 7 from both sides
$m = \frac{1}{2}$ divide both sides by 6

When you solve an equation which brackets, the first step is to expand the brackets.

**EXAMPLE**

Solve $4(2x - 8) = 3(1 - x) + 7$

$8(x+4) = 3(1-x)+7$
$8x+32 = 3-3x+7$
$8x+32 = 10-3x$
$11x+32 = 10$
$11x = -22$
$x = -2$

multiply out the brackets collect like terms add $3x$ to both sides subtract 32 from both sides divide both sides by 11

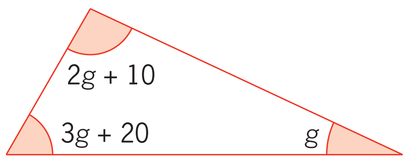
**EXAMINER'S TIP:**

→ Equations in the exam will often have solutions that are negative or fractions.

You can check your answer by substituting your solution into each side of the equation and seeing if you get the same number. Try it with this example and you should get −48 on each side.

**EXAMPLE**

Form and solve an equation to find $g$.




Because angles in a triangle add to 180°

$g + 2g + 10 + 3g + 20 = 180$
$6g + 30 = 180$
$6g = 150$
$g = 25$

**EXAMINER'S TIP:**

→ When questions say 'Form and solve', the answer by itself will not score full marks

# Exercise A6

**1** Solve

**a** $5x+2=2x+17$ **b** $8x-9=2x+15$ **c** $5x+4=3x+11$

**d** $20-3x=2x+5$ **e** $9x+15=5x-1$ **f** $8x+9=5x$

**g** $\frac{7y+6}{4}=12$ **h** $1=\frac{3m+8}{2}$ **i** $\frac{6-x}{5}=5$

**2** The angles of a quadrilateral are $5x$, $10x$, $4x+12$ and $8x+24$.

**a** Form and solve an equation in $x$ and write down the sizes of all four angles.

**b** What sort of quadrilateral is this?

**3** Solve

**a** $2(x+3)=12$ **b** $x=3(x-4)$ **c** $3(y-2)=2(y+11)$

**d** $6(z+1)=2(1-z)$ **e** $4(x+1)+3(2x+5)=-11$ **f** $3(1-2k)-5(1-k)=11$

**4** Sam, Kabir and Patrick collect game cards.
Sam has $x$ cards, Kabir has 10 cards more than Sam and Patrick has twice as many cards as Kabir.
Altogether they have 182 cards.

**a** Write an expression in terms of $x$ for the number of cards that

**i** Kabir has

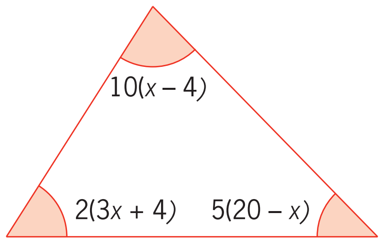
**ii** Patrick has.

**b** Form and solve an equation in $x$ to find out how many cards Sam has.

**5** **a** Form and solve an equation to find $x$.

**b** Find the size of each angle of the triangle.

**c** What type of triangle is this?




**6** Catherine is solving an equation but has made an error.

$$4(x-1)=6-2(3x+5)$$
$$4x-4=6-6x+10$$
$$4x-4=16-6x$$
$$10x=20$$
$$x=2$$

**a** Show by substitution that her solution is wrong.

**b** Describe her error.

**c** Solve the equation correctly.